Name: ______________________________ Date:_________

PC: Rational Zeros Theorem Ms. Loughran

**Rational Zeros Theorem**

If the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of $P$ is of the form

$$\frac{p}{q}$$

where $p$ is a factor of the constant coefficient $a_0$
and $q$ is a factor of the leading coefficient $a_n$

1. List all possible (prz) rational zeros of $P(x) = x^3 - 3x + 2$.

$$prz = \frac{\pm 1, \pm 2}{\pm 1} = \pm 1, \pm 2$$

2. Find the zeros of $P(x) = x^3 - 3x + 2$

$P(1) = 1^3 - 3(1) + 2 = 0$ therefore 1 is a zero of $P(x)$

$\{1 \text{ (double)}, -2\}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$P(x) = (x-1)(x^2 + x - 2)$

$P(x) = (x-1)(x+2)(x-1) = (x-1)^2(x+2)$

$P(x) = 0$

$(x-1)^2(x+2) = 0$

$x = 1 \text{ (double)}$ | $x = -2$

3. Factor the polynomial $P(x) = 2x^3 + x^2 - 13x + 6$ grouping is not going to work here

prz: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

$P(1) \neq 0$

$P(-1) \neq 0$

$P(2) = 0$ therefore 2 is a zero of $P(x)$ and $(x-2)$ is a factor of $P(x)$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$P(x) = (x-2)(2x^2+5x-3)$

$P(x) = (x-2)(2x^2+6x-x-3)$

$P(x) = (x-2)(2x(x+3)-1(x+3))$

$P(x) = (x-2)(2x-1)(x+3)$

4. Let $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$

(a) Find the zeros of $P(x)$. $\{-2, 5, 1 \pm \sqrt{2}\}$

(b) Sketch the graph of $P$ **without using your calculator.**

a) prz: $\dfrac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1} = \pm 1, \pm 2, \pm 5, \pm 10$

$P(-2) = 0$

$P(5) = 0$

$$\begin{array}{r|rrrrr} -2 & 1 & -5 & -5 & 23 & 10 \\ & & -2 & 14 & -18 & -10 \\ \hline 5 & 1 & -7 & 9 & 5 & 0 \\ & & 5 & -10 & -5 & \\ \hline & 1 & -2 & -1 & 0 & \end{array}$$

time out:

$x^3 - 7x^2 + 9x + 5$

need to keep going

$P(x) = (x+2)(x-5)(x^2-2x-1)$

$0 = (x+2)(x-5)(x^2-2x-1) \rightarrow$ use QF or CS

$x = -2$

$x = 5$

CS: $x^2 - 2x - 1 = 0$

$x^2 - 2x + 1 = 1 + 1$

$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

OR

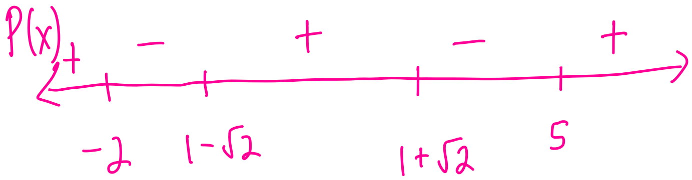
QF $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

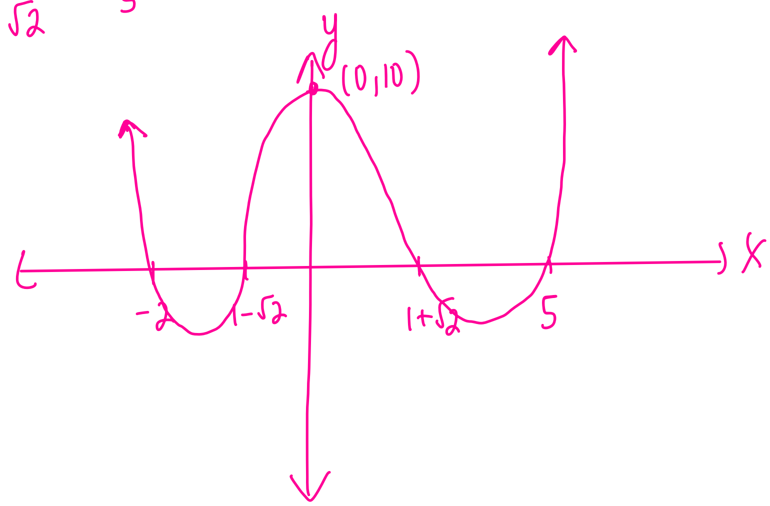
$x = \dfrac{2 \pm \sqrt{8}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$$P(x) = \overset{+}{(x+2)}\overset{-}{(x-5)}\overset{-}{(x^2-2x-1)}$$

