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PC: Operations on Matrices

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Two matrices are equal if they have the same order  $m \times n$  and their corresponding entries are equal.

1. Solve for  $a_{11}$ ,  $a_{12}$ ,  $a_{21}$ , and  $a_{22}$  in the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

### Matrix Addition

You can add two matrices (of the same order) by adding their corresponding entries. The sum of two matrices of different orders is undefined.

2.  $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} =$

3.  $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$

4.  $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} =$

5. The sum of  $A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$  is

## Scalar Multiplication

In work with matrices, numbers are usually referred to as scalars. For our purposes, scalars will always be real numbers. You can multiply a matrix  $A$  by a scalar  $c$  by multiplying each entry in  $A$  by  $c$ .

The symbol  $-A$  represents the scalar product  $(-1)A$ . Moreover, if  $A$  and  $B$  are of the same order,  $A - B$  represents the sum of  $A$  and  $(-1)B$ . That is,

$$A - B = A + (-1)B \quad (\text{Subtraction of matrices})$$

6. For the following matrices, find (a)  $3A$   
(b)  $-B$   
(c)  $3A - B$

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

### Properties of Matrix Addition and Scalar Multiplication

Let  $A$ ,  $B$ , and  $C$  be  $m \times n$  and let  $c$  and  $d$  be scalars.

1.  $A + B = B + A$  ( )

2.  $A + (B + C) = (A + B) + C$  ( )

3.  $(cd)A = c(dA)$  ( )

4.  $IA = A$  ( )

5.  $c(A + B) = cA + cB$  ( )

6.  $(c + d)A = cA + dA$  ( )

7. 
$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} =$$

8. Solve for  $X$  in the equation  $3X + A = B$ , where

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}.$$

## Matrix Multiplication

To find the entries of the product, multiply each row of  $A$  by each column of  $B$ . Note that the number of columns of  $A$  must be equal to the number of rows of  $B$ .

9. Find the product  $AB$  where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}.$$

10.  $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} =$

11.  $\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

$$12. \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$$

$$13. \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} =$$

$$14. \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} =$$

15. Find the product of  $AB$ . If  $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$ .

The  $m \times n$  matrix that consists of 1's on its main diagonal and 0's elsewhere is called the **identity matrix of order  $n$**  and is denoted by:

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

An identity matrix must be square.

16. Solve the matrix equation  $AX = B$  for  $X$ , where

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 4 \\ 2 \end{bmatrix}.$$

**Exercises**  
(courtesy of our textbook)

In Exercises 1–4, find  $x$  and  $y$ .

$$1. \begin{bmatrix} x & -2 \\ 7 & y \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ 7 & 22 \end{bmatrix}$$

$$2. \begin{bmatrix} -5 & x \\ y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$$

$$3. \begin{bmatrix} 16 & 4 & 5 & 4 \\ -3 & 13 & 15 & 6 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ -3 & 13 & 15 & 3x \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$$

$$4. \begin{bmatrix} x+2 & 8 & -3 \\ 1 & 2y & 2x \\ 7 & -2 & y+2 \end{bmatrix} = \begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & 11 \end{bmatrix}$$

In Exercises 5–10, find (a)  $A + B$ , (b)  $A - B$ , (c)  $3A$ , and (d)  $3A - 2B$ .

$$5. A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$$

$$7. A = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix}$$

$$8. A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix}$$

$$9. A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ 1 & 1 & -2 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ -3 & 4 & 9 & -6 & -7 \end{bmatrix}$$

$$10. A = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

In Exercises 11–14, solve for  $X$  given

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$11. X = 3A - 2B$$

$$12. 2X = 2A - B$$

$$13. 2X + 3A = B$$

$$14. 2A + 4B = -2X$$

In Exercises 15–20, find (a)  $AB$ , (b)  $BA$ , and, if possible, (c)  $A^2$ . (Note:  $A^2 = AA$ .)

$$15. A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$$

$$16. A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$$

$$17. A = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$

$$18. A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$$

$$19. A = \begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix}$$

$$20. A = [3 \quad 2 \quad 1], B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

In Exercises 21–28, find  $AB$ , if possible.

$$21. A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$$

$$22. A = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$$

$$23. A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

$$24. A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$25. A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 7 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$26. A = \begin{bmatrix} 10 \\ 12 \end{bmatrix}, B = [6 \quad -2 \quad 1 \quad 6]$$

$$27. A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 & -11 & 4 \\ 8 & 16 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$28. A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 6 & 13 & 8 & -17 \end{bmatrix}, B = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}$$