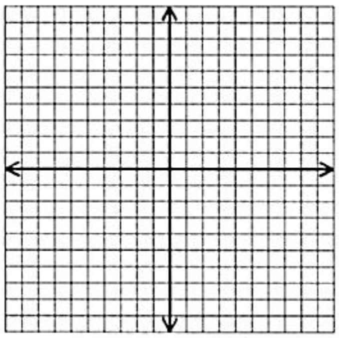
Name:_______________________________________ Date:_____________

PC: Transformations of Functions Ms. Loughran

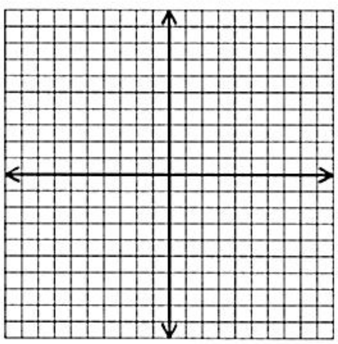
Do Now Activity

You can use your graphing calculator.

1. Graph $y = x^2$.

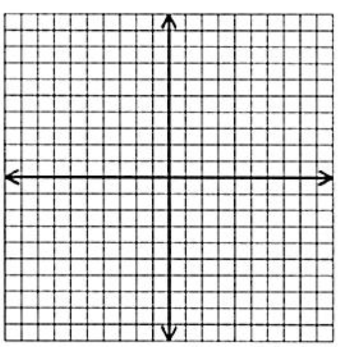


2. Graph $y = (x+4)^2$ **and describe how it is related to** $y = x^2$.



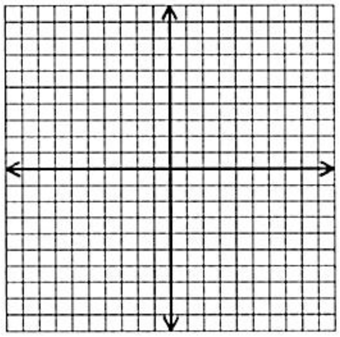
shifts x^2 graph to the left 4 units

3. Graph $y = (x-2)^2$ **and describe how it is related to** $y = x^2$.



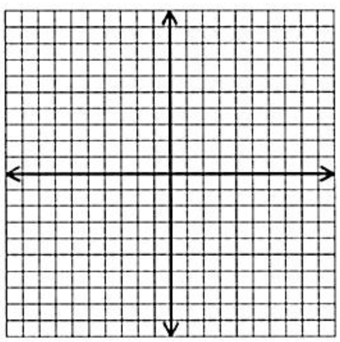
shifted x^2 graph 2 units right

4. **Graph** $y = x^2 + 4$ **and describe how it is related to** $y = x^2$.



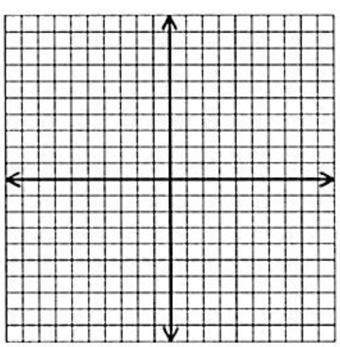
shifts x^2 ↑ 4 units

5. **Graph** $y = x^2 - 2$ **and describe how it is related to** $y = x^2$.



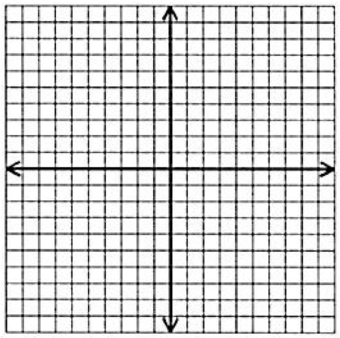
shifts x^2 graph ↓ 2 units

6. **Graph** $y = -x^2$ **and describe how it is related to** $y = x^2$.



reflects x^2 graph over the x-axis

7. **Graph $y = (-x)^2$ and describe how it is related to $y = x^2$.**



reflects x^2 graph over the y-axis

Use what you have discovered in questions 1 – 7 to fill in the following blanks:

- $f(x) + a$ is $f(x)$ shifted $\underline{\quad \uparrow \quad}$ a units
- $f(x) - a$ is $f(x)$ shifted $\underline{\quad \downarrow \quad}$ a units
- $f(x + a)$ is $f(x)$ shifted ___left___ a units
- $f(x - a)$ is $f(x)$ shifted ___right___ a units
- $-f(x)$ is $f(x)$ reflected over the ___x-axis___
- $f(-x)$ is $f(x)$ reflected over the ___y-axis___