Zeros: $-2,\ 5,\ 1 \pm \sqrt{2}$

y-int: $(0, 10)$
(let $x = 0$)



# Homework 12-11

Name: ________________________________ Date: ________________
PC: Ms. Loughran

Do Now:

1. Determine all the factors of $x^3 - 4x^2 - 11x + 30$ given that $x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$(x-2)(x^2-2x-15)$

$(x-2)(x-5)(x+3)$

2. Find a polynomial function of degree 4 that has integer coefficients and zeros $1, -1, 2, \frac{1}{2}$.

$P(x) = (x-1)(x+1)(x-2)(2x-1)$

3. Use the remainder theorem to determine if $x + 2$ is a factor of $p(x) = x^5 + 2x^4 - 3x^3 - 6x^2 - 6x - 12$. Justify your answer.

$p(-2) = (-2)^5 + 2(-2)^4 - 3(-2)^3 - 6(-2)^2 - 6(-2) - 12$

$\cancel{-32} + \cancel{32} + \cancel{24} - \cancel{24} + \cancel{12} - \cancel{12} = 0$

Yes $(x+2)$ is a factor b/c $p(-2) = 0$

4. If $p(x) = 2x^3 + cx^2 - 5x - 6$ and $x + 2$ is a factor of $p(x)$, find the value of $c$.

$p(-2) = 0$

$0 = 2(-2)^3 + c(-2)^2 - 5(-2) - 6$

$0 = -16 + 4c + 10 - 6$

$0 = -12 + 4c$

$12 = 4c$

$c = 3$

4, 11-16

## Exercises

**1–6** ■ List all possible rational zeros given by the Rational Zeros Theorem (but don't check to see which actually are zeros).

**1.** $P(x) = x^3 - 4x^2 + 3$

**2.** $Q(x) = x^4 - 3x^3 - 6x + 8$

**3.** $R(x) = 2x^5 + 3x^3 + 4x^2 - 8$

3: $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

**4.** $S(x) = 6x^4 - x^2 + 2x + 12$

**5.** $T(x) = 4x^4 - 2x^2 - 7$

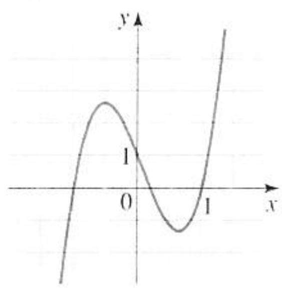
**6.** $U(x) = 12x^5 + 6x^3 - 2x - 8$

**7–10** ■ A polynomial function $P$ and its graph are given.

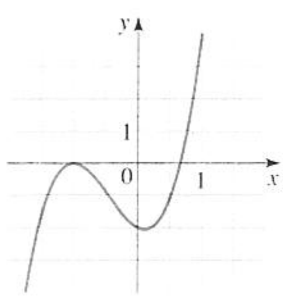
**(a)** List all possible rational zeros of $P$ given by the Rational Zeros Theorem.

