

Name: _____
PC Inequalities

Date: _____
Ms. Loughran

Do Now:

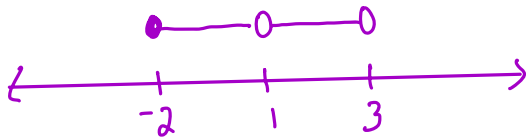
Solve the following inequalities and place solutions in notation indicated.

1. $\frac{x^2 + x - 2}{x^2 - 4x + 3} \leq 0$ (set builder)

$$\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-3)}(x-1)} \leq 0$$

$$\frac{x+2}{x-3} \leq 0$$

$$\{x \mid -2 \leq x < 1 \text{ or } 1 < x < 3\}$$



test
 $x=0$

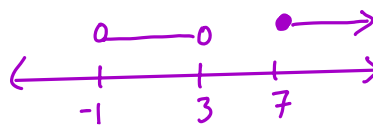
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2. $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$ (interval)

$$\frac{(x+3)(x-3)}{(x+1)(x-3)} + \frac{(x+2)(x+1)}{(x-3)(x+1)} \geq 0$$

$$\frac{x^2 - 9 - x^2 + x + 2}{(x+1)(x-3)} \geq 0$$

$$\frac{x-7}{(x+1)(x-3)} \geq 0$$



test
 $x=0$

- + - +

$$(-1, 3) \text{ or } [7, \infty)$$

Answer Key to Review sheet for Exam 1 Q1

① $x^2 - 9x + 14 < 0$
 $(x-7)(x-2) < 0$



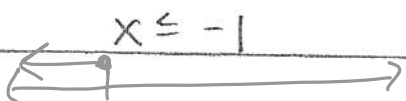
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SB: $\{x \mid 2 < x < 7\}$
 IN: $(2, 7)$

② $4 - 3x \leq -(1 + 8x)$
 $4 - 3x \leq -1 - 8x$
 $+8x \quad +8x$

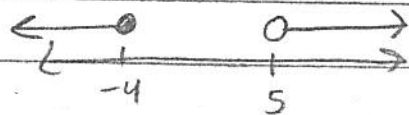
$4 + 5x \leq -1$
 $-4 \quad -4$

$5x \leq -5$



SB: $\{x \mid x \leq -1\}$
 IN: $(-\infty, -1]$

③ $\frac{x+4}{5-x} \leq 0$



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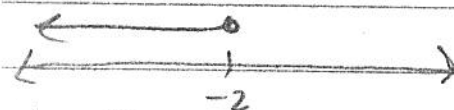
SB: $\{x \mid x \leq -4 \text{ or } x > 5\}$
 IN: $(-\infty, -4] \cup (5, \infty)$

④ $\frac{3x}{2} \leq \frac{3x-6}{4}$

$\frac{3x^{(2)}}{2^{(2)}} \leq \frac{3x-6}{4}$
 $\frac{3x-6}{4} \leq 0$

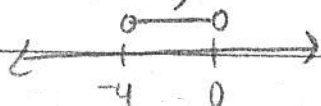
$\frac{6x}{4} - \frac{3x-6}{4} \leq 0$

$\frac{3x+6}{4} \leq 0$



SB: $\{x \mid x \leq -2\}$
 IN: $(-\infty, -2]$

⑤ $x^2 + 4x < 0$
 $x(x+4) < 0$



SB: $\{x \mid -4 < x < 0\}$
 IN: $(-4, 0)$

⑥ $1 + \frac{2}{x+1} \leq \frac{2}{x}$

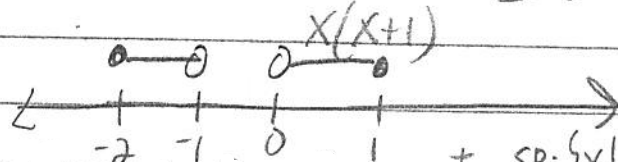
$1 + \frac{2^{(x)}}{x+1^{(2)}} - \frac{2^{(x+1)}}{x^{(x+1)}} \leq 0$