Quarter Exam Review Sheet Key (Q1)

1. $f(x) = x^3 - 3x + 6$

(a) $f(-2) = (-2)^3 - 3(-2) + 6 = -8 + 6 + 6 = 4$

(b) $f(x-2) = (x-2)^3 - 3(x-2) + 6$

$(x-2)(x-2)(x-2) - 3x + 6 + 6$

$(x^2 - 4x + 4)(x-2) - 3x + 6 + 6$

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 - 3x + 12$

$x^3 - 6x^2 + 9x + 4$

2. (a) $(g \circ f)(x$

$g(x-1) = (x-1)^2 = x^2 - 2x + 1$

(b) $(f \circ h \circ g)$

$f(h(x^2)$

$f(\sqrt{x^2 - 25}) = \sqrt{x^2-25} - 1$

(c) $\dfrac{(x+h)^2 - 5(x+h) + 4 - (x^2 - 5x + 4)}{h}$ $h \neq 0$

$\dfrac{\cancel{x^2} + 2xh + h^2 \cancel{-5x} - 5h \cancel{+4} \cancel{-x^2} \cancel{+5x} \cancel{-4}}{h}$

$\dfrac{2xh + h^2 - 5h}{h} = 2x + h - 5$

(3) $f(x) = 3x - 8$

$y = 3x - 8$

$x = 3y - 8$

$x + 8 = 3y$

$\frac{x+8}{3} = y$

$\frac{x+8}{3} = f^{-1}(x)$

(4) $f(x) = \sqrt{3x-6}$

$y = \sqrt{3x-6}$

$x = \sqrt{3y-6}$

$x^2 = 3y - 6$

$\frac{x^2+6}{3} = \frac{3y}{3}$

$y = \frac{x^2+6}{3}$

(5) (a) $m = \frac{7-(-4)}{-2-4} = \frac{11}{-6}$

(b) $-7x + 4y = 12$

$4y = 7x + 12$

$y = \frac{7}{4}x + 3$

$m = 7/4$

(c) $y + 5 = 3(x-3)$

$y + 5 = 3x - 9$

$y = 3x - 14$

$m = 3$

slope of line $\perp$ is $-1/3$

(d) $y = 2x - 5$

$m = 2$

slope of $\parallel$ line is 2

(6) (a) $y + 4 = 3(x-1)$ point slope

$y + 4 = 3x - 3$

$y = 3x - 7$ slope intercept form

$0 = 3x - y - 7$ standard form

(b) $m = \frac{7-(-4)}{-2-4} = \frac{11}{-6}$

point slope: $y-7 = -\frac{11}{6}(x+2)$ or $y+4 = -11/6\,(x-4)$

$y-7 = -11/6\,x - \frac{22}{6}$

$y = -11/6\,x - 22/6 + 7$

slope intercept: $y = -11/6\,x + \frac{10}{3}$

$6\left(y = -\frac{11}{6}x + \frac{10}{3}\right)$

$6y = -11x + 20$

standard form: $11x + 6y - 20 = 0$

(7) (a) $7x-2$

$g(x) = 7x$

$f(x) = x-2$

$f(g(x))$

