Name:________________________________ Date:_______
PC: Remainder Theorem and Factor Theorem Ms. Loughran

Do now:

1. Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$.
   (a) Find the quotient and remainder when $P(x)$ is divided by $x+2$.
   (b) Find $P(-2)$.

a)
$$\begin{array}{r|rrrrrr} -2 & 3 & 5 & -4 & 0 & 7 & 3 \\ & & -6 & 2 & 4 & -8 & 2 \\ \hline & 3 & -1 & -2 & 4 & -1 & 5 \end{array}$$

Quotient: $3x^4 - x^3 - 2x^2 + 4x - 1$
Remainder: 5

b) $P(-2) = 3(-2)^5 + 5(-2)^4 - 4(-2)^3 + 7(-2) + 3$
$P(-2) = 5$

Remainder Theorem:
If the polynomial $P(x)$ is divided by $x-c$, then the remainder is the value $P(c)$.

1. Let $P(x) = x^3 - 2x^2 + 3x - 1$. Find $P(3)$ using 2 different methods.

$P(3) = 3^3 - 2(3)^2 + 3(3) - 1 = 17$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & 3 & -1 \\ & & 3 & 3 & 18 \\ \hline & 1 & 1 & 6 & 17 \end{array}$$

remainder

$P(3) = 17$

You could also do long division, dividing $x-3$ and look for the remainder.

Factor Theorem:
A polynomial $P(x)$ has a factor of $x-c$ if and only if $P(c) = 0$.

* when the remainder is zero when plugging in c, then $x-c$ is a factor. *

2. Show that $x-2$ is a factor of $P(x) = x^3 - 3x^2 + 7x - 10$.

$P(2) = 2^3 - 3(2)^2 + 7(2) - 10 = 0$

or

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 7 & -10 \\ & & 2 & -2 & 10 \\ \hline & 1 & -1 & 5 & 0 \end{array}$$

a remainder of 0 means that $x-2$ is a factor of $P(x)$

or

You could use LD to show that the remainder is 0.

3. (a) Use the factor theorem to show that $x+3$ is a factor of $P(x) = x^3 - x^2 - 8x + 12$.
(b) Factor $P(x)$ completely.

a) $P(-3) = (-3)^3 - (-3)^2 - 8(-3) + 12 = 0$

b)
$$\begin{array}{r|rrrr} -3 & 1 & -1 & -8 & 12 \\ & & -3 & 12 & -12 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$(x+3)(x^2-4x+4)$

$(x+3)(x-2)(x-2)$ or $(x+3)(x-2)^2$

4. Let $P(x) = x^3 - 7x + 6$.
(a) Show that $P(1) = 0$.
(b) Factor $P(x)$ completely.

a)
$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

Since the remainder is 0
$P(1) = 0$

b) $(x-1)(x^2+x-6)$

$(x-1)(x+3)(x-2)$

5. Find a polynomial of degree 4 that has zeros $-3, 0, 1,$ and 5.

$P(x) = x(x+3)(x-1)(x-5)$ factored form

If the question asks for the polynomial in expanded form, you have to multiply it out.

$P(x) = x(x+3)(x-1)(x-5)$

$P(x) = (x^2+3x)(x^2-6x+5)$

$P(x) = x^4 - 6x^3 + 5x^2 + 3x^3 - 18x^2 + 15x$

$P(x) = x^4 - 3x^3 - 13x^2 + 15x$

Practice Section A

1–6 ▪ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express P in the form $P(x) = D(x) \cdot Q(x) + R(x)$.

1. $P(x) = 3x^2 + 5x - 4, \quad D(x) = x + 3$
2. $P(x) = x^3 + 4x^2 - 6x + 1, \quad D(x) = x - 1$
3. $P(x) = 2x^3 - 3x^2 - 2x, \quad D(x) = 2x - 3$
4. $P(x) = 4x^3 + 7x + 9, \quad D(x) = 2x + 1$
5. $P(x) = x^4 - x^3 + 4x + 2, \quad D(x) = x^2 + 3$
6. $P(x) = 2x^5 + 4x^4 - 4x^3 - x - 3, \quad D(x) = x^2 - 2$

7–12 ▪ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

7. $P(x) = x^2 + 4x - 8, \quad D(x) = x + 3$
8. $P(x) = x^3 + 6x + 5, \quad D(x) = x - 4$
9. $P(x) = 4x^2 - 3x - 7, \quad D(x) = 2x - 1$
10. $P(x) = 6x^3 + x^2 - 12x + 5, \quad D(x) = 3x - 4$
11. $P(x) = 2x^4 - x^3 + 9x^2, \quad D(x) = x^2 + 4$
12. $P(x) = x^5 + x^4 - 2x^3 + x + 1, \quad D(x) = x^2 + x - 1$

13–22 ▪ Find the quotient and remainder using long division.

13. $\dfrac{x^2 - 6x - 8}{x - 4}$
14. $\dfrac{x^3 - x^2 - 2x + 6}{x - 2}$
15. $\dfrac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$
16. $\dfrac{x^3 + 3x^2 + 4x + 3}{3x + 6}$
17. $\dfrac{x^3 + 6x + 3}{x^2 - 2x + 2}$
18. $\dfrac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$
19. $\dfrac{6x^3 + 2x^2 + 22x}{2x^2 + 5}$
20. $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$
21. $\dfrac{x^6 + x^4 + x^2 + 1}{x^2 + 1}$
22. $\dfrac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$

23–36 ▪ Find the quotient and remainder using synthetic division.

23. $\dfrac{x^2 - 5x + 4}{x - 3}$
24. $\dfrac{x^2 - 5x + 4}{x - 1}$
25. $\dfrac{3x^2 + 5x}{x - 6}$
26. $\dfrac{4x^2 - 3}{x + 5}$
27. $\dfrac{x^3 + 2x^2 + 2x + 1}{x + 2}$
28. $\dfrac{3x^3 - 12x^2 - 9x + 1}{x - 5}$
29. $\dfrac{x^3 - 8x + 2}{x + 3}$
30. $\dfrac{x^4 - x^3 + x^2 - x + 2}{x - 2}$