$\frac{x(x+1) + 2x - 2(x+1)}{x(x+1)} \leq 0$

$\frac{x^2+x + 2x - 2x - 2}{x(x+1)} \leq 0$

$\frac{x^2+x-2}{x(x+1)} \leq 0$

$\frac{(x+2)(x-1)}{x(x+1)} \leq 0$



SB: $\{x \mid -2 \leq x < -1 \text{ or } 0 < x \leq 1\}$
 IN: $[-2, -1) \cup (0, 1]$

$$(7) \frac{4x}{2x+3} > 2$$

$$(8) \frac{x+1}{x} < 3$$

$$(9) \frac{x^2-2x-3}{(x-1)^2} < 0$$

$$\frac{4x}{2x+3} - \frac{2(2x+3)}{1(2x+3)} > 0$$

$$\frac{x+1}{x} - 3 < 0$$

$$\frac{(x-4)(x+2)}{(x-1)^2} < 0$$

$$\frac{4x}{2x+3} \ominus \frac{4x+6}{2x+3} > 0$$

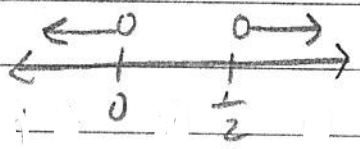
$$\frac{x+1}{x} - \frac{3x}{x} < 0$$

$$\frac{4x-4x-6}{2x+3} > 0$$

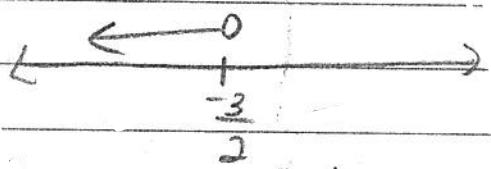
$$\frac{-2x+1}{x} < 0$$



$$\frac{-6}{2x+3} > 0$$



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SB: $\{x \mid x < 0 \text{ or } x > \frac{1}{2}\}$
 IN: $(-\infty, 0) \cup (\frac{1}{2}, \infty)$

SB: $\{x \mid -2 < x < 1 \text{ or } 1 < x < 4\}$
 IN: $(-2, 1) \cup (1, 4)$

SB: $\{x \mid x < -\frac{3}{2}\}$
 IN: $(-\infty, -\frac{3}{2})$

(10) $56x^3 - 28x^2 + 7x$
 $7x(8x^2 - 4x + 1)$ $ac=8$ $b=-4$
 can not be factored any further

(11) $x^3 - 3x^2 - 4x + 12$
 $x^2(x-3) - 4(x-3)$
 $(x^2-4)(x-3)$
 $(x-2)(x+2)(x-3)$

