Name: ____ Date: ___________

PreCalculus Even More Difference Quotient Practice Ms. Loughran

Do Now: Question 3

Find $\frac{f(x+h)-f(x)}{h}$ for each of the following.

1. $f(x)=4x^2-2x+3$

2. $f(x)=-16x^2+9x+10$

3. $f(x)=4-3x-7x^2$

$(x^2+2xh+h^2)$

$$\frac{4-3(x+h)-7(x+h)^2-(4-3x-7x^2)}{h}$$

$$\frac{\cancel{4}-\cancel{3x}-3h-\cancel{7x^2}-14xh-7h^2-\cancel{4}+\cancel{3x}+\cancel{7x^2}}{h}$$

$$\frac{\cancel{h}(-3-14x)}{\cancel{h}}$$

$$-3-14x$$

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PC: Composition of Functions Ms. Loughran

Two functions can be combined to form a new function.

Given: $f(x) = x^2 + 4$ and $g(x) = 2x$, then the composite function $f \circ g$, read " f following g," can be defined by $[f \circ g](x) = f(g(x))$. "f of g of x"

If we want to find $f(g(4))$, we can go about it two ways:

First way:

Since $g(4) = 8$ then $f(g(4)) = f(8) = 8^2 + 4 = 68$

Therefore $f(g(4)) = 68$

Second way:

Since $g(x) = 2x$, then $f(g(x)) = f(2x) = (2x)^2 + 4 = 4x^2 + 4$

Now we can plug 4 into that rule so $f(g(4)) =$

$4(4)^2 + 4 = 68$

Given: $f(x) = x^2 + 4$ and $g(x) = 2x$

Example 1: Using the functions given above find: (a) $g(f(x))$

$$g(f(x))$$

$$g(x^2+4) = 2(x^2+4)$$

$$= 2x^2+8$$

(b) $g(f(4))$

$$f(4) = 4^2+4 = 20$$

$$g(20) = 2(20) = 40$$

$$\boxed{40}$$

Notice that $[f \circ g](x) \neq [g \circ f](x)$. Therefore, the operation of composition is not commutative.

Example 2:

Let $f(x) = 3x - 5$ and $g(x) = 2 - x^2$. Find:

(a) $[f \circ g](0) = 1$

$g(0) = 2 - 0^2 = 2$

$f(2) = 3(2) - 5 = 1$

(b) $g(f(0))$

$f(0) = 3(0) - 5 = -5$

$g(-5) = 2 - (-5)^2 = -23$

(c) $[f \circ f](4)$

$f(4) = 3(4) - 5 = 7$

$f(7) = 3(7) - 5 = 16$

(d) $[g \circ g](3)$

$g(3) = 2 - (3)^2 = -7$

$g(-7) = 2 - (-7)^2 = -47$

(e) $f(g(-2))$

$g(-2) = 2 - (-2)^2 = -2$

$f(-2) = 3(-2) - 5 = -11$

(f) $g(f(x))$

$g(3x-5) = 2 - (3x-5)^2$

$= 2 - (9x^2 - 30x + 25)$

$= 2 - 9x^2 + 30x - 25$

$= -9x^2 + 30x - 23$

(g) $(f \circ g)(x)$

$f(2 - x^2)$

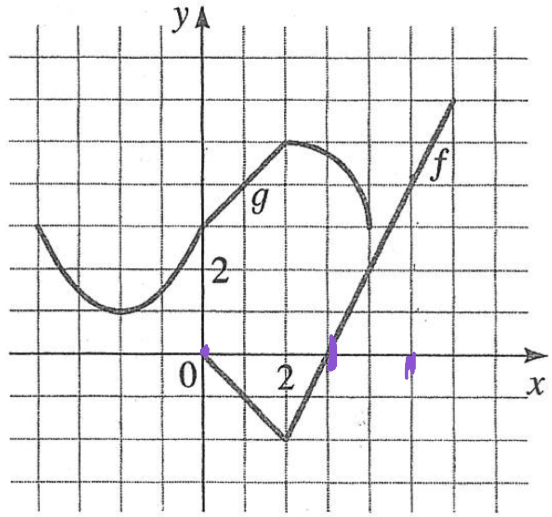
$3(2 - x^2) - 5$

$6 - 3x^2 - 5$

$1 - 3x^2$

Practice:

For 1-6, use the given graphs of *f* and *g* to evaluate the expression.



