Name:______________________________________ Date:____________

PC: Circles Ms. Loughran

Do Now:

1. Find the length of the line segment determined by points $A(x, y)$ and $C(h, k)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-h)^2 + (y-k)^2}$

An equation of the circle with center (h, k) and radius r is

If we replace d with the radius

$r = \sqrt{(x-h)^2 + (y-k)^2}$

$r^2 = (x-h)^2 + (y-k)^2$

This is called the standard form for the equation of the circle. If the center of the circle is the origin, then the equation is

$(0,0)$

(h,k)

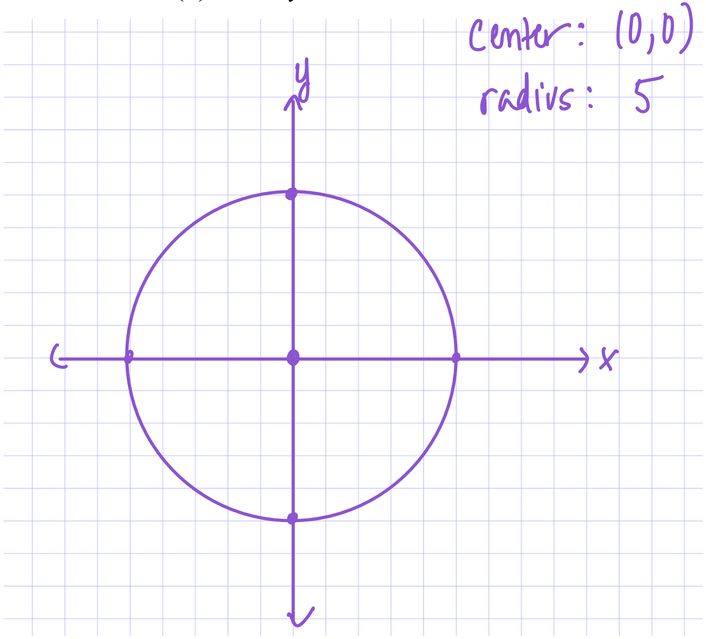
$(x-h)^2 + (y-k)^2 = r^2$ with center of (h,k) radius of r

$(x-0)^2 + (y-0)^2 = r^2$

$x^2 + y^2 = r^2$

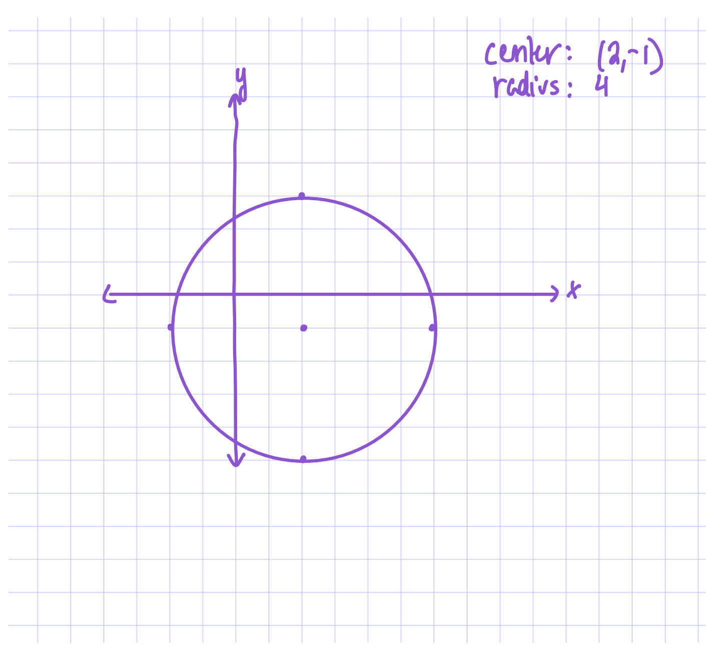
1. Graph each equation.

