

Name: _____
 PC: Simplifying Rational Expressions

Date: _____
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A **rational expression** is a ratio of polynomials. A rational expression is said to be **undefined** if its denominator is zero.

1. For what value(s) of x does the expression have no meaning? undefined

(a) $\frac{7}{x-3}$ $x=3$
 (b) $\frac{12}{x+8}$ $x=-8$
 (c) $\frac{x-3}{x^2-16}$ $x^2-16=0$
 $(x-4)(x+4)=0$
 $x=\pm 4$
 (d) $\frac{x-2}{x^2+4}$ $x^2+4=0$
 $x^2=-4$
 $x=\pm\sqrt{-4}=\pm 2i$
 none

2. Find the value(s) of the variable for which each rational expression is not defined.

(a) $\frac{x^2-49}{2x^2-4x}$
 $2x^2-4x=0$
 $2x(x-2)=0$
 $x=0, 2$
 (b) $\frac{5}{c^2-25}$
 $c^2-25=0$
 $(c+5)(c-5)=0$
 $c=\pm 5$
 (c) $\frac{x-3}{x^2+9}$
 none
 $x^2+9=0$
 $x^2=-9$
 $x=\pm 3i$
 (d) $\frac{6}{3x^2-8x+4}$
 $3x^2-8x+4=0$
 $3x^2-6x-2x+4=0$
 $3x(x-2)-2(x-2)=0$
 $(3x-2)(x-2)=0$
 $x=\frac{2}{3}, 2$

Simplify each expression.

3. $\frac{x^2+6x}{x} = \frac{x(x+6)}{x} = x+6$
 Restrictions: $x \neq 0$

4. $\frac{x^2}{x^2+3x} = \frac{x^{\cancel{2}}}{x^{\cancel{2}}(x+3)} = \frac{x}{x+3}$
 Restrictions: $x \neq 0, -3$

5. $\frac{5b^2-5ab}{2a^2-2ab} = \frac{5b(b-a)}{2a(a-b)} = \frac{-5b}{2a}$
 Restrictions: $a \neq 0$, $a-b=0$, $a \neq b$
 $a \neq 0, b$

6. $\frac{x^2+2x-3}{x^2-1} = \frac{(x-1)(x+3)}{(x-1)(x+1)} = \frac{x+3}{x+1}$
 Restrictions: $x \neq \pm 1$

7. $\frac{5x^2-15x}{27x-3x^2} = \frac{5x(x-3)}{3x(9-x)} = \frac{5(x-3)}{3(9-x)}$
 Restrictions: $x \neq 0, 9$

$$8. \frac{x^2 - x - 6}{3x^2 - 15x + 18} = \frac{\cancel{(x-3)}(x+2)}{3\cancel{(x-3)}(x-2)} = \frac{x+2}{3(x-2)} \quad \begin{array}{l} \underline{R} \\ x \neq 3, 2 \end{array}$$

$$9. \frac{y^2 - 3y - 18}{2y^2 + 5y + 3} = \frac{(y-6)(y+3)}{(y+1)(2y+3)} \quad \begin{array}{l} \underline{R} \\ y \neq -\frac{3}{2}, -1 \end{array}$$

$2y^2 + 3y + 2y + 3$
 $y(2y+3) + 1(2y+3)$

$$10. \frac{x^2 - y^2}{x^2 - 6y - xy + 6x} = \frac{\cancel{(x-y)}(x+y)}{(x+b)\cancel{(x-y)}} \quad \begin{array}{l} \underline{R} \\ x \neq -b, y \\ x-y=0 \\ x \neq y \end{array}$$

$x^2 - xy + bx - by$
 $x(x-y) + b(x-y)$

$$(1-x)(1+y) \quad 11. \frac{1-x^2}{x^3-1} = -(x^2-1) = -(x+1)(x-1)$$

to be continued...

$$12. \frac{x^2 + 2x + xy + 2y}{x^2 + 4x + 4}$$

$$13. \frac{x^3 + 27}{x^3 - 3x^2 + 9x}$$

Steps for Simplifying Rational Expressions:

1. Completely factor the numerator and denominator
2. Cancel common factors

***Note** $\frac{a-b}{b-a} = -1$

****Don't forget to write restrictions.****