1. $f(g(2))$

$f(5)$

4

2. $g(f(0))$

$g(0)$

3

3. $(g \circ f)(4)$

$f(4) = 2$

$g(2) = 5$

4. $(f \circ g)(4)$

$g(4) = 3$

$f(3) = 0$

5. $(g \circ g)(-2)$

$g(-2) = 1$

$g(1) = 4$

6. $(f \circ f)(4)$

$f(4) = 2$

$f(2) = -2$

7. For each of the following, find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

(a) $f(x) = 2x + 3,\ g(x) = 4x - 1$

(b) $f(x) = 6x - 5,\ g(x) = \frac{x}{2}$

(c) $f(x) = x^3 + 2,\ g(x) = \sqrt[3]{x}$

(d) $f(x) = x^2,\ g(x) = \sqrt{x - 3}$

(e) $f(x) = x^2,\ g(x) = x - 1$

8. Find $f(g(h(x)))$

(a) $f(x) = x - 1,\ \ g(x) = \sqrt{x},\ \ h(x) = x + 1$

$h(x) = x+1$

$g(x+1) = \sqrt{x+1}$

$f(\sqrt{x+1}) = \sqrt{x+1} - 1$

(b) $f(x)=\frac{1}{x}$, $g(x)=x^3$, $h(x)=x^2+2$

$h(x)=x^2+2$

$g(x^2+2)=(x^2+2)^3$

$f(x^2+2)^3=\frac{1}{(x^2+2)^3}$

(c) $f(x)=x^4+1$, $g(x)=x-5$, $h(x)=\sqrt{x}$

(d) $f(x) = \sqrt{x}, \quad g(x) = \dfrac{x}{x-1}, \quad h(x) = \sqrt[3]{x}$

Difference Quotient Homework

① $f(x)=3x+2$

$$\frac{3(x+h)+2-(3x+2)}{h}=\frac{\cancel{3x}+3h\cancel{+2}-\cancel{3x}\cancel{-2}}{h}=\frac{3h}{h}=3 \quad h\neq 0$$

② $f(x)=x^2+1$

$$\frac{(x+h)^2+1-(x^2+1)}{h}=\frac{\cancel{x^2}+2xh+h^2\cancel{+1}\cancel{-x^2}\cancel{-1}}{h}=\frac{2xh+h^2}{h}=2x+h$$

$h\neq 0$

③ $f(x)=x^2+3x-4$

$$\frac{(x+h)^2+3(x+h)-4-(x^2+3x-4)}{h}=\frac{\cancel{x^2}+2xh+h^2\cancel{+3x}+3h\cancel{-4}\cancel{-x^2}\cancel{-3x}\cancel{+4}}{h}$$

$$\frac{2xh+h^2+3h}{h}=2x+h+3 \quad h\neq 0$$

④ $f(x)=2x^2-5x+3$

$$\frac{2(x+h)^2-5(x+h)+3-(2x^2-5x+3)}{h}=\frac{\cancel{2x^2}+4xh+2h^2\cancel{-5x}-5h\cancel{+3}\cancel{-2x^2}\cancel{+5x}\cancel{-3}}{h}$$

$$\frac{4xh+2h^2-5h}{h}=4x+2h-5 \quad h\neq 0$$

⑤ $f(x) = 3 - 5x + 4x^2$

$$\frac{3 - 5(x+h) + 4(x+h)^2 - (3 - 5x + 4x^2)}{h}$$

$$\frac{\cancel{3} \cancel{-5x} - 5h + \cancel{4x^2} + 8xh + 4h^2 \cancel{-3} \cancel{+5x} \cancel{-4x^2}}{h} = \frac{-5h + 8xh + 4h^2}{h} = -5 + 8x + 4h$$

$h \neq 0$

⑥ $f(x) = x^3 - 6x^2 + 12x - 8$

$$\frac{(x+h)^3 - 6(x+h)^2 + 12(x+h) - 8 - (x^3 - 6x^2 + 12x - 8)}{h}$$

$$\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-6x^2} - 12xh - 6h^2 \cancel{+12x} + 12h \cancel{-8} \cancel{-x^3} \cancel{+6x^2} \cancel{-12x} \cancel{+8}}{h}$$

$$\frac{3x^2h + 3xh^2 + h^3 - 12xh - 6h^2 + 12h}{h}$$

$3x^2 + 3xh + h^2 - 12x - 6h + 12 \quad h \neq 0$