(a) $x^2 + y^2 = 25$



$(x-h)^2 + (y-k)^2 = r^2$

(b) $(x-2)^2 + (y+1)^2 = 16$



$$(x-h)^2+(y-k)^2=r^2$$

2. Find an equation of the circle with radius 3 and center $\overset{(h,k)}{(-1,4)}$.

$$(x+1)^2+(y-4)^2=9$$

3. Find the center and radius of the circle whose equation is $(x+2)^2+(y-3)^2=10$.

center: $(-2,3)$
radius $=\sqrt{10}$

4. Write an equation of the circle whose diameter has endpoints (0,0) and (6,8).

midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

midpt: $\left(\frac{0+6}{2}, \frac{0+8}{2}\right)$

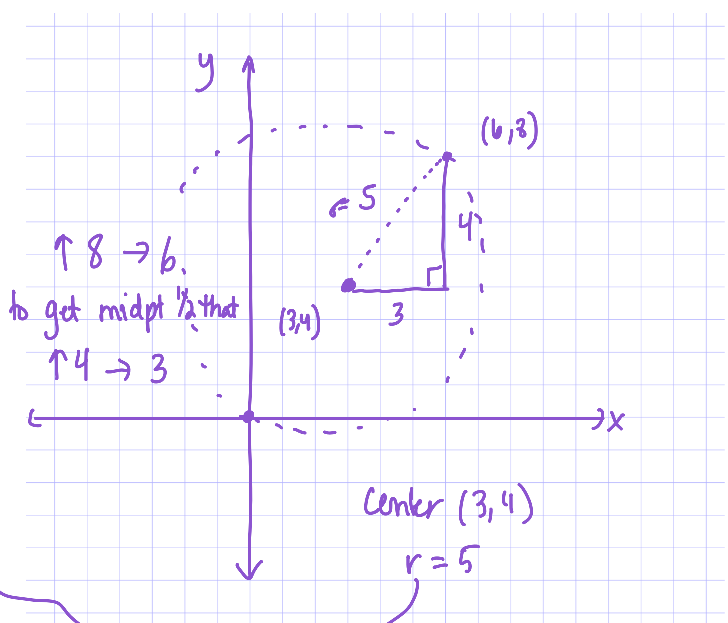
$(3,4)$ ← center of circle

$(3,4)$, $(0,0)$

$r=\sqrt{(3-0)^2+(4-0)^2}$

$r=\sqrt{9+16}$

$r=\sqrt{25}=5$



$$(x-3)^2+(y-4)^2=25$$

5. Points P(1,-5) and Q(-3,3) are the endpoints of a diameter of a circle. Find the center, radius, and equation of the circle.

midpt: $\left(\frac{1+(-3)}{2}, \frac{-5+3}{2}\right)$

: $(-1,-1)$ ← center

$r = \sqrt{(1-(-1))^2 + (-5-(-1))^2}$

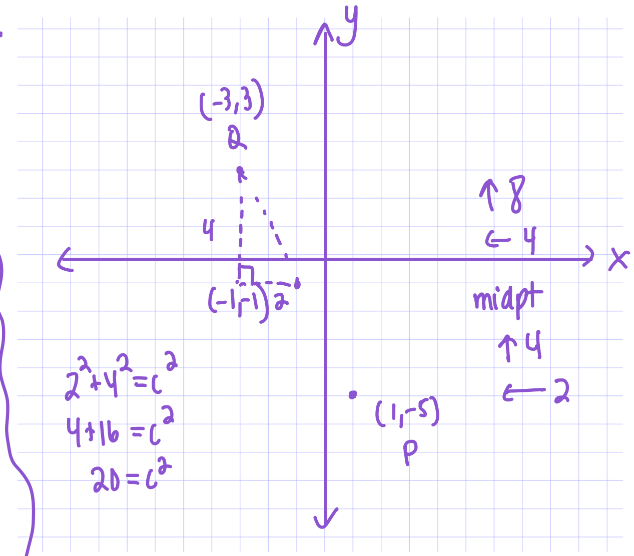
$r = \sqrt{4+16}$

$r = \sqrt{20}$

$r^2 = 20$

$(x-(-1))^2 + (y-(-1))^2 = 20$

$(x+1)^2 + (y+1)^2 = 20$



6. Find the center and radius of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$.

* would be really to have in that center-radius form

$(x-h)^2 + (y-k)^2 = r^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$

$(x+2)^2 + (y-3)^2 = 25$

center: $(-2,3)$

radius = 5

7. Find the center and radius of the circle whose equation is $x^2 + y^2 + 2x - 6y + 7 = 0$.

$x^2 + 2x + 1 + y^2 - 6y + 9 = -7 + 1 + 9$

$(x+1)^2 + (y-3)^2 = 3$

center: $(-1,3)$

$r = \sqrt{3}$

8. Find the center and radius of the circle whose equation is. $x^2 + y^2 + 6y + 2 = 0$

$$x^2 + y^2 + 6y + 9 = -2 + 9$$

$$x^2 + (y+3)^2 = 7$$

center: $(0, -3)$

$r = \sqrt{7}$

9. Find the center and radius of the circle whose equation is $x^2 + y^2 - 4x + 10y + 13 = 0$.

$$x^2 - 4x + 4 + y^2 + 10y + 25 = -13 + 4 + 25$$

$$(x-2)^2 + (y+5)^2 = 16$$

center: $(2, -5)$

$r = 4$

10. Find the center and radius of the circle whose equation is $9x^2 + 12x + 9y^2 - 77 = 0$.

$\frac{1}{2}\left(\frac{4}{3}\right) = \frac{2}{3}$

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$$\frac{9x^2}{9} + \frac{12x}{9} + \frac{9y^2}{9} = \frac{77}{9}$$

