Name:______________________________ Date:__________

PC: Quadratic Functions Ms. Loughran

Standard form: $y = f(x) = ax^2 + bx + c\,,\ a \neq 0$

- If $a > 0$, then the parabola opens upward; if $a < 0$, then the parabola opens downward.
- The vertex of the parabola is the point $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, and the axis of symmetry is $x = \frac{-b}{2a}$.
- To find the y-intercept, let $x = 0$ and solve for y.
- To find the x-intercept, let $y = 0$ and solve for x. (This will result in a quadratic equation which might have 0, 1 or 2 solutions.)

Vertex form: $y = f(x) = a(x-h)^2 + k,\ a \neq 0$

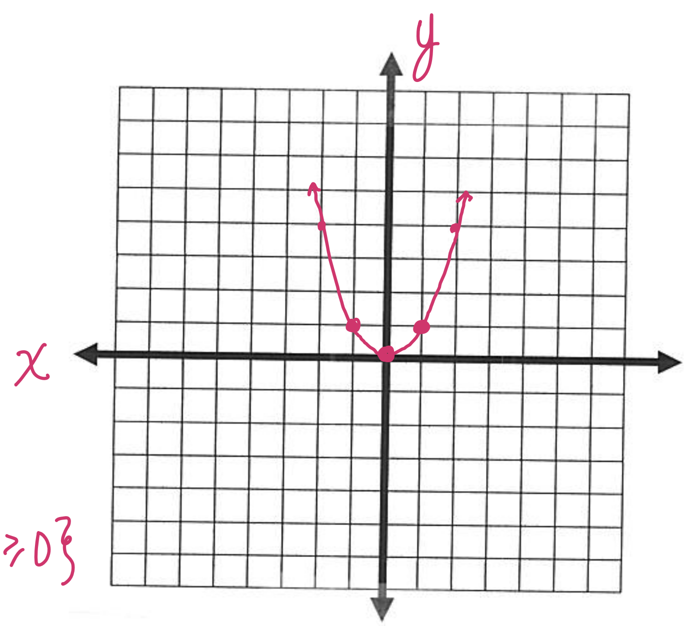
- If $a > 0$, then the parabola opens upward; if $a < 0$, then the parabola opens downward.
- The vertex of the parabola is the point (h, k) and $x = h$ is the axis of symmetry.
- To find the y-intercept, let $x = 0$ and solve for y.
- To find the x-intercept, let $y = 0$ and solve for x. (This will result in a quadratic equation which might have 0, 1 or 2 solutions.)

General Graph for $y = x^2$

key pts

$(-1,1)$
$(0,0)$
$(1,1)$

you can also use $(2,4)$ $(-2,4)$



Domain: $(-\infty,\infty)$ or $\mathbb{R}$
Range: $[0,\infty)$ or $\{y \mid y \geq 0\}$

x-int: $(0,0)$
y-int: $(0,0)$

$y = a(x-h)^2 + k$

Examples:

1. Given the quadratic function $f(x) = -x^2 + 6x - 5$, find the axis of symmetry, vertex, x- and y-intercepts and graph it.

$f(x) = -x^2 + 6x - 5$

$f(x) = -(x^2 - 6x + 9 - 9 + 5)$ (−4)

