Name:______________________________

PC: Factoring and Solving Higher Degree Polynomials

Date:______________

Ms. Loughran

Find the complete factorization (CF) and all zeros (Z) of the following polynomials using the information given.

1. $P(x) = x^4 + 6x^3 + 7x^2 - 12x - 18$; zero: -3 (a double zero)

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 7 & -12 & -18 \\ & & -3 & -9 & 6 & 18 \\ \hline -3 & 1 & 3 & -2 & -6 & 0 \\ & & -3 & 0 & 6 & \\ \hline & 1 & 0 & -2 & 0 & \end{array}$$

CF: $P(x) = (x+3)^2(x^2-2)$

$(x+3)^2(x^2-2) = 0$

$x = -3$ (double) | $x^2 - 2 = 0$, $x^2 = 2$, $x = \pm\sqrt{2}$

Zeros: $\{-3 \text{ (double)}, \pm\sqrt{2}\}$

2. $P(x) = x^4 - x^3 - 5x^2 - x - 6$; zeros: 3, -2

$$\begin{array}{r|rrrrr} 3 & 1 & -1 & -5 & -1 & -6 \\ & & 3 & 6 & 3 & 6 \\ \hline -2 & 1 & 2 & 1 & 2 & 0 \\ & & -2 & 0 & -2 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

CF: $P(x) = (x-3)(x+2)(x^2+1)$

Zeros: $\{3, -2, \pm i\}$

$0 = (x-3)(x+2)(x^2+1)$

$x = 3$ | $x = -2$ | $x^2 + 1 = 0$, $x^2 = -1$, $x = \pm\sqrt{-1}$, $x = \pm i$

3. $P(x) = x^4 - 5x^3 + 3x^2 + 15x - 18$; zeros: 3, 2

$$
\begin{array}{r|rrrrr}
3 & 1 & -5 & 3 & 15 & -18 \\
 & & 3 & -6 & -9 & 18 \\
\hline
2 & 1 & -2 & -3 & 6 & 0 \\
 & & 2 & 0 & -6 & \\
\hline
 & 1 & 0 & -3 & 0 &
\end{array}
$$

CF: $P(x) = (x-3)(x-2)(x^2-3)$

$0 = (x-3)(x-2)(x^2-3)$

$x = 3$ | $x = 2$ | $x^2 - 3 = 0$, $x^2 = 3$, $x = \pm\sqrt{3}$

Zeros: $\{3, 2, \pm\sqrt{3}\}$

4. $P(x) = x^4 + 4x^3 - 7x^2 - 36x - 18$; zeros: ± 3

$$
\begin{array}{r|rrrrr}
3 & 1 & 4 & -7 & -36 & -18 \\
 & & 3 & 21 & 42 & 18 \\
\hline
-3 & 1 & 7 & 14 & 6 & 0 \\
 & & -3 & -12 & -6 & \\
\hline
 & 1 & 4 & 2 & 0 &
\end{array}
$$

CF: $P(x) = (x-3)(x+3)(x^2+4x+2)$

$0 = (x-3)(x+3)(x^2+4x+2)$ — * Could solve using the QF or by completing the square

$x = 3$ | $x = -3$ | $x^2 + 4x + 2 = 0$

$x^2 + 4x + 4 = -2 + 4$

$(x+2)^2 = 2$

$x + 2 = \pm\sqrt{2}$

$x = -2 \pm \sqrt{2}$

Zeros: $\{\pm 3, -2 \pm \sqrt{2}\}$

5. $P(x)=x^4+3x^3+3x^2+x$; zero: -1

$P(x)=x(x^3+3x^2+3x+1)$

-1	1	3	3	1
		-1	-2	-1
	1	2	1	0

zeros: $\{0, -1 \text{ (triple)}\}$

$P(x)=x(x+1)(x^2+2x+1)$

CF: $P(x)=x(x+1)(x+1)(x+1)=x(x+1)^3$

$0=x(x+1)^3$

$x=0$ | $x=-1$ (triple) or (multiplicity of 3)

