Name:_____ Date:_____

PC: Decomposition of Functions Ms. Loughran

Do Now:

Given $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{x}$, find

(a) $f(g(4))$

$g(4)=\frac{1}{4}$

$f\left(\frac{1}{4}\right)=\sqrt{\frac{1}{4}}=\frac{\sqrt{1}}{\sqrt{4}}=\frac{1}{2}$

(b) $f(g(x))$

$g(x)=\frac{1}{x}$

$f\left(\frac{1}{x}\right)=\sqrt{\frac{1}{x}}=\frac{\sqrt{1}}{\sqrt{x}}=\frac{1}{\sqrt{x}}$

Homework 10-06

7. For each of the following, find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

(a) $f(x)=2x+3,\ g(x)=4x-1$

$(f \circ g)x$

$f(4x-1)=2(4x-1)+3$

$=8x-2+3$

$=8x+1$

$(g \circ f)(x)$

$g(2x+3)=4(2x+3)-1$

$=8x+12-1$

$=8x+11$

(b) $f(x)=6x-5,\ g(x)=\frac{x}{2}$

$(f \circ g)x$

$g(x)=\frac{x}{2}$

$f\left(\frac{x}{2}\right)=6\left(\frac{x}{2}\right)-5$

$=3x-5$

$(g \circ f)x$

$f(x)=6x-5$

$g(6x-5)=\frac{6x-5}{2}$

(c) $f(x)=x^3+2,\ g(x)=\sqrt[3]{x}$

$(f \circ g)(x)$

$g(x)=\sqrt[3]{x}$

$f\left(\sqrt[3]{x}\right)=\left(\sqrt[3]{x}\right)^3+2$

$=x+2$

$(g \circ f)(x)$

$f(x)=x^3+2$

$g(x^3+2)=\sqrt[3]{x^3+2}$

(d) $f(x)=x^2,\ g(x)=\sqrt{x-3}$

$(f \circ g)(x)$

$g(x)=\sqrt{x-3}$

$f\left(\sqrt{x-3}\right)=\left(\sqrt{x-3}\right)^2$

$=x-3$

$(g \circ f)(x)$

$g(x^2)=\sqrt{x^2-3}$

(e) $f(x)=x^2,\ g(x)=x-1$

$(f\circ g)(x)$

$g(x)=x-1$

$f(x-1)=(x-1)^2=x^2-2x+1$

$(g\circ f)x$

$f(x)=x^2$

$g(x^2)=x^2-1$

8. Find $f(g(h(x)))$

(c) $f(x)=x^4+1,\ \ g(x)=x-5,\ \ h(x)=\sqrt{x}$

$h(x)=\sqrt{x}$

$g(\sqrt{x})=\sqrt{x}-5$

$f(\sqrt{x}-5)=(\sqrt{x}-5)^4+1$

(d) $f(x)=\sqrt{x},\ \ g(x)=\dfrac{x}{x-1},\ \ h(x)=\sqrt[3]{x}$

$h(x)=\sqrt[3]{x}$

$g(\sqrt[3]{x})=\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}$

$f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\sqrt{\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$

A composite function is a function that brings together two or more functions. For instance, let h be given by

$$h(x)=\sqrt{x^2+2x+2}$$

If we let $f(x)=x^2+2x+2$ and $g(x)=\sqrt{x}$, then $(g\circ f)(x)=$

$f(x)=x^2+2x+2$

$g(x^2+2x+2)=\sqrt{x^2+2x+2}$

Thus the given function h has been ***decomposed*** into the composition of the two functions f and g. Such decompositions are not unique. More than one decomposition is possible.

We could have decomposed h into $f(x)=\sqrt{x+2}$ and $g(x)=x^2+2x$.

$f(g(x))=$

$g(x)=x^2+2x$

$f(x^2+2x)=\sqrt{x^2+2x+2}$

We are going to avoid using the identity function ($f(x)=x$) in our decompositions.

1. Find the functions f and g so that $h(x)=f(g(x))$

(a) $h(x)=(3x+1)^2$

$f(x)=(x+1)^2$

$g(x)=3x$

check $f(g(x))$

$f(3x)=(3x+1)^2$

$f(x)=x^2$

$g(x)=3x+1$

(b) $h(x) = \sqrt{1-4x}$

$f(x) = \sqrt{x}$

$g(x) = 1-4x$

OR

$f(x) = \sqrt{1-x}$

$g(x) = 4x$

(c) $h(x) = \sqrt[4]{x+9}$

$f(x) = \sqrt[4]{x}$

$g(x) = x+9$

OR

$f(y) = \sqrt[4]{x+1}$

$g(x) = x+8$

Practice

Express the function in the form $f \circ g$ = $F(g(x))$

1. $F(x) = (x-9)^5$

$f(x) = x^5$

$g(x) = x-9$

2. $F(x) = \sqrt{x}+1$

$f(x) = x+1$

$g(x) = \sqrt{x}$

3. $F(x) = \dfrac{x^2}{x^2+4}$

$f(x) = \dfrac{x}{x+4}$

$g(x) = x^2$

4. $F(x) = \dfrac{1}{x+3}$

$f(x) = \dfrac{1}{x}$

$g(x) = x+3$

5. $F(x) = |1 - x^3|$

$f(x) = |x|$

$g(x) = 1 - x^3$

6. $F(x) = \sqrt{1 + \sqrt{x}}$

$f(x) = \sqrt{x}$

$g(x) = 1 + \sqrt{x}$

Express the function in the form $f \circ g \circ h$ $= f(g(h(x)))$

7. $F(x) = \dfrac{1}{x^2 + 1}$

$f(x) = \dfrac{1}{x}$

$g(x) = x + 1$

$h(x) = x^2$

check

$g(x^2) = x^2 + 1$

$f(x^2 + 1) = \dfrac{1}{x^2 + 1}$