$f(x) = -(x-3)^2 - (-4)$

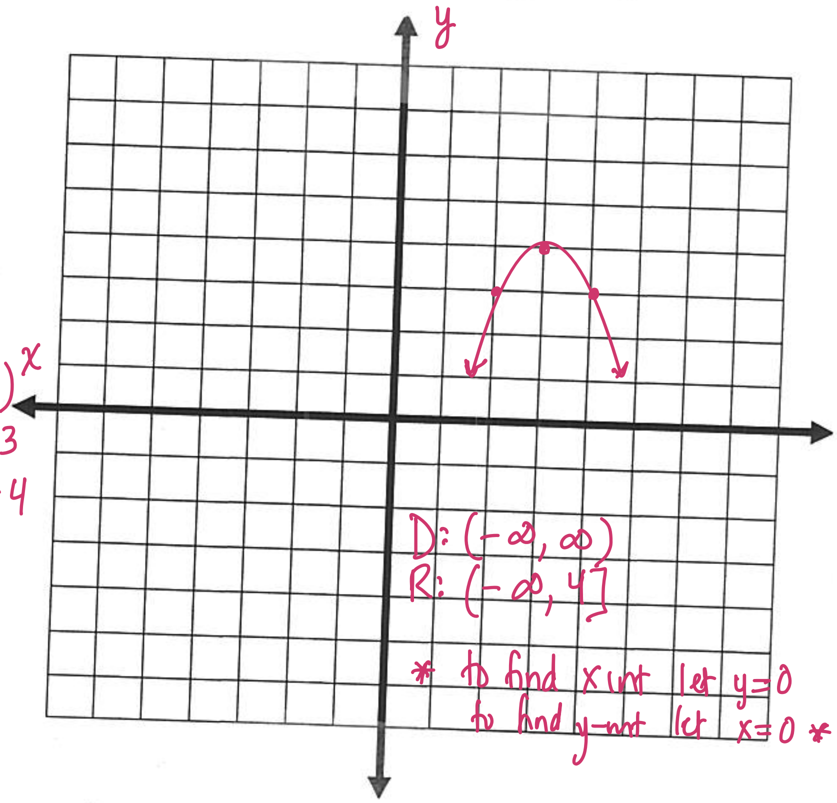
$f(x) = -(x-3)^2 + 4$

vertex: (3, 4)

a of s: $x = 3$

x^2 → 3 right, → reflection over x-axis, (negate y) → ↑4

x^2	3 right	reflection over x-axis (negate y)	↑4
(−1, 1)	(2, 1)	(2, −1)	(2, 3)
(0, 0)	(3, 0)	(3, 0)	(3, 4)
(1, 1)	(4, 1)	(4, −1)	(4, 3)



2. Given the quadratic function $f(x) = (x-4)^2$, find the axis of symmetry, vertex, x- and y-intercepts and graph it.

$f(x) = (x-4)^2$ is a transformation of $y = x^2$ 4 units to the right

x^2	4 units right →
(−1, 1)	(3, 1)
(0, 0)	(4, 0)
(1, 1)	(5, 1)

vertex: (4, 0)

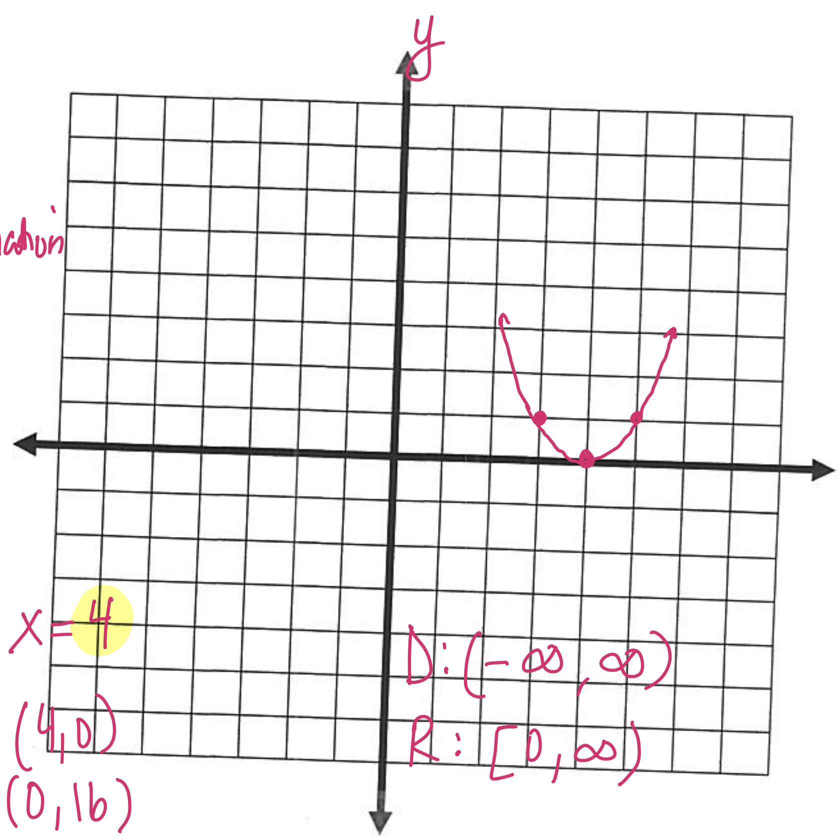
axis of symmetry $x = 4$

x-int: (4, 0)