6. $P(x)=x^4+6x^3+2x^2-18x-15$; zeros: $-1, -5$

-1	1	6	2	-18	-15
		-1	-5	3	15
-5	1	5	-3	-15	0
		-5	0	15	
	1	0	-3	0	

CF: $P(x)=(x+1)(x+5)(x^2-3)$

zeros: $\{-1, -5, \pm\sqrt{3}\}$

$0=(x+1)(x+5)(x^2-3)$

$x=-1$ | $x=-5$ | $x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}$

7. $P(x)=x^4+2x^3-7x^2-18x-18$; zeros: ± 3

3	1	2	-7	-18	-18
		3	15	24	18
-3	1	5	8	6	0
		-3	-6	-6	
	1	2	2	0	

CF: $P(x)=(x-3)(x+3)(x^2+2x+2)$

$0=(x-3)(x+3)(x^2+2x+2)$

zeros: $\{\pm 3, -1\pm i\}$

$x=3$ | $x=-3$ | $x^2+2x+2=0$

$x^2+2x+1=-2+1$

$(x+1)^2=-1$

$x+1=\pm\sqrt{-1}$

$x+1=\pm i$

$x=-1\pm i$

8. $P(x)=-x^5+5x^4-3x^3-15x^2+18x$; zeros: 3, 2

$P(x)=-x(x^4-5x^3+3x^2+15x-18)$

3	1	-5	+3	+15	-18
		3	-6	-9	18
2	1	-2	-3	6	0
		2	0	-6	
	1	0	-3	0	

CF: $P(x)=-x(x-3)(x-2)(x^2-3)$

zeros: $\{0, 3, 2, \pm\sqrt{3}\}$

$0=-x(x-3)(x-2)(x^2-3)$

$x=0$ | $x=3$ | $x=2$ | $x^2=3$, $x=\pm\sqrt{3}$

9. $P(x)=3x^4-11x^3-3x^2-6x+8$; zeros: $4, \frac{2}{3}$

4	3	-11	-3	-6	8
		12	4	4	-8
$\frac{2}{3}$	3	1	1	-2	0
		2	2	2	
	3	3	3	0	

$\div 3$

factor is $3x-2$

Zeros: $\left\{4, \frac{2}{3}, \frac{-1 \pm i\sqrt{3}}{2}\right\}$

CF: $P(x)=(x-4)(3x-2)(x^2+x+1)$

$0=(x-4)(3x-2)(x^2+x+1)$

$x=4 \quad | \quad x=\frac{2}{3} \quad | \quad x^2+x+1=0$

$x=\frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2(1)}$

$x=\frac{-1 \pm \sqrt{-3}}{2}=\frac{-1 \pm i\sqrt{3}}{2}$

10. $P(x)=2x^5-5x^4+x^3+4x^2-4x$; zeros: $2, -1$

$P(x)=x(2x^4-5x^3+x^2+4x-4)$

2	2	-5	1	4	-4
		4	-2	-2	4
-1	2	-1	-1	2	0
		-2	3	-2	
	2	-3	2	0	

Zeros: $\left\{0, 2, -1, \frac{3 \pm i\sqrt{7}}{4}\right\}$

CF: $P(x)=x(x-2)(x+1)(2x^2-3x+2)$

$0=x(x-2)(x+1)(2x^2-3x+2)$

$x=0 \quad | \quad x=2 \quad | \quad x=-1 \quad | \quad x=\frac{3 \pm \sqrt{9-4(2)(2)}}{2(2)}=\frac{3 \pm \sqrt{-7}}{4}=\frac{3 \pm i\sqrt{7}}{4}$

Set A

Homework 12-08

(46) $P(-3)=100$

-3	-2	7	40	0	-7	10	112
		6	-39	-3	9	-6	-12
	-2	13	1	-3	2	4	(100)

(58) $P(x)=x(x+2)(x-2)(x-4)$ or $x^4-4x^3-4x^2+16x$

(60) $P(x)=x(x+2)(x-1)(x+1)(x-2)$
or
x^5-5x^3+4x

(45) $P(3)=2159$

3	1	0	0	0	0	-3	0	-1
		3	9	27	81	243	720	2160
	1	3	9	27	81	240	720	(2159)