$$\underbrace{x^2 + \frac{4}{3}x + \frac{4}{9}} + y^2 = \frac{77}{9} + \frac{4}{9}$$

$$\left(x + \frac{2}{3}\right)^2 + y^2 = \frac{81}{9}$$

$$\left(x + \frac{2}{3}\right)^2 + y^2 = 9$$

center: $\left(-\frac{2}{3}, 0\right)$

$r = 3$

Homework 04-12

Name: Key

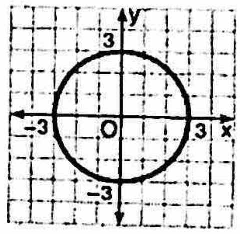
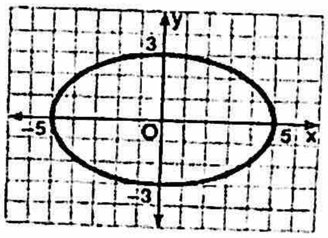
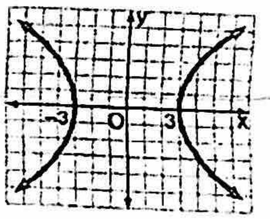
PC: Special Relations

Date:____________

Ms. Loughran

The General Equation $ax^2 + by^2 = c$

Depending on the values of the coefficients, the general equation $ax^2 + by^2 = c$, where $a, b, c \neq 0$, describes the graph of a *circle*, *ellipse*, or *hyperbola*.

Values of Coefficients	Name of Graph	Example	
$a = b$ and have the same sign as c	*circle*	$2x^2 + 2y^2 = 18$ or $x^2 + y^2 = 9$ circle with center at origin and radius = 3	
$a \neq b$ and have the same sign as c	*ellipse*	$9x^2 + 25y^2 = 225$ ellipse with center at origin and x-intercepts = ±5 y-intercepts = ±3	
a, b have different signs	*hyperbola*	$x^2 - y^2 = 9$ hyperbola with center at origin and x-intercepts = ±3 no y-intercepts	

Recall: **The equation of a parabola contains only one square term: either $y = ax^2 + bx + c$ or $x = ay^2 + by + c$**
The equation of a straight line contains no square terms: $ax + by = c$

EXERCISES

In 1-14, identify the graph of the given relation as

(1) a circle
(2) an ellipse
(3) a hyperbola
(4) a parabola

1. $4y^2 = 25 - 4x^2$ 1
2. $2x^2 + 3y^2 = 24$ (2)
3. $x^2 = y^2 + 9$ 3
4. $x^2 = 6 - y$ 4
5. $4x^2 - 100 = 25y^2$ 3
6. $3y^2 = 6 - x^2$ (2)
7. $3x^2 + 2y^2 = 6$ 2
8. $4x^2 + 16y^2 = 25$ 2
9. $x^2 + y = 9$ 4
10. $2x^2 = 5 - 2y^2$ 1
11. $y^2 = 6 - 3x^2$ 2
12. $2x^2 - 9 = 2y^2$ 3
13. $4x^2 - 4y^2 = 9$ 3
14. $x^2 - \frac{y^2}{16} = 1$ 3

15. Which of the following is the equation of a hyperbola?
(1) $x^2 = 10 - y^2$
(2) $x = y^2 - 9$
(3) $y^2 = x^2 - 1$
(4) $4x^2 + y^2 = 9$

16. The graph of which equation is an ellipse?
(1) $3x^2 - 4y^2 = 7$
(2) $\frac{y + 6}{x - 1} = 3$
(3) $y = 2x^2 + 3x - 5$
(4) $x^2 + 5y^2 = 2$

17. Which is an equation of a circle?
(1) $2x^2 - 2y^2 = 18$
(2) $2x^2 + 3y^2 = 36$
(3) $3x^2 + 3y^2 = 21$
(4) $x^2 = y^2 + 16$

18. Which equation has a hyperbola as its graph?
(1) $x^2 = 10 + y$
(2) $x^2 = 10 - y^2$
(3) $3x^2 = 10 - 2y^2$
(4) $3x^2 = 10 + 2y^2$

19. Which equation has an ellipse as its graph?
(1) $2x^2 = 8 - 3y$
(2) $2x^2 = 8 + 3y^2$
(3) $2x^2 = 8 - 3y^2$
(4) $2x = 8 - 3y$

20. Which is an equation of a circle?
(1) $2x^2 + y^2 = 7$
(2) $x = \frac{y}{8}$
(3) $x^2 - y^2 = 10$
(4) $5(x^2 + y^2) = 12$

21. Which is an equation of a parabola?
(1) $x^2 = 3 + y^2$ (3) $x = 3 + y$
(2) $x = 3 + y^2$ (4) $y^2 = 3x^2 + 3$

22. The graph of the relation $ay = bx^2 + c$ in which neither a nor b is 0 is
(1) a parabola (3) an ellipse
(2) a straight line (4) a hyperbola

