Name:_____ Date:_____

PC : More Transformations Ms. Loughran

Do Now:

Let $y = f(x)$ be given by the graph below. For each of the following functions, choose the letter (a) – (i) corresponding to the graph.

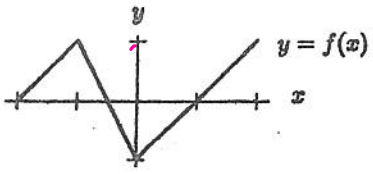
(1) $y = 2f(x)$ vertical stretch by a factor of 2 (i)

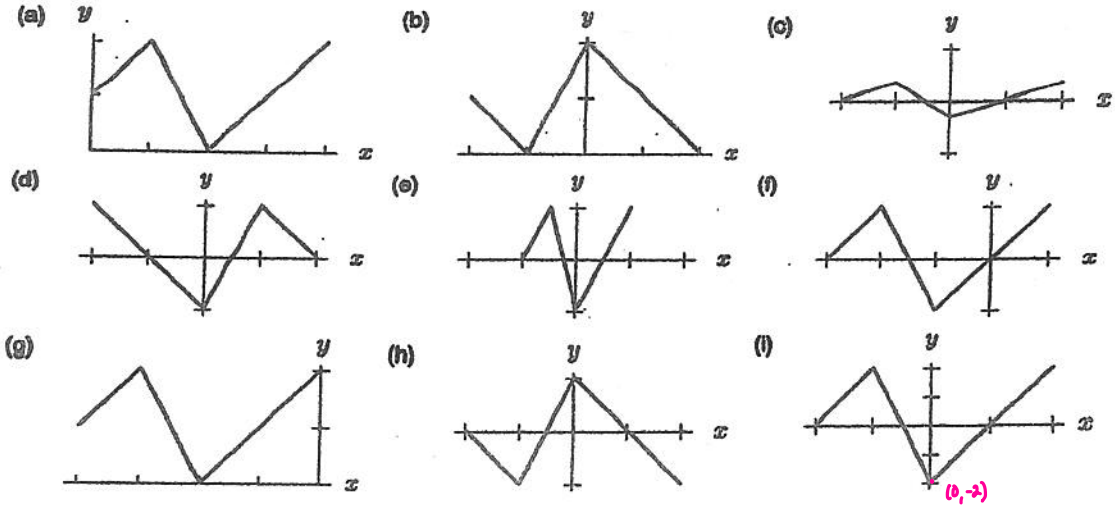
(2) $y = \frac{1}{3}f(x)$ vertical shrink by a factor of $\frac{1}{3}$ (c)

(3) $y = -f(x)+1$ reflection over the x-axis ↑1 (b)

(4) $y = f(x+2)+1$ left 2 ↑1 (g)

(5) $y = f(-x)$ reflection over the y-axis (d)





Name:_____ Date:_____

PC: Even and Odd Functions

Let f be a function.

f is even if $f(-x) = f(x)$ for all x in the domain of f

f is odd if $f(-x) = -f(x)$ for all x in the domain of f

The graph of an even function is symmetric with respect to the y-axis.

The graph of an odd function is symmetric with respect to the origin.

Examples:

Determine whether the functions are even, odd, or neither even nor odd.

1. $f(x) = x^5 + x$

$f(-x) = (-x)^5 + (-x)$

$f(-x) = -x^5 - x$ ← compare

Since $f(-x) = -f(x)$

ODD

2. $g(x) = 1 - x^4$

$g(-x) = 1 - (-x)^4$

$g(-x) = 1 - x^4$ ← compare

Since $g(-x) = g(x)$

EVEN

3. $h(x) = 2x - x^2$

$h(-x) = 2(-x) - (-x)^2$

$h(-x) = -2x - x^2$ ← compare

Since $h(-x) = h(x)$ or $-h(x)$

NEITHER

4. $f(x) = 3x^3 + 2x^2 + 1$

$f(-x) = 3(-x)^3 + 2(-x)^2 + 1$

$f(-x) = -3x^3 + 2x^2 + 1$ ← compare

Since $f(-x) \neq f(x)$ or $-f(x)$

NEITHER

5. $g(x) = x + \frac{1}{x}$

$g(-x) = (-x) + \frac{1}{-x}$

$g(-x) = -x - \frac{1}{x}$ ← compare

Since $g(-x) = -g(x)$

ODD

6. $h(x) = x^4 - 4x^2$

$h(-x) = (-x)^4 - 4(-x)^2$

$h(-x) = x^4 - 4x^2$ ← compare

Since $h(-x) = h(x)$

EVEN

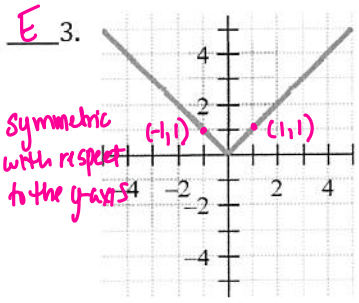
Exercises

1. If a function is even, its graph is symmetric with respect to the y-axis.
   This also means that $f(-x) = f(x)$
   If a function contains the points (x, y) and $(-x, y)$, it is even.

2. If a function is odd, its graph is symmetric with respect to the origin.
   This also means that $f(-x) = -f(x)$
   If a function contains the points (x, y) and $(-x, -y)$, it is odd.