(57) $(x-1)(x+1)(x-3)=P(x)$
or
$x^3-3x^2-x+3=P(x)$

(59) $(x-1)(x+1)(x-3)(x-5)=P(x)$
$P(x)=x^4-8x^3+4x^2+8x-15$

(67) a) $6(-1) \quad -17(-1) \quad +12(-1)+26 = 3$

b) NO b/c $(1)^{567} - 3(1)^{400} + 1^{9} + 2 \neq 0$

Set B

(13)

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$(x+1)(x^2+5x+6)$

$(x+1)(x+3)(x+2)$

(17)

$$\begin{array}{r|rrrr} -2 & -1 & 0 & 7 & 6 \\ & & 2 & -4 & -6 \\ \hline & -1 & 2 & 3 & 0 \end{array}$$

$(x+2)(-x^2+2x+3)$

$-(x+2)(x^2-2x-3)$

$-(x+2)(x-3)(x+1)$

(19)

$$\begin{array}{r|rrrr} 5 & 6 & -25 & -29 & 20 \\ & & 30 & 25 & -20 \\ \hline & 6 & 5 & -4 & 0 \end{array}$$

$(x-5)(6x^2+5x-4)$

$(x-5)(6x^2+8x-3x-4)$

$(x-5)[2x(3x+4)-1(3x+4)]$

$(x-5)(2x-1)(3x+4)$

(20)

$$\begin{array}{r|rrrr} -2 & 12 & -22 & -100 & -16 \\ & & -24 & 92 & 16 \\ \hline & 12 & -46 & -8 & 0 \end{array}$$

