Name:______________________________________ Date:___________

PC: Difference Quotients Ms. Loughran

Difference quotient is an expression of the form:

$$\frac{f(x+h)-f(x)}{h}, \; h \neq 0$$

It represents the slope of the line between two points, $(x, f(x))$ and $(x+h, f(x+h))$. It is going to be used in Calculus, so we need to get really comfortable with it.

Do Now: 1part a only

$f(1) = 4(1)^2 = 4$

$f(a) = 4a^2$

1. Given $f(x) = 4x^2$, find the following and simplify.

(a). $f(x+h)$

$4(x+h)^2$

$4(x^2+2xh+h^2)$

$4x^2+8xh+4h^2$

$(x+h)(x+h)$

$x^2+xh+xh+h^2$

$x^2+2xh+h^2$

(b). $f(x+h)-f(x)$

$\cancel{4x^2}+8xh+4h^2-\cancel{4x^2}$

$8xh+4h^2$

(c). $\frac{f(x+h)-f(x)}{h}$

$\frac{8xh+4h^2}{h}$

$\frac{4\cancel{h}(2x+h)}{\cancel{h}} = 4(2x+h)$ or $8x+4h$, $h \neq 0$

2. Given $f(x) = 2x^2 - x$, find the following and simplify.

(a). $f(x+h)$

$2(x+h)^2-(x+h)$

$2(x^2+2xh+h^2)-x-h$

$2x^2+4xh+2h^2-x-h$

(b). $f(x+h)-f(x)$

$2x^2+4xh+2h^2-x-h-(2x^2-x)$

$\cancel{2x^2}+4xh+2h^2\cancel{-x}-h\cancel{-2x^2}\cancel{+x}$

$4xh+2h^2-h$

(c). $\frac{f(x+h)-f(x)}{h}$

$\frac{4xh+2h^2-h}{h}$

$\frac{\cancel{h}(4x+2h-1)}{\cancel{h}}$

$4x+2h-1, \; h \neq 0$

3. Given $f(x) = 9 - \frac{1}{2}x^2$, find the following and simplify.

(a). $f(x+h)$

$9-\frac{1}{2}(x+h)^2$

$9-\frac{1}{2}(x^2+2xh+h^2)$

$9-\frac{1}{2}x^2-xh-\frac{1}{2}h^2$

(b). $f(x+h)-f(x)$

$9-\frac{1}{2}x^2-xh-\frac{1}{2}h^2-(9-\frac{1}{2}x^2)$

$\cancel{9}\cancel{-\frac{1}{2}x^2}-xh-\frac{1}{2}h^2\cancel{-9}\cancel{+\frac{1}{2}x^2}$

$-xh-\frac{1}{2}h^2$

(c). $\frac{f(x+h)-f(x)}{h}$

$\frac{-xh-\frac{1}{2}h^2}{h}$

$\frac{\cancel{h}(-x-\frac{1}{2}h)}{\cancel{h}} = -x-\frac{1}{2}h, \; h \neq 0$

$h \neq 0$

4. Given $f(x) = 1 - x^2$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$$f(x+h) - f(x)$$

$$\frac{1-(x+h)^2 - (1-x^2)}{h}$$

$$\frac{1-(x^2+2xh+h^2)-1+x^2}{h} = \frac{\cancel{1}\cancel{-x^2}-2xh-h^2\cancel{-1}\cancel{+x^2}}{h}$$

$$= \frac{-2xh-h^2}{h} = \frac{\cancel{h}(-2x-h)}{\cancel{h}}$$

$$= -2x-h, h \neq 0$$

If you let $h = 0$, what does your answer become?

$-2x$

5. Given $C(x) = 2x^2 - 4x + 3$, find and simplify $\dfrac{C(x+h) - C(x)}{h}$

$$\frac{2(x+h)^2 - 4(x+h) + 3 - (2x^2 - 4x + 3)}{h}$$

$$\frac{2(x^2+2xh+h^2) - 4x - 4h + 3 - 2x^2 + 4x - 3}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-4x} - 4h \cancel{+3} \cancel{-2x^2} \cancel{+4x} \cancel{-3}}{h} = \frac{4xh + 2h^2 - 4h}{h} = \frac{\cancel{h}(4x+2h-4)}{\cancel{h}}$$

$$4x+2h-4, h \neq 0$$

If you let $h = 0$, what does your answer become?

$4x + 2(0) - 4 = 4x - 4$

6. Given $p(q) = q^2 + 2q - 5$, find and simplify $\dfrac{p(q+h) - p(q)}{h}$

$$\frac{(q+h)^2 + 2(q+h) - 5 - (q^2 + 2q - 5)}{h}$$

$$\frac{\cancel{q^2} + 2qh + h^2 + \cancel{2q} + 2h \cancel{-5} \cancel{-q^2} \cancel{-2q} \cancel{+5}}{h} = \frac{2qh + h^2 + 2h}{h} = \frac{\cancel{h}(2q+h+2)}{\cancel{h}}$$

$$= 2q + h + 2, h \neq 0$$

If you let $h = 0$, what does your answer become?

$2q + 0 + 2 = 2q + 2$