Name:_______________________________ Date:___________

PC: Inverses Ms. Loughran

The functions f and g are **inverse functions** if $f(g(x)) = g(f(x)) = x$.

Example1:

Let $f(x) = 2x+1$ and $g(x) = \frac{x-1}{2}$, are f and g inverse functions? Yes

$f(g(x))$

$f\left(\frac{x-1}{2}\right) = \cancel{2}\left(\frac{x-1}{\cancel{2}}\right)+1$

$x-1+1$

x

$g(f(x))$

$g(2x+1) = \frac{2x+\cancel{1}-\cancel{1}}{2}$

$\frac{2x}{2}$

x

The symbol f^{-1} is often used for the inverse of function *f*. The inverse "undoes" or reverses what the function has done. The inverse of a function interchanges the domain and range. That is for every point (a,b) on the graph of f, there is a point (b,a) on the graph of the inverse of f. The graphs of a function and its inverse are symmetric with respect to the line $y = x$.

Example 2:

Let $g(x) = \{(1,2),(2,2),(3,2),(4,2),(5,2)\}$, find the inverse of g? Is the inverse also a function?

g is not 1-1

inverse of g: $\{(2,1),(2,2),(2,3),(2,4),(2,5)\}$

the inverse is not a function b/c there is repitition in the domain (x's repeating)

b/c the inverse is NOT a function, we can refer to it as $g^{-1}(x)$

Example 3:

Let $f(x) = 2x-3$, find the inverse of f? Is the inverse also a function?

f is 1-1

A function whose inverse is also a function is called one to one. (can also be written as 1-1) It is easy to detect a one to one function from its graph using the **horizontal line test.** A function is 1-1 if and only if no horizontal line intersects the graph more than once.

method 1: replace function name with y, switch x and y, solve for the new y

$y = 2x-3$

$x = 2y-3$

$x+3 = 2y$

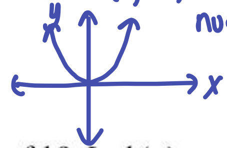
$\frac{x+3}{2} = y$ ← passes the VLT so it is a function so we can call it $f^{-1}(x)$

method 2 "unwrapping method"

f	inverse
mult. by 2	add 3
subtract 3	divide by 2

$\frac{x+3}{2} = y = f^{-1}(x)$

* You would have also looked at the graph of $h(x) = x^2$ not 1-1 b/c it fails HLT



Example 4:

Let $h(x) = x^2$, find the inverse of h? Is $h(x)$ one to one?

$y = x^2$

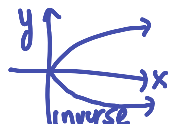
$x = y^2$

$\pm\sqrt{x} = y$

graph this

$y_1 = \sqrt{x}$

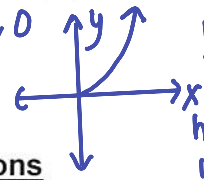
$y_2 = -\sqrt{x}$



not a function, therefore $h(x)$ is not 1-1 *

We can make the inverse of h from example 4 a function by restricting its domain.

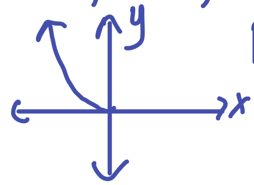
$h(x) = x^2,\ x \geq 0$



now it passes the HLT so $h(x) = x^2,\ x \geq 0$ is 1-1

or

$h(x) = x^2,\ x \leq 0$



passes the HLT so $h(x) = x^2,\ x \leq 0$ is 1-1

Practice:

Inverse Relations

Find the inverse for each relation.

this is a 1-1 function

1. { (1, -3), (-2, 3), (5, 1), (6, 4) }

$\{(-3,1), (3,-2), (1,5), (4,6)\}$

2. { (-5, 7), (-6, -8), (1, -2), (10, 3) }

Finding Inverses

Find an equation for the inverse for each of the following relations.

3. $y = 3x + 2$
4. $y = -5x - 7$
5. $y = 12x - 3$
6. $y = -8x + 16$
7. $y = \frac{2}{3}x - 5$
8. $y = -\frac{3}{4}x + 5$
9. $y = -\frac{5}{8}x + 10$
10. $y = \frac{1}{2}x + 8$
11. $y = x^2 + 5$
12. $y = x^2 - 4$
13. $y = (x + 3)^2$
14. $y = (x - 6)^2$
15. $y = \sqrt{x - 2},\ y \geq 0$
16. $y = \sqrt{x + 5},\ y \geq 0$
17. $y = \sqrt{x} + 8,\ y \geq 8$
18. $y = \sqrt{x} - 7,\ y \geq -7$

Verifying Inverses

Verify that f and g are inverse functions.