(b) $\frac{12}{\sqrt{x+12}}$

$h(x) = x+12$

$g(x) = \sqrt{x}$

$f(x) = 12/x$

$f(g(h(x)))$

(8) $f(g(x)) = g(f(x)) = x$

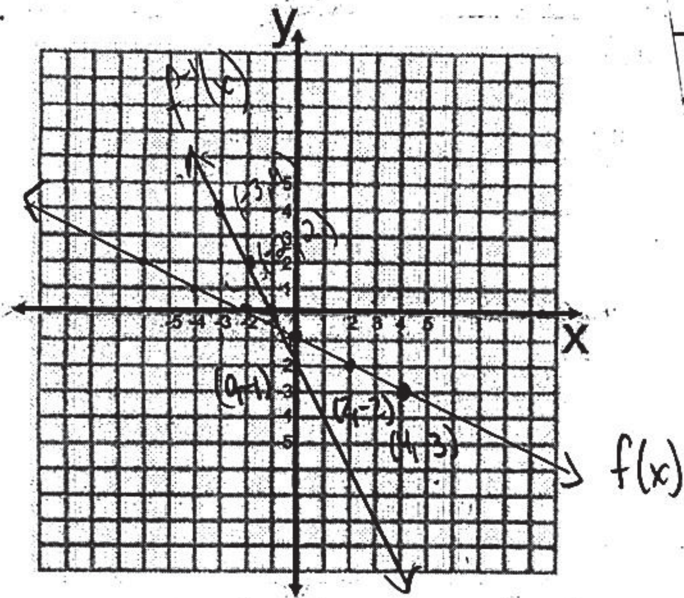
$f(x^2+5) = g(\sqrt{x-5})$

$\sqrt{x^2+5-5} = (\sqrt{x-5})^2 + 5$

$\sqrt{x^2} = x-5+5$

$x = x$

(9)



(10) $\dfrac{5k^2-20k}{12+5k-2k^2} = \dfrac{5k\cancel{(k-4)}}{-(2k+3)\cancel{(k-4)}}$

$-2k^2+5k+12$

$-(2k^2-5k-12)$ $ac=-24$, $b=-5$

$-(2k^2-8k+3k-12)$

$-(2k(k-4)\ 3(k-4))$

$-(2k+3)(k-4)$

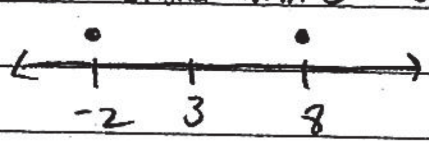
$\dfrac{5k}{-(2k+3)}$ $k \neq 4, -3/2$

(b) $\dfrac{5h^{\cancel{3}\,2}}{h^2-h} = \dfrac{5h^2}{h-1}$ $h \neq 0, 1$

$\cancel{h}(h-1)$

(11) (a) $|x-3| = 5$

x's distance from 3 = 5



$\{-2, 8\}$

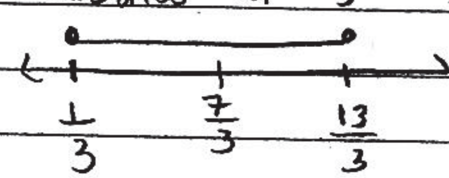
(b) $|7-3x| \le 6$

$|3x-7| \le 6$

$3\,|x-\frac{7}{3}| \le 6$

$|x-7/3| \le \frac{6}{3}$

x's distance from $7/3 \le 6/3$



$\left[\frac{1}{3}, \frac{13}{3}\right]$

12 (a)
$$\frac{1-\frac{1}{1-x}}{4+\frac{3}{x^2-1}} \qquad x \neq \pm 1, \pm \tfrac{1}{2}$$

$$\frac{x^2-1+x+1}{4x^2-4+3} = \frac{x^2+x}{4x^2-1} = \frac{x(x+1)}{(2x-1)(2x+1)}$$

(b)
$$\frac{4-x^{-2}}{2x^{-1}-x^{-2}} = \frac{4-\frac{1}{x^2}}{\frac{2}{x}-\frac{1}{x^2}} = \frac{\frac{\cancel{(2x-1)}(2x+1)}{4x^2-1}}{\cancel{2x-1}} \qquad x \neq 0, \tfrac{1}{2}$$

$$= 2x+1$$

(c)
$$\frac{y}{2+y} + \frac{2}{3-y} - \frac{3y+1}{y^2-y-6} \qquad y \neq 3, -2$$

$$(y-3)(y+2)$$

$$\frac{y^2-3y-2y-4-3y-1}{(y-3)(y+2)} = \frac{y^2-8y-5}{(y-3)(y+2)}$$

(d)
$$\frac{a^2-ab}{ab+2b^3} \div \frac{a^2+ab}{ab+b^2}$$

$$a \neq -2b^2, -b, 0$$

$$b \neq 0$$

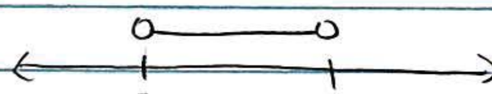
$$\frac{\cancel{a}(a-b)}{\cancel{b}(a+2b^2)} \cdot \frac{\cancel{b(a+b)}}{\cancel{a(a+b)}} = \frac{a-b}{a+2b^2}$$

