Name:______________________________ Date:___________

PC: Decomposition of Functions Ms. Loughran

Do Now:

Given $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{x}$, find

(a) $f(g(4))$

$g(4) = \frac{1}{4}$

$f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

(b) $f(g(x))$

$f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} = \frac{\sqrt{1}}{\sqrt{x}} = \frac{1}{\sqrt{x}}$

A composite function is a function that brings together two or more functions. For instance, let h be given by

$$h(x)=\sqrt{x^2+2x+2}$$

If we let $f(x)=x^2+2x+2$ and $g(x)=\sqrt{x}$, then $(g\circ f)(x)=$

$g(x^2+2x+2) = \sqrt{x^2+2x+2}$

Thus the given function h has been ***decomposed*** into the composition of the two functions f and g. Such decompositions are not unique. More than one decomposition is possible.

We could have decomposed h into $f(x)=\sqrt{x+2}$ and $g(x)=x^2+2x$.

$f(g(x))=$ $f(x^2+2x) = \sqrt{x^2+2x+2}$

We are going to avoid using the identity function ($f(x)=x$) in our decompositions.

These functions we create are not unique meaning there are infinite ways to decompose, let's come up with 2 ways for each.

1. Find the functions f and g so that $h(x) = f(g(x))$

(a) $h(x) = (3x+1)^2$

$g(x) = 3x+1$
$f(x) = x^2$

check:
$f(g(x))$
$f(3x+1) = (3x+1)^2$

$g(x) = 3x$
$f(x) = (x+1)^2$

$f(g(x))$
$f(3x) = (3x+1)^2$

(b) $h(x) = \sqrt{1-4x}$

$g(x) = 1-4x$
$f(x) = \sqrt{x}$

check
$f(1-4x)$
$= \sqrt{1-4x}$

$g(x) = 4x$
$f(x) = \sqrt{1-x}$

(c) $h(x) = \sqrt[4]{x+9}$

$g(x) = x+9$
$f(x) = \sqrt[4]{x}$

check
$f(g(x))$
$f(x+9)$
$\sqrt[4]{x+9}$

$g(x) = x+8$
$f(x) = \sqrt[4]{x+1}$

Practice

Express the function in the form $f \circ g$ $= f(g(x))$

1. $F(x) = (x-9)^5$

$g(x) = x-9$
$f(x) = x^5$

or

$g(x) = x-8$
$f(x) = (x-1)^5$

4. $F(x) = \dfrac{1}{x+3}$

$g(x) = x+3$
$f(x) = \frac{1}{x}$

or

$g(x) = x+2$
$f(x) = \frac{1}{x+1}$

2. $F(x) = \sqrt{x} + 1$

$g(x) = \sqrt{x}$
$f(x) = x+1$

or

$g(x) = \sqrt{x} + 2$
$f(x) = x - 1$

5. $F(x) = |1 - x^3|$

$g(x) = 1-x^3$
$f(x) = |x|$

or

$g(x) = x^3$
$f(x) = |1-x|$

3. $F(x) = \dfrac{x^2}{x^2+4}$

$g(x) = x^2$
$f(x) = \frac{x}{x+4}$

or

$g(x) = x^2-4$
$f(x) = \frac{x+4}{x+8}$

6. $F(x) = \sqrt{1+\sqrt{x}}$

$g(x) = 1+\sqrt{x}$
$f(x) = \sqrt{x}$

or

$g(x) = \sqrt{x}$
$f(x) = \sqrt{1+x}$

Express the function in the form $f \circ g \circ h$

7. $F(x) = \dfrac{1}{x^2 + 1}$

check

$h(x) = x^2$

$g(x) = x + 1$

$f(x) = \dfrac{1}{x}$

$f(g(h(x)))$

$g(x^2) = x^2 + 1$

$f(x^2 + 1) = \dfrac{1}{x^2 + 1}$

$h(x) = x^2 + 2$

$g(x) = x - 1$

$f(x) = \dfrac{1}{x}$

Homework 10-12

7. For each of the following, find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

(a) $f(x) = 2x + 3,\ g(x) = 4x - 1$

$(f \circ g)x$

$f(4x-1) = 2(4x-1)+3$

$= 8x - 2 + 3$

$= 8x + 1$

$(g \circ f)(x)$

$g(2x+3) = 4(2x+3) - 1$

$= 8x + 12 - 1$

$= 8x + 11$

(b) $f(x) = 6x - 5,\ g(x) = \frac{x}{2}$

$(f \circ g)x$

$g(x) = \frac{x}{2}$

$f\left(\frac{x}{2}\right) = \cancel{6}^{3}\left(\frac{x}{\cancel{2}}\right) - 5$

$= 3x - 5$

$(g \circ f)x$

$f(x) = 6x - 5$

$g(6x-5) = \frac{6x-5}{2} = \frac{6x}{2} - \frac{5}{2}$

$3x - \frac{5}{2}$

(c) $f(x) = x^3 + 2,\ g(x) = \sqrt[3]{x}$

$(f \circ g)(x)$

$g(x) = \sqrt[3]{x}$

$f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 + 2$

$= x + 2$

$(g \circ f)(x)$

$f(x) = x^3 + 2$

$g(x^3+2) = \sqrt[3]{x^3+2}$

(d) $f(x) = x^2,\ g(x) = \sqrt{x-3}$

$(f \circ g)(x)$

$g(x) = \sqrt{x-3}$

$f(\sqrt{x-3}) = (\sqrt{x-3})^2$

$= x - 3$

$(g \circ f)(x)$

$g(x^2) = \sqrt{x^2 - 3}$

(e) $f(x)=x^2$, $g(x)=x-1$

$(f \circ g)(x)$

$g(x)=x-1$

$f(x-1)=(x-1)^2=x^2-2x+1$

$(g \circ f)x$

$f(x)=x^2$

$g(x^2)=x^2-1$

8. Find $f(g(h(x)))$

(c) $f(x)=x^4+1$, $g(x)=x-5$, $h(x)=\sqrt{x}$

$h(x)=\sqrt{x}$

$g(\sqrt{x})=\sqrt{x}-5$

$f(\sqrt{x}-5)=(\sqrt{x}-5)^4+1$

(d) $f(x)=\sqrt{x}$, $g(x)=\dfrac{x}{x-1}$, $h(x)=\sqrt[3]{x}$

$h(x)=\sqrt[3]{x}$

$g(\sqrt[3]{x})=\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}$

$f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\sqrt{\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$