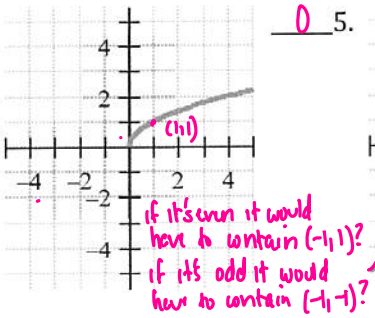
Determine whether each function graphed is even, odd, or neither

E 3.



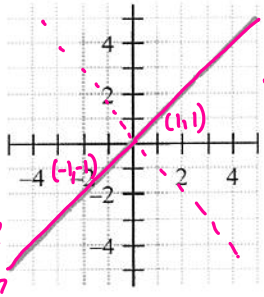
symmetric with respect to the y-axis

N 4.



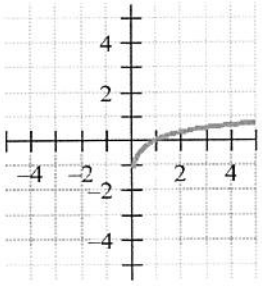
if it's even it would have to contain (-1,1)?
if it's odd it would have to contain (-1,-1)?

O 5.

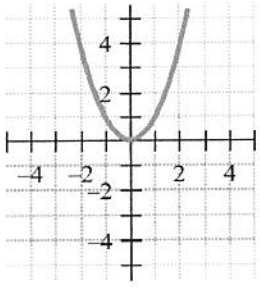


Symmetric with respect to the origin

N 6.



E 7.



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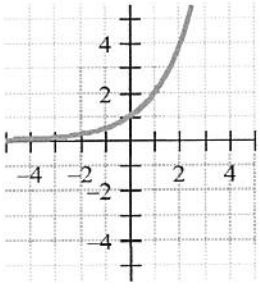
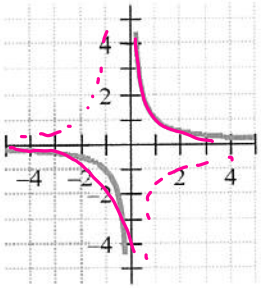
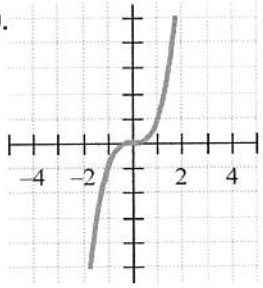


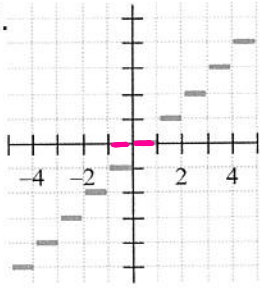
O 9.



O 10.



N 11.



Determine algebraically whether each of the following functions is even, odd or neither.

12. $f(x) = 4x + 5$

$f(-x) = 4(-x) + 5$

$f(-x) = -4x + 5$

NEITHER

13. $f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

$f(-x) = -x^3 + x$

ODD

14. $f(x)=x^2-6$

$f(-x)=(-x)^2-6$

$f(-x)=x^2-6$

EVEN

15. $f(x)=x^3-x-2$

$f(-x)=(-x)^3-(-x)-2$

$f(-x)=-x^3+x-2$

NEITHER

16. $f(x)=\dfrac{x^4-x}{x^5-x}$

$f(-x)=\dfrac{(-x)^4-(-x)}{(-x)^5-(-x)}$

$f(-x)=\dfrac{x^4+x}{-x^5+x}$ neither

If the numerator is multiplied by -1 and the denominator is multiplied by -1, then the function even.
If just the numerator or just the denominator is multiplied by -1, then the function is odd.

17. $f(x)=\dfrac{x^3-x}{x^5}$

$f(-x)=\dfrac{(-x)^3-(-x)}{(-x)^5}$

$f(-x)=\dfrac{-x^3+x}{-x^5}=\dfrac{+(x^3-x)}{+x^5}$

EVEN

18. $f(x)=(x-4)^2$ $= x^2-8x+16$

$f(-x)=(-x-4)^2=x^2+8x+16$

NEITHER

19. $f(x)=x^4-x^2+4$

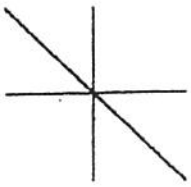
$f(-x)=(-x)^4-(-x)^2+4$

$f(-x)=x^4-x^2+4$

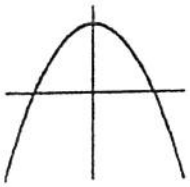
EVEN

More Practice

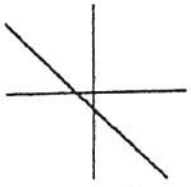
1. Indicate which of the following functions are even, which are odd, and which are neither.



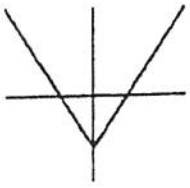
Graph (a)



Graph (b)



Graph (c)



Graph (d)

2. Algebraically, determine whether each function is odd, even, or neither.

a) $f(x)=3x^4-5x^2+17$

b) $f(x)=|x|$

c) $f(x)=12x^7+6x^3-2x$

d) $f(x)=4x^3-7$

e) $f(x)=x^2+2x+2$

f) $f(x)=\dfrac{x^2-5}{2x^3+x}$