(12) $3x^2 - 75$
 $3(x^2 - 25)$
 $3(x-5)(x+5)$

$$\textcircled{3} ax^2 + 15 - 5ax - 3x$$

reorganize

$$ax^2 - 5ax - 3x + 15$$

$$ax(x-5) - 3(x-5)$$

$$(ax-3)(x-5)$$

$$\textcircled{14} 6x^2 - 11x - 10 \quad ac = -60$$

$$b = -11$$

$$6x^2 - 15x + 4x - 10$$

$$3x(2x-5) + 2(2x-5)$$

$$(3x+2)(2x-5)$$

$$\textcircled{15} a^8 - b^8$$

$$(a^4 + b^4)(a^4 - b^4)$$

$$(a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$$

$$(a^4 + b^4)(a^2 + b^2)(a+b)(a-b)$$

$$\textcircled{16} x^2 - 8x + 4$$

not factorable

prime

$$\textcircled{17} x^4 - x^2 - 12$$

$$\text{let } y = x^2$$

$$y^2 - y - 12$$

$$(y-4)(y+3)$$

$$(x^2-4)(x^2+3)$$

$$(x-2)(x+2)(x^2+3)$$

$$\textcircled{18} 16x^2y^2 - 25$$

$$(4xy-5)(4xy+5)$$

$$\textcircled{19} x+1+y+xy$$

reorganize

$$x+1+xy+y$$

$$1(x+1) + y(x+1)$$

$$(1+y)(x+1)$$

$$\textcircled{20} x^5 + 27x^2$$

$$x^2(x^3+27)$$

$$x^2(x+3)(x^2-3x+9)$$

$$\textcircled{21} 8x^3 - 125y^3$$

$$(2x-5y)(4x^2+10xy+25y^2)$$

$$\textcircled{22} (x^2-3x)^2 - 38(x^2-3x) - 80$$

$$\text{let } y = x^2 - 3x$$

$$y^2 - 38y - 80$$

$$(y-40)(y+2)$$

$$(x^2-3x-40)(x^2-3x+2)$$

$$(x-8)(x+5)(x-1)(x-2)$$

$$\textcircled{23} x^6 + 8$$

$$(x^2+2)(x^4-2x^2+4)$$

$$\textcircled{24} x^2(x^2-1) - 9(x^2-1)$$

$$\textcircled{25} 4(x^2-1)^2 - 13(x^2-1) - 12$$

$$\text{let } u = x^2 - 1$$

$$(x^2-1)(x^2-9)$$

$$(x-1)(x+1)(x-3)(x+3)$$

(26) $6 - 7t^2 + t^4$ rearranged

$$t^4 - 7t^2 + 6$$

$$(t^2-1)(t^2-6)$$

$$(t-1)(t+1)(t^2-6)$$

(27) $7x^2 + 10xy + 3y^2$

$$7x^2 + 7xy + 3xy + 3y^2$$

$$7x(x+y) + 3y(x+y)$$

$$(7x+3y)(x+y)$$

(28) $x^9 - x^6 - x^3 + 1$

$$x^6(x^3-1) - 1(x^3-1)$$

$$(x^6-1)(x^3-1)$$

$$(x^2-1)(x^4+x^2+1)(x-1)(x^2+x+1)$$

$$(x+1)(x-1)(x^4+x^2+1)(x-1)(x^2+x+1)$$

$$4y^2 - 13y - 12$$

$$4y^2 - 16y + 3y - 12$$

$$4y(y-4) + 3(y-4)$$

$$(4y+3)(y-4)$$

$$(4(x^2-1)+3)(x^2-1-4)$$

$$(4x^2-4+3)(x^2-5)$$

$$(4x^2-1)(x^2-5)$$

$$(2x-1)(2x+1)(x^2-5)$$

(29) $8a^2 - 33ab + 4b^2$

$$8a^2 - 32ab - ab + 4b^2$$

$$8a(a-4b) - b(a-4b)$$

$$(8a-b)(a-4b)$$

14. $6x^2 - 11x - 10$

15. $a^8 - b^8$

16. $x^2 - 8x + 4$

17. $x^4 - x^2 - 12$

18. $16x^2y^2 - 25$

19. $x + 1 + y + xy$

20. $x^5 + 27x^2$

21. $8x^3 - 125y^3$

22. $(x^2 - 3x)^2 - 38(x^2 - 3x) - 80$

23. $x^6 + 8$

24. $x^2(x^2 - 1) - 9(x^2 - 1)$

25. $4(x^2 - 1)^2 - 13(x^2 - 1) - 12$

26. $6 - 7t^2 + t^4$

27. $7x^2 + 10xy + 3y^2$

28. $x^9 - x^6 - x^3 + 1$

29. $8a^2 - 33ab + 4b^2$

15. $a^8 - b^8$

$(a^4 + b^4)(a^4 - b^4)$

$(a^2 + b^2)(a^2 - b^2)$

$(a^4 + b^4)(a^2 + b^2)(a - b)(a + b)$

28. $x^9 - x^6 - x^3 + 1$

$x^6(x^3 - 1) - 1(x^3 - 1)$

$(x^6 - 1)(x^3 - 1)$

$(x^2 - 1)(x^4 + x^2 + 1)(x - 1)(x^2 + x + 1)$

25. $4(x^2 - 1)^2 - 13(x^2 - 1) - 12$

let $y = x^2 - 1$

$4y^2 - 13y - 12$

$4y^2 - 16y + 3y - 12$

$4y(y - 4) + 3(y - 4)$

$(4y + 3)(y - 4)$

$(4(x^2 - 1) + 3)(x^2 - 1 - 4)$

$(4x^2 - 4 + 3)(x^2 - 5)$

$(4x^2 - 1)(x^2 - 5)$

$(2x + 1)(2x - 1)(x^2 - 5)$

$27x^3y^3 - 216d^3$

$27(x^3y^3 - 8d^3)$

$27(xy - 2d)(x^2y^2 + 2dxy + 4d^2)$

$(3xy - 6d)(9x^2y^2 + 18xyd + 36d^2)$
 $3(xy - 2d) \cdot 9(x^2y^2 + 2xyd + 4d^2)$
 $27(xy - 2d)(x^2y^2 + 2xyd + 4d^2)$

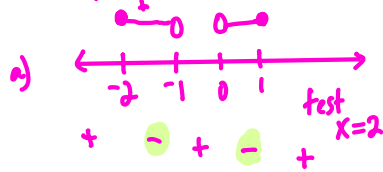
6. $1 + \frac{2}{x+1} \leq \frac{2}{x}$

$\frac{x(x+1)}{x(x+1)} + \frac{2(x)}{x+1} + \frac{-2(x+1)}{x(x+1)} \leq 0$

$\frac{x^2 + x + 2x - 2x - 2}{x(x+1)} \leq 0$

$\frac{x^2 + x - 2}{x(x+1)} \leq 0$

$\frac{(x-1)(x+2)}{x(x+1)} \leq 0$



b) $\{x \mid -2 \leq x < -1 \text{ or } 0 < x \leq 1\}$

c) $[-2, -1) \text{ or } (0, 1]$