$(x+2)(12x^2-46x-8)$

$(x+2)(2(6x^2-23x-4))$

$2(x+2)[6x^2-24x+x-4]$

$2(x+2)[6x(x-4)+1(x-4)]$

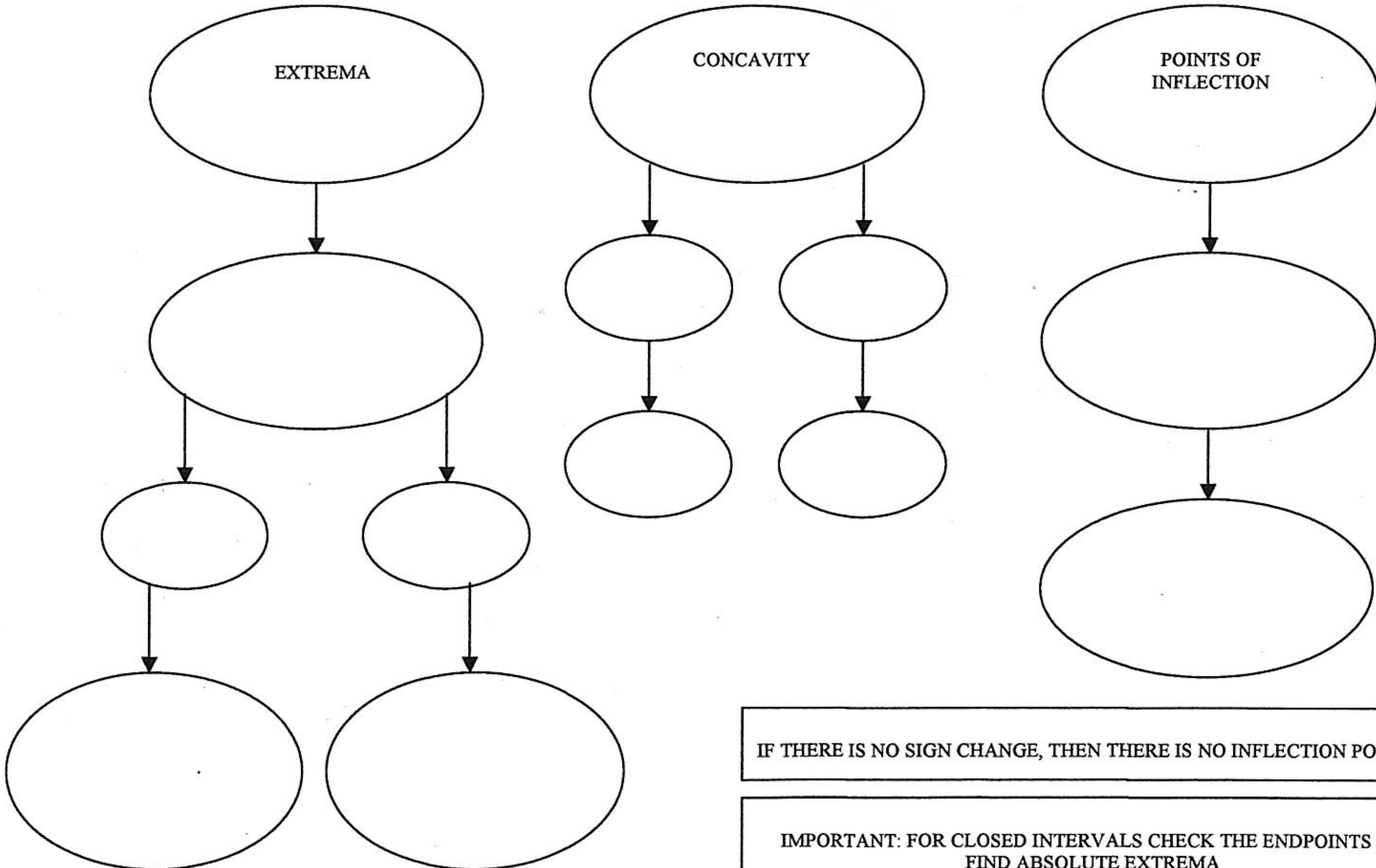
$\rightarrow 2(x+2)(x-4)(6x+1)$

Do Now:

Determine if each of the following statements is true or false. If you decide a statement is false, provide a counterexample to show why it is false and then rewrite the statement in order to make it true. Unless otherwise specified, assume each function is defined and continuous for all real numbers.

1. A critical point (or critical number) of a function f of a variable x is the x-coordinate of a relative maximum or minimum value of the function.
2. A continuous function on a closed interval can have only one maximum value.
3. If $f''(x)$ is always positive, then the function f must have a relative minimum value.
4. If a function f has a local minimum value at $x =$ c, then $f'(c) = 0$.

Connecting f' and f'' with the graph of f



IF THERE IS NO SIGN CHANGE, THEN THERE IS NO INFLECTION POINT.

IMPORTANT: FOR CLOSED INTERVALS CHECK THE ENDPOINTS TO FIND ABSOLUTE EXTREMA

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

---

2. $\int \frac{1}{x^2}\,dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

---

4. $\int(\sin(2x)+\cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim_{x\to 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) nonexistent

---

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

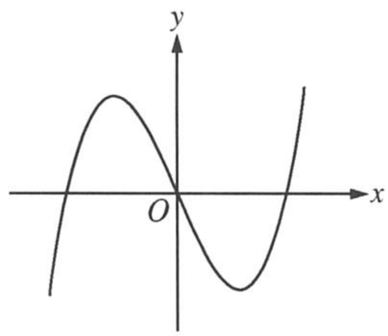
7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

---

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

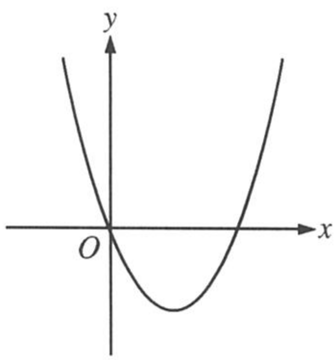
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



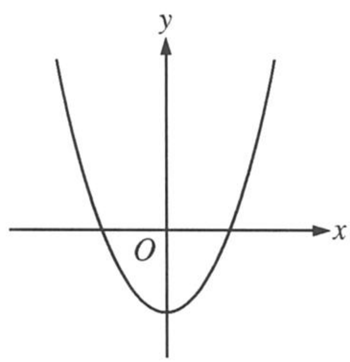
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