23. If a, b, and c are positive unequal numbers, the graph of $ax^2 + by^2 = c$ is
(1) a circle (3) an ellipse
(2) a parabola (4) a hyperbola

24. The graph of $ax^2 + by^2 = c$, in which a, b, and c are real numbers, is an ellipse if
(1) $a = b, a > 0, b < 0, c > 0$
(2) $a = b, a > 0, b > 0, c < 0$
(3) $a \neq b, a > 0, b > 0, c > 0$
(4) $a \neq b, a > 0, b < 0, c > 0$

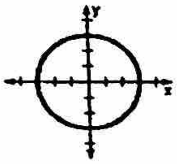
25. If $a \neq 0$, $b \neq 0$, and $c \neq 0$, the graph of $ax^2 + by^2 = c$ can not be
(1) an ellipse (3) a parabola
(2) a circle (4) a hyperbola

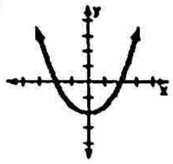
26. The graph of the equation $\frac{x^2}{4} + \frac{y^2}{16} = 1$ passes through the point whose coordinates are
(1) (0, 0) (2) (0, 2) (3) (0, 4) (4) (4, 0)

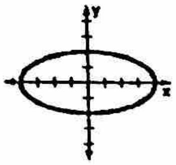
27. Which relation is a function?
(1) $\{(x, y)| x^2 + y = 4\}$ (3) $\{(x, y)| x^2 - y^2 = 4\}$
(2) $\{(x, y)| x^2 + y^2 = 4\}$ (4) $\{(x, y)| x^2 + 4y^2 = 4\}$

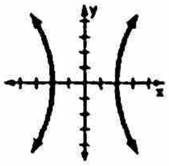
28. If the replacement set is the set of real numbers, what is the domain of the relation represented by $\{(x, y)| x^2 + 4y^2 = 16\}$?
(1) $\{y| -2 \leq y \leq 2\}$ (3) $\{x| -4 \leq x \leq 4\}$
(2) $\{y| -2 < y < 2\}$ (4) $\{x| -4 < x < 4\}$

29. Which is the graph of a quadratic relation for which the domain consists of all the real numbers?

(1)


(3)


(2)


(4)


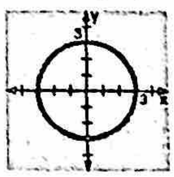
30. If the graphs of the equations $x^2 + y^2 = 9$ and $y = 3$ are drawn on the same set of axes, what is the total number of points common to both graphs?
(1) 1 (2) 2 (3) 3 (4) 0

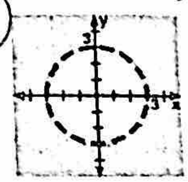
31. When drawn on the same set of axes, the points of intersection of the graphs of $x^2 + y^2 = 16$ and $x = 2$ are located in quadrants
(1) I and III (3) II and III
(2) I and IV (4) II and IV

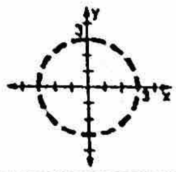
32. The graphs of the equations $x^2 + y^2 = 25$ and $y = x^2$ are drawn on the same set of axes. The total number of points common to these graphs is
(1) 1 (2) 2 (3) 3 (4) 4

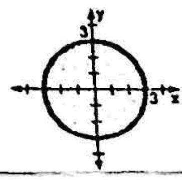
33. The graph of $x^2 + y^2 = 25$ and the graph of $x - 4 = 0$ are drawn on the same set of axes. A point of intersection of the graphs is
(1) (5, 0) (2) (−4, −3) (3) (4, −3) (4) (−3, 4)

34. What is the graph of the solution set of $x^2 + y^2 > 9$?

(1)


(3)


(2)


(4)


35. Each equation in column A has one of the geometric figures in column B as its graph. List the numbers 1-5 on your answer paper and after each number write the letter that indicates the corresponding graph.

Column A

(e) (1) $x^2 + y^2 - 4 = 0$
(f) (2) $4x^2 + y^2 - 1 = 0$
(d) (3) $x^2 - y - 4 = 0$
(a) (4) $x^2 + 4y^2 = 0$
(c) (5) $x^2 - 4y^2 = 0$

Column B

a. The point (0, 0)
b. Two straight lines parallel to the y-axis
c. Two straight lines intersecting at the origin
d. A parabola that crosses the y-axis at (0, −4)
e. A circle whose center is the origin and whose radius is 2
f. An ellipse that crosses the y-axis at (0, 1) and (0, −1)
g. A hyperbola that crosses the y-axis at (0, 2) and (0, −2)

$x^2 + 4y^2 = 1$