(e) $\dfrac{3x^2+14x-5}{2x^2-9x-5} \cdot \dfrac{2x^2-5x-3}{3x^2-10x+3}$ $\quad x \neq -\frac{1}{2}, 5, \frac{1}{3}, 3$

$$\frac{\cancel{(3x-1)}(x+5)}{\cancel{(2x+1)}(x-5)} \cdot \frac{\cancel{(2x+1)}\cancel{(x-3)}}{\cancel{(3x-1)}\cancel{(x-3)}} = \frac{x+5}{x-5}$$

13 (a) $x^2+9x+14<0$

$(x+7)(x+2)<0$



		-7		-2	
x+7	−		−		+
x+2	−		+		+
P	+		−		+

(a) $\{x \mid -7<x<-2\}$
(b) $(-7,-2)$

(b) $\dfrac{3x}{4} \leq \dfrac{3x-6}{8}$

$\dfrac{3x(2)}{4(2)} - \dfrac{3x-6}{8} \leq 0$

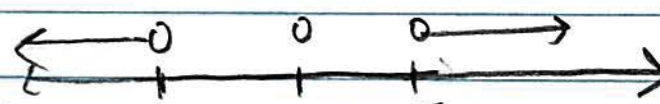
$\dfrac{6x-3x+6}{8} \leq 0$

$\dfrac{3x+6}{8} \leq 0$

		-2	
3x+6	−		+
8	+		+
Q	−		+

(a) $\{x \mid x \leq -2\}$
(b) $(-\infty, -2]$

(c) $\dfrac{\overset{(x-5)(x+2)}{x^2-3x-10}}{(x-1)^2} > 0$



		-2		1		5	
(x-5)(x+2)	+		−		−		+
$(x-1)^2$	+		+		+		+
Q	+		−		−		+

(a) $\{x \mid x<-2 \lor x>5\}$
(b) $(-\infty,-2) \cup (5,\infty)$

14 (a) $28x^3 - 49x^2 + 21x$

$7x(4x^2 - 7x + 3)$ $ac = 12$ $b = -7$

$7x(4x^2 - 4x - 3x + 3)$

$7x(4x(x-1) - 3(x-1))$

$7x(4x-3)(x-1)$

(b) $2x^3 + 3x^2 - 2x - 3$

$x^2(2x+3) - 1(2x+3)$

$(x^2-1)(2x+3)$

$(x-1)(x+1)(2x+3)$

(c) $75x^2 - 3$

$3(25x^2 - 1)$

$3(5x-1)(5x+1)$

(d) $8x^3 + 27$

$(2x+3)(4x^2 - 6x + 9)$

(e) $x^6 - 64y^3$

$(x^2 - 4y)(x^4 + 4x^2y + 16y^2)$

(f) $x^4 - 6x^2 - 27$

$(x^2 - 9)(x^2 + 3)$

$(x-3)(x+3)(x^2+3)$

Name:______________________________ Date:___________

PC: Transformations Ms. Loughran

Given each original function, ***describe*** each transformation in terms of the original function.

1. $y = x^2$
   (a) $y = x^2 - 2$
   (b) $y = (x-2)^2$
   (c) $y = x^2 + 2$
   (d) $y = (x+2)^2$
   (e) $y = (-x)^2$
   (f) $y = -x^2$
   (g) $y = -(x+1)^2$
   (h) $y = (x-1)^2 + 3$
   (i) $y = (x+3)^2 - 1$
   (j) $y = 2 - (x-4)^2$

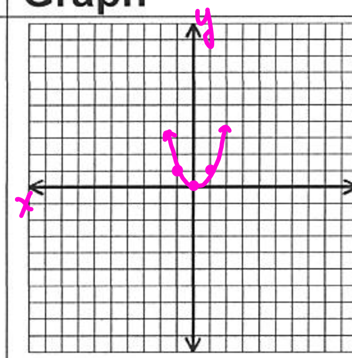
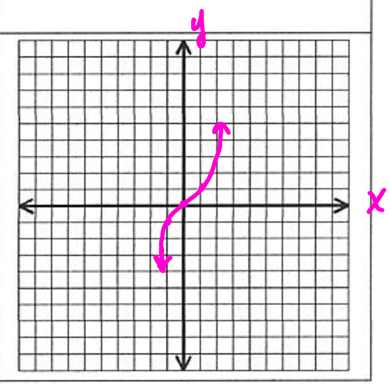
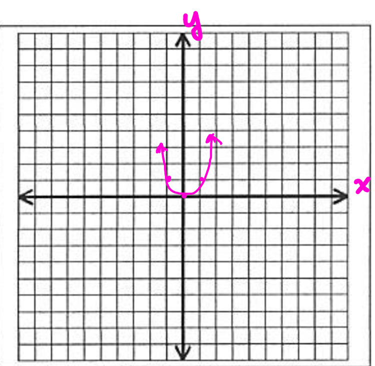
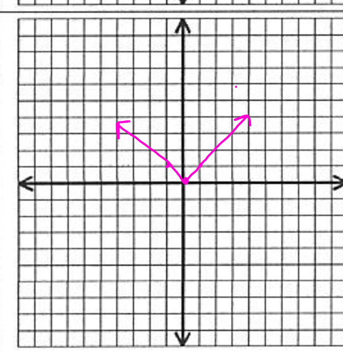
(j) $y = -(x-4)^2 + 2$

