Do Now: #23 from Set B of the Remainder and Factor Theorems packet

Show that the given binomial $x - c$ is a factor of $p(x)$, and then factor $p(x)$ completely.

23. $p(x) = x^6 + 6x^5 + 8x^4 - 6x^3 - 9x^2$ $(x + 3)$

$p(x) = x^2(x^4 + 6x^3 + 8x^2 - 6x - 9)$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 8 & -6 & -9 \\ & & -3 & -9 & 3 & 9 \\ \hline & 1 & 3 & -1 & -3 & 0 \end{array}$$

$p(x) = x^2(x+3)(x^3 + 3x^2 - x - 3)$

$p(x) = x^2(x+3)\left(x^2(x+3) - 1(x+3)\right)$

$p(x) = x^2(x+3)(x^2 - 1)(x+3)$

$p(x) = x^2(x+3)^2(x+1)(x-1)$

Name:_____ Date:_____

PC: Polynomial Practice Ms. Loughran

1. If $f(3)=0$, then ___$x-3$___ is a factor of $f(x)$.

2. If $x+2$ is a factor of $f(x)$, then ___-2___ is a zero of $f(x)$.

3. If $f(x)=(x-2)(x+1)(3x-1)$, then the zeros of $f(x)$ are: ___$2, -1, \frac{1}{3}$___

4. If $f(5)=0$, then a factor of $f(x)$ is: ___$x-5$___

5. If $2x-3$ is a factor of $f(x)$, then $f($ ___$3/2$___ $)=0$

6. Show in 2 ways that $y-1$ is a factor of y^3-3y^2+3y-1.

7. Show 2 ways that $x-2$ is a factor of x^5-32.

8. Factors of x^3+x^2-4x-4 are $(x-2),(x+2)$ and $(x+1)$. What are the zeros of the polynomial? $\{\pm 2, -1\}$

9. Given the zeros of $x^3-6x^2+11x-6$ are 1, 2, and 3. What are the factors of the polynomial? Check by multiplication.

10. Show that -3 is a zero of $f(x)=x^3+7x^2+7x-15$.

11. Given that $(x-1)$ is a factor of $f(x)=3x^3-4x^2-9x+10$ find all zeros of $f(x)$.

12. One root of $x^3+8x^2+11x-20=0$ is -5. Find the complete solution set of this equation.

13. Show that $(x+1)$ is a factor of $x^3-2x^2+3=0$. Use this information to find the solution set of this equation.

6. Show in 2 ways that $y-1$ is a factor of y^3-3y^2+3y-1.

plug in 1 and show that you get 0 out

$(1)^3-3(1)+3(1)-1=0$

or

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

← remainder is 0 so $y-1$ is a factor

or

you would do LD to show that the remainder is 0.

7. Show 2 ways that $x-2$ is a factor of x^5-32.

plug in 2 for x and show we get zero

$2^5-32=0$

or

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

← remainder is 0 so $x-2$ is a factor

or

you could use LD to show that you get a remainder of 0

8. Factors of x^3+x^2-4x-4 are $(x-2),(x+2)$ and $(x+1)$. What are the zeros of the polynomial?

$\{\pm 2, -1\}$

9. Given the zeros of $x^3-6x^2+11x-6$ are 1, 2, and 3. What are the factors of the polynomial? Check by multiplication.

$(x-1)(x-2)(x-3)$

check:

$(x^2-3x+2)(x-3)$

$x^3-3x^2+2x-3x^2+9x-6$

$x^3-6x^2+11x-6$

10. Show that -3 is a zero of $f(x)=x^3+7x^2+7x-15$.

$f(-3)=(-3)^3+7(-3)^2+7(-3)-15=0$

or use SD or LD to show that the remainder is 0.

11. Given that $(x-1)$ is a factor of $f(x)=3x^3-4x^2-9x+10$ find all zeros of $f(x)$.

$$\begin{array}{r|cccc} 1 & 3 & -4 & -9 & 10 \\ & & 3 & -1 & -10 \\ \hline & 3 & -1 & -10 & 0 \end{array}$$

$$(x-1)(3x^2-x-10)=0$$

$$(x-1)(3x^2-6x+5x-10)=0$$

$$(x-1)(3x(x-2)+5(x-2))=0$$

$$(x-1)(x-2)(3x+5)=0$$

$$x=1,\ 2,\ -\frac{5}{3}$$

12. One root of $x^3+8x^2+11x-20=0$ is -5. Find the complete solution set of this equation.

$$\begin{array}{r|rrr} -5 & 1 & 8 & 11 & -20 \\ & & -5 & -15 & 20 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$$(x+5)(x^2+3x-4)=0$$

$$(x+5)(x+4)(x-1)=0$$

$$\{-5,-4,1\}$$

13. Show that $(x+1)$ is a factor of $x^3-2x^2+3=0$. Use this information to find the solution set of this equation.

$$\begin{array}{r|rrr} -1 & 1 & -2 & 0 & 3 \\ & & -1 & 3 & -3 \\ \hline & 1 & -3 & 3 & 0 \end{array}$$

Quad Form: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$(x^2-3x+3)(x+1)=0$$

$x^2-3x+3=0$ | $x=-1$

$$x=\frac{3\pm\sqrt{(-3)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{-3}}{2}$$

$$x=\frac{3\pm i\sqrt{3}}{2}$$

$$\left\{-1,\ \frac{3\pm i\sqrt{3}}{2}\right\}$$

$\rightarrow (4x+1)$

14. One zero of $4x^3 - 11x^2 + 5x + 2$ is $-\frac{1}{4}$. Find the complete **factorization** of this polynomial and find the remaining zeros. (**THE COMPLETE FACTORIZATION OF A POLYNOMIAL WILL INCLUDE FACTORS WITH ONLY INTEGRAL COEFFICIENTS.**)

$$\begin{array}{r|cccc} -\frac{1}{4} & 4 & -11 & 5 & 2 \\ & & -1 & 3 & -2 \\ \hline & 4 & -12 & 8 & 0 \end{array}$$

$\div 4$

$$(4x+1)(x^2-3x+2)$$

complete factorization: $(4x+1)(x-2)(x-1)$

$$(4x+1)(x-2)(x-1) = 0$$

remaining zeros: $\{2, 1\}$