**(b)** From the graph, determine which of the possible rational zeros actually turn out to be zeros.

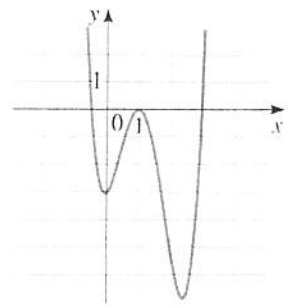
**7.** $P(x) = 5x^3 - x^2 - 5x + 1$



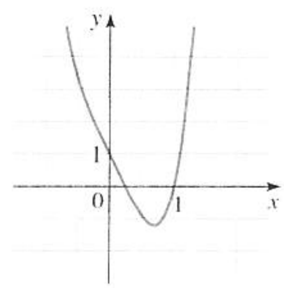

**8.** $P(x) = 3x^3 + 4x^2 - x - 2$




**9.** $P(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$




**10.** $P(x) = 4x^4 - x^3 - 4x + 1$




**11–40** ■ Find all [redacted] zeros of the polynomial.

**11.** $P(x) = x^3 + 3x^2 - 4$

**12.** $P(x) = x^3 - 7x^2 + 14x - 8$

**13.** $P(x) = x^3 - 3x - 2$

**14.** $P(x) = x^3 + 4x^2 - 3x - 18$

**15.** $P(x) = x^3 - 6x^2 + 12x - 8$

**16.** $P(x) = x^3 - x^2 - 8x + 12$

**17.** $P(x) = x^3 - 4x^2 + x + 6$

**18.** $P(x) = x^3 - 4x^2 - 7x + 10$

**19.** $P(x) = x^3 + 3x^2 + 6x + 4$

20. $P(x) = x^3 - 2x^2 - 2x - 3$

21. $P(x) = x^4 - 5x^2 + 4$

22. $P(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

23. $P(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$

24. $P(x) = x^4 - x^3 - 23x^2 - 3x + 90$

25. $P(x) = 4x^4 - 25x^2 + 36$

26. $P(x) = x^4 - x^3 - 5x^2 + 3x + 6$

27. $P(x) = x^4 + 8x^3 + 24x^2 + 32x + 16$

28. $P(x) = 2x^3 + 7x^2 + 4x - 4$

29. $P(x) = 4x^3 + 4x^2 - x - 1$

30. $P(x) = 2x^3 - 3x^2 - 2x + 3$

31. $P(x) = 4x^3 - 7x + 3$

32. $P(x) = 8x^3 + 10x^2 - x - 3$

33. $P(x) = 4x^3 + 8x^2 - 11x - 15$

34. $P(x) = 6x^3 + 11x^2 - 3x - 2$

35. $P(x) = 2x^4 - 7x^3 + 3x^2 + 8x - 4$

36. $P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

37. $P(x) = x^5 + 3x^4 - 9x^3 - 31x^2 + 36$

38. $P(x) = x^5 - 4x^4 - 3x^3 + 22x^2 - 4x - 24$

39. $P(x) = 3x^5 - 14x^4 - 14x^3 + 36x^2 + 43x + 10$

40. $P(x) = 2x^6 - 3x^5 - 13x^4 + 29x^3 - 27x^2 + 32x - 12$

**41–50** ■ Find all the real zeros of the polynomial. Use the quadratic formula if necessary, as in Example 3(a).

41. $P(x) = x^3 + 4x^2 + 3x - 2$

42. $P(x) = x^3 - 5x^2 + 2x + 12$

43. $P(x) = x^4 - 6x^3 + 4x^2 + 15x + 4$

44. $P(x) = x^4 + 2x^3 - 2x^2 - 3x + 2$

45. $P(x) = x^4 - 7x^3 + 14x^2 - 3x - 9$

46. $P(x) = x^5 - 4x^4 - x^3 + 10x^2 + 2x - 4$

47. $P(x) = 4x^3 - 6x^2 + 1$

48. $P(x) = 3x^3 - 5x^2 - 8x - 2$

49. $P(x) = 2x^4 + 15x^3 + 17x^2 + 3x - 1$

50. $P(x) = 4x^5 - 18x^4 - 6x^3 + 91x^2 - 60x + 9$

**51–58** ■ A polynomial $P$ is given.

**(a)** Find all the real zeros of $P$.

**(b)** Sketch the graph of $P$.

51. $P(x) = x^3 - 3x^2 - 4x + 12$

52. $P(x) = -x^3 - 2x^2 + 5x + 6$

53. $P(x) = 2x^3 - 7x^2 + 4x + 4$

54. $P(x) = 3x^3 + 17x^2 + 21x - 9$

55. $P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

56. $P(x) = -x^4 + 10x^2 + 8x - 8$

57. $P(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$

58. $P(x) = x^5 - x^4 - 6x^3 + 14x^2 - 11x + 3$