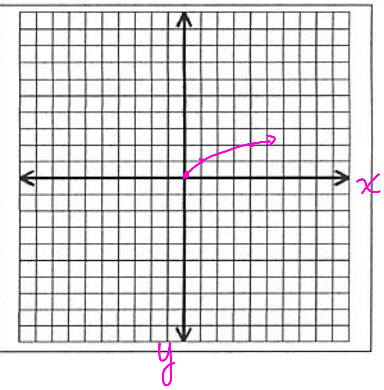
2. $y = |x|$
   (a) $y = |x| - 2$ ↓2
   (b) $y = |x-2|$ right 2
   (c) $y = |x| + 2$ ↑2
   (d) $y = |x+2|$ left 2
   (e) $y = -|x|$ reflect over x-axis
   (f) $y = -|x+1|$ left 1 unit, reflect over x-axis
   (g) $y = -|x| + 1$ reflect over x-axis, ↑1
   (h) $y = |x+3| - 2$ left 3, ↓2
   (i) $y = -|x| - 2$ reflect over x-axis ↓2
   (j) $y = -|x-1| + 3$ right one, reflect over x-axis ↑3

3. $y = \sqrt{x}$
   (a) $y = \sqrt{x-1}$
   (b) $y = \sqrt{x} + 2$
   (c) $y = \sqrt{x+2}$
   (d) $y = -\sqrt{x}$
   (e) $y = -\sqrt{x+1}$
   (f) $y = \sqrt{x} - 3$
   (g) $y = -\sqrt{x} + 2$
   (h) $y = -\sqrt{x-3} + 1$
   (i) $y = -4 - \sqrt{x}$
   (j) $y = \sqrt{x-1} + 2$

4. $y = x^3$
   (a) $y = (x-1)^3$
   (b) $y = x^3 - 4$
   (c) $y = -x^3$
   (d) $y = -(x+2)^3$
   (e) $y = (-x)^3$
   (f) $y = 2 + x^3$
   (g) $y = -4 - x^3$

Parent Functions

Function	Domain and Range	Key Points	Graph
Quadratic Equation: $f(x) = x^2$	D: $(-\infty, \infty)$ R: $[0, \infty)$	$(-1, 1)$ $(0, 0)$ $(1, 1)$	
Cubic Equation: $f(x) = x^3$	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	$(-1, -1)$ $(0, 0)$ $(1, 1)$	
Quartic Equation: $f(x) = x^4$	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	$(-1, 1)$ $(0, 0)$ $(1, 1)$	
Absolute Value Equation: $f(x) = \|x\|$	D $(-\infty, \infty)$ R: $[0, \infty)$	$(-1, 1)$ $(0, 0)$ $(1, 1)$	

Square Root Equation: $f(x)=\sqrt{x}$	D: $[0,\infty)$ R: $[0,\infty)$	$(0,0)$ $(1,1)$ $(4,2)$	

Transformation Rules

- $f(x) + a$ is $f(x)$ shifted upward a units
- $f(x) - a$ is $f(x)$ shifted downward a units
- $f(x + a)$ is $f(x)$ shifted left a units
- $f(x - a)$ is $f(x)$ shifted right a units
- $-f(x)$ is $f(x)$ flipped upside down ("reflected about the x-axis")
- $f(-x)$ is the mirror of $f(x)$ ("reflected about the y-axis")

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$$f(x) = \begin{cases} -2 & x \neq 5 \;\; \begin{matrix} \nearrow x<5 \\ \searrow x>5 \end{matrix} \\ -3 & x = 5 \end{cases}$$