19. $f(x) = x + 6,\ g(x) = x - 6$
20. $f(x) = 5x + 2,\ g(x) = \frac{x - 2}{5}$
21. $f(x) = -3x - 9,\ g(x) = -\frac{1}{3}x - 3$
22. $f(x) = 2x - 7,\ g(x) = \frac{x + 7}{2}$
23. $f(x) = -4x + 8,\ g(x) = -\frac{1}{4}x + 2$
24. $f(x) = \frac{1}{2}x - 7,\ g(x) = 2x + 14$

3. $y = 3x + 2$

← passes the VLT and the HLT so it's one to one

$x = 3y + 2$

$x - 2 = 3y$

or

$\frac{x-2}{3} = y$ ← passes the VLT so $y = 3x + 2$ is 1-1

5. $y = 12x - 3$ ← 1-1 b/c its inverse is also a function b/c it passes the HLT

$y = \frac{x+3}{12}$

13. $y = (x + 3)^2$

	inverse
add 3	$\sqrt{\ }$
squaring	$-$

not 1-1

$y = \pm\sqrt{x} - 3$

$x = (y+3)^2$

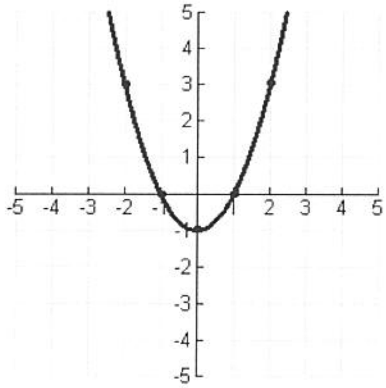
$\pm\sqrt{x} = y + 3$

$\pm\sqrt{x} - 3 = y$

Graphing Inverses

Graph the inverse for each relation below (put your answer on the same graph).

25.



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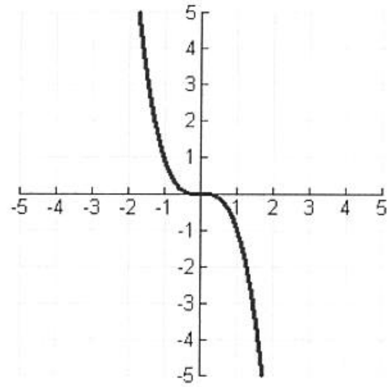

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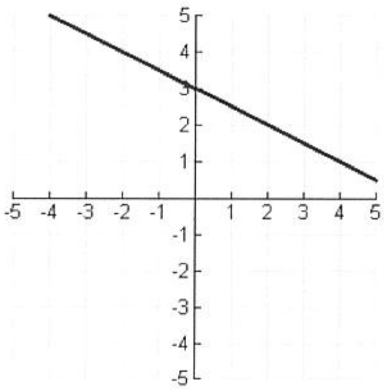

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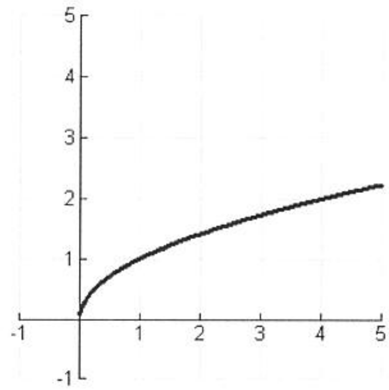

Name: Key

PC: Decomposition Homework 2

Date: ____________

Ms. Loughran

Find functions f and g such that $h(x) = (f \circ g)(x)$.

1. $h(x) = (3x^2 - 2)^4$ $g(x) = 3x^2 - 2$ $f(x) = x^4$	2. $h(x) = (x^2 - x + 1)^8$ $g(x) = x^2 - x + 1$ $f(x) = x^8$
3. $h(x) = \sqrt[3]{x-4}$ $g(x) = x - 4$ $f(x) = \sqrt[3]{x}$	4. $h(x) = \frac{1}{x+2}$ $g(x) = x + 2$ $f(x) = \frac{1}{x}$
5. $h(x) = \sqrt{x^2 + 6}$ $g(x) = x^2 + 6$ $f(x) = \sqrt{x}$	6. $h(x) = 2^{6x+7}$ $g(x) = 6x + 7$ $f(x) = 2^x$
7. $h(x) = \sqrt[4]{x+1}$ $g(x) = x + 1$ $f(x) = \sqrt[4]{x}$	8. $h(x) = (5x - 8)^6$ $g(x) = 5x - 8$ $f(x) = x^6$

Express the function in the form $(f \circ g \circ h)(x)$

9. $F(x) = \sqrt[3]{(x+4)^2}$

$h(x) = x + 4$

$g(x) = x^2$

$f(x) = \sqrt[3]{x}$

10. $F(x) = (5x - 8)^6$

$h(x) = 5x - 7$

$g(x) = x - 1$

$f(x) = x^6$

$h(x) = 5x$

$g(x) = x - 8$

$f(x) = x^6$