y-int: (0, 16)

let $x = 0$

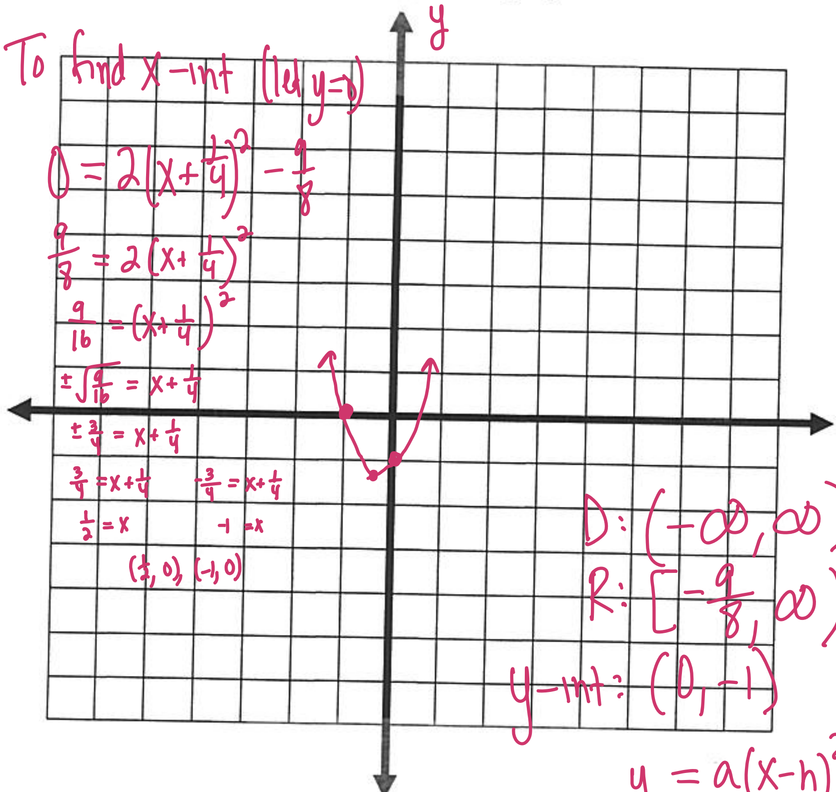
$y = (0-4)^2 = 16$



How does the graph in question 2 compare to the general graph of $y = x^2$?

4 units right

3. Given the quadratic function, $f(x) = 2x^2 + x - 1$ find the axis of symmetry, vertex, x- and y-intercepts and graph it.



$f(x) = 2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right)$

$f(x) = 2\left(x + \frac{1}{4}\right)^2 + 2\left(-\frac{9}{16}\right)$

$f(x) = 2\left(x + \frac{1}{4}\right)^2 - \frac{9}{8}$

$\frac{1}{2}b$

$v: \left(-\frac{1}{4}, -\frac{9}{8}\right)$

a of s: $x = -\frac{1}{4}$

To find x-int (let y=0)

$0 = 2\left(x+\frac{1}{4}\right)^2 - \frac{9}{8}$

$\frac{9}{8} = 2\left(x+\frac{1}{4}\right)^2$

$\frac{9}{16} = \left(x+\frac{1}{4}\right)^2$

$\pm\sqrt{\frac{9}{16}} = x + \frac{1}{4}$

$\pm\frac{3}{4} = x + \frac{1}{4}$

$\frac{3}{4} = x + \frac{1}{4}$ $\quad -\frac{3}{4} = x + \frac{1}{4}$

$\frac{1}{2} = x$ $\quad -1 = x$

$(\frac{1}{2}, 0)$, $(-1, 0)$

D: $(-\infty, \infty)$

R: $[-\frac{9}{8}, \infty)$

y-int: $(0, -1)$

plug in:

$x = 0$ $\quad f(0) = 2(0)^2 + 0 - 1 = -1$

$x = -1$ $\quad f(-1) = 2(-1)^2 - 1 - 1 = 0$

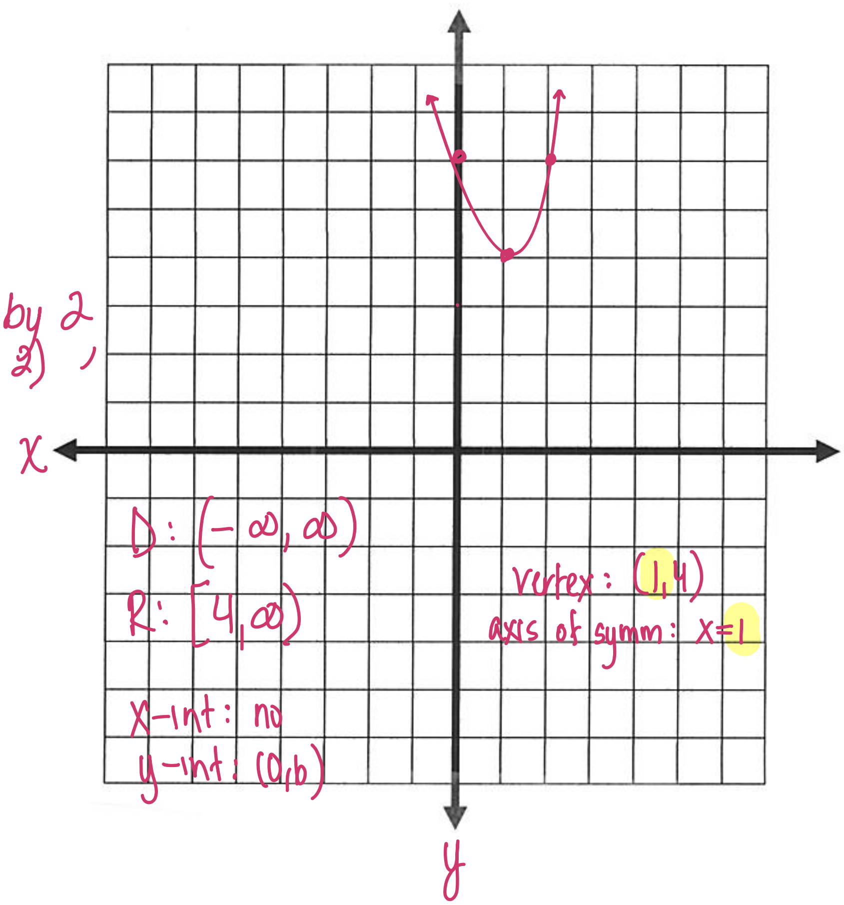
$y = a(x-h)^2 + k$

4. Given the quadratic function $f(x) = 2(x-1)^2 + 4$, find the axis of symmetry, vertex, x- and y-intercepts and graph it.

$f(x) = 2(x-1)^2 + 4$ is a transformation of $y = x^2$

right one, vertical stretch by 2 (mult. y's by 2), ↑4

x^2	right 1	vertical stretch	↑4
(-1,1)	(0,1)	(0,2)	(0,6)
(0,0)	(1,0)	(1,0)	(1,4)
(1,1)	(2,1)	(2,2)	(2,6)



D: $(-\infty, \infty)$

R: $[4, \infty)$

vertex: (1,4)

axis of symm: x=1

x-int: no

y-int: (0,6)

5. Use the information to write the vertex form equation of each parabola

(a) $y=-x^2-14x-59$
(b) $y=x^2-12x+46$
(c) $y=x^2-6x+5$
(d) $y=x^2+16x+71$
(e) $y=x^2-2x-5$
(f) $y=x^2+4x$
(g) $y=2x^2+36x+170$
(h) $y=2x^2+12x-2$
(i) $y=2x^2-12x-23$

b) $y=x^2-12x+46$
$y=x^2-12x+36-36+46$
$\frac{1}{2}b$
$y=(x-6)^2+10$
v: (6,10)
a of s: x=6

g) $y=2x^2+36x+170$
$y=2(x^2+18x+81-81+85)$ 4
$y=2(x+9)^2+2(4)$
$y=2(x+9)^2+8$
v: (-9,8)
a of s: x=-9

For each of the following, find the axis of symmetry, vertex, x- and y-intercepts and sketch the graph on a separate piece of graph paper.

6. $y=(x-5)^2-4$

7. $f(x)=x^2+6x+5$

8. $y=x^2+4x+5$

9. $f(x)=-x^2+8x$

10. $y=4x^2-8x+3$

11. $y=x^2-6x+13$