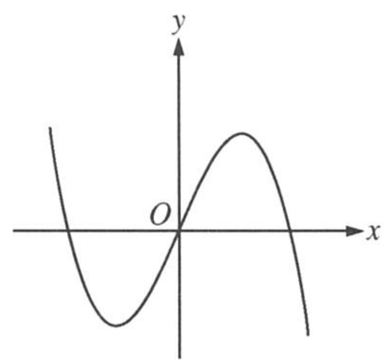
(A)



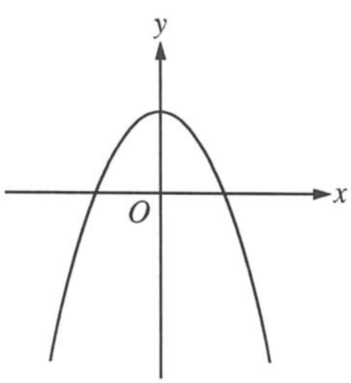
(B)



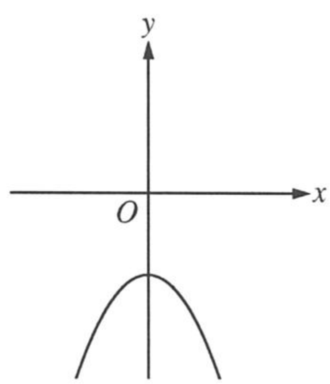
(C)



(D)



(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\dfrac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

---

13. If $f(x) = x^2 + 2x$, then $\dfrac{d}{dx}\left(f(\ln x)\right) =$

(A) $\dfrac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \dfrac{2}{x}$ (E) $\dfrac{2x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

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15. $\displaystyle\int \frac{x}{x^2 - 4}\, dx =$

(A) $\dfrac{-1}{4\left(x^2 - 4\right)^2} + C$

(B) $\dfrac{1}{2\left(x^2 - 4\right)} + C$

(C) $\dfrac{1}{2}\ln\left|x^2 - 4\right| + C$

(D) $2\ln\left|x^2 - 4\right| + C$

(E) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

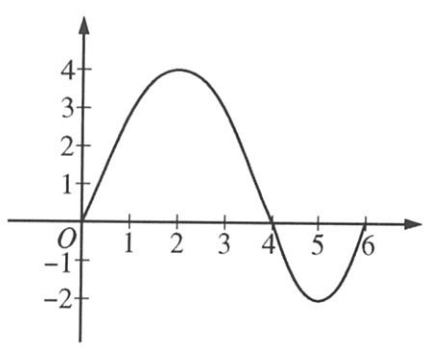
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

---

19. What are all horizontal asymptotes of the graph of $y = \frac{5 + 2^x}{1 - 2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

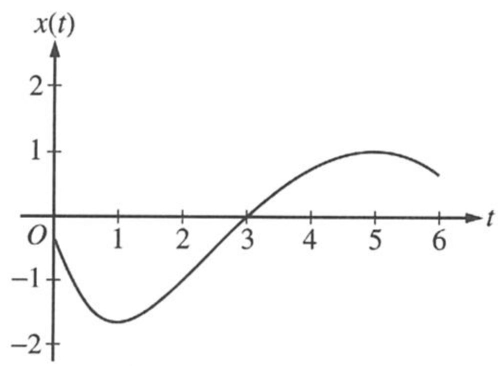
(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

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26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

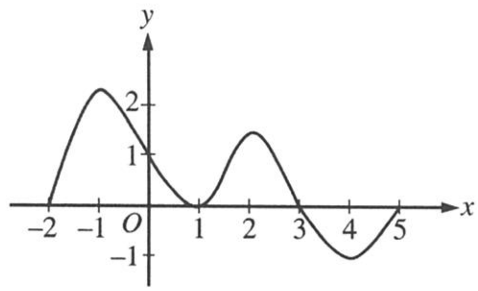
(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

END OF PART A OF SECTION I



Graph of f'

76. The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$