Do Now #1 from yesterday's Do Now sheet

Name: ______________________________________ Date: __________
A2H: Vertex Form Ms. Loughran

Do Now:

Put the following quadratics in vertex form, and then find all of the following for it:

(a) the vertex
(b) the axis of symmetry
(c) the x-intercepts, if any
(d) the y-intercepts
(e) the domain
(f) the range

$y = a(x-h)^2 + k$

1. $y = -2x^2 - 4x + 11$ $\quad -\frac{13}{2}$

$y = -2(x^2 + 2x + 1 - 1 - \frac{11}{2})$

$y = -2(x+1)^2 - 2(-\frac{13}{2})$

$y = -2(x+1)^2 + 13$

vertex: $(-1, 13)$

axis of symmetry: $x = -1$

D: $(-\infty, \infty)$

R: $(-\infty, 13]$

x-int: (let $y=0$)

$0 = -2(x+1)^2 + 13$

$-13 = -2(x+1)^2$

$\frac{13}{2} = (x+1)^2$

$\pm\sqrt{\frac{13}{2}} = x+1$

$-1 \pm \sqrt{\frac{13}{2}} = x$

$(-1 \pm \sqrt{\frac{13}{2}}, 0)$

y-int (let $x=0$)

$y = -2(0+1)^2 + 13$

$y = -2 + 13$

$y = 11$

$(0, 11)$

2. $y = 3x^2 - 5x + 1$ $\quad \frac{12}{36}$

$y = 3(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36} + \frac{1}{3})$

$y = 3(x - \frac{5}{6})^2 + 3(\frac{-13}{36})$ $\quad \frac{-13}{12}$

$y = 3(x - \frac{5}{6})^2 - \frac{13}{12}$

Vertex: $(\frac{5}{6}, -\frac{13}{12})$

axis of symmetry: $x = \frac{5}{6}$

D: $(-\infty, \infty)$

R: $[-\frac{13}{12}, \infty)$

x-int: (let $y=0$)

$0 = 3(x - \frac{5}{6})^2 - \frac{13}{12}$

$\frac{13}{12} = 3(x - \frac{5}{6})^2$

$\frac{13}{36} = (x - \frac{5}{6})^2$

$\pm\sqrt{\frac{13}{36}} = x - \frac{5}{6}$

$\frac{5}{6} \pm \frac{\sqrt{13}}{6} = x$

$(\frac{5}{6} \pm \frac{\sqrt{13}}{6}, 0)$

y-int (let $x=0$)

$y = 3(0 - \frac{5}{6})^2 - \frac{13}{12}$

$y = 3(\frac{25}{36}) - \frac{13}{12}$ $\quad \frac{25}{12}$

$y = \frac{25}{12} - \frac{13}{12}$

$y = 1$

$(0, 1)$

Quarter Exam Review Sheet Key (Q1)

1. $f(x) = x^3 - 3x + 6$

(a) $f(-2) = (-2)^3 - 3(-2) + 6 = -8 + 6 + 6 = 4$

(b) $f(x-2) = (x-2)^3 - 3(x-2) + 6$

$(x-2)(x-2)(x-2) \; -3x + 6 + 6$

$(x^2 - 4x + 4)(x-2) - 3x + 6 + 6$

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 \; - 3x + 12$

$x^3 - 6x^2 + 9x + 4$

2. (a) $(g \circ f)(x$

$g(x-1) = (x-1)^2 = x^2 - 2x + 1$

(b) $(f \circ h \circ g)$

$f(h(x^2)$

$f(\sqrt{x^2-25}) = \sqrt{x^2-25} - 1$

(c) $\dfrac{(x+h)^2 - 5(x+h) + 4 - (x^2 - 5x + 4)}{h}$ $\quad h \neq 0$

$\dfrac{\cancel{x^2} + 2xh + h^2 \cancel{-5x} - 5h \cancel{+4} \cancel{-x^2} \cancel{+5x} \cancel{-4}}{h}$

$\dfrac{2xh + h^2 - 5h}{h} = 2x + h - 5$

3 $f(x) = 3x - 8$

$y = 3x - 8$

$x = 3y - 8$

$x + 8 = 3y$

$\frac{x+8}{3} = y$

$\frac{x+8}{3} = f^{-1}(x)$

4 $f(x) = \sqrt{4x-3}$

$y = \sqrt{4x-3}$

$x = \sqrt{4y-3}$

$x^2 = 4y - 3$

$x^2 + 3 = 4y$

$\frac{x^2+3}{4} = f^{-1}(x)$

5 (a) $m = \frac{7-(-4)}{-2-4} = \frac{11}{-6}$

(b) $-7x + 4y = 12$

$4y = 7x + 12$

$y = \frac{7}{4}x + 3$

$m = 7/4$

(c) $y + 5 = 3(x - 3)$

$y + 5 = 3x - 9$

$y = 3x - 14$

$m = 3$

slope of line $\perp$ is $-1/3$

(d) $y = 2x - 5$

$m = 2$

slope of $\parallel$ line is 2

6 (a) $y + 4 = 3(x - 1)$ point slope

$y + 4 = 3x - 3$

$y = 3x - 7$ slope intercept form

$3x - y = 7$ standard form

(b) $m = \frac{7-(-4)}{-2-4} = \frac{11}{-6}$

point slope: $y-7 = -\frac{11}{6}(x+2)$ or $y+4 = -11/6\,(x-4)$

$y-7 = -11/6\,x - \frac{22}{6}$

$y = -11/6\,x - 22/6 + 7$

slope intercept: $y = -11/6\,x + \frac{10}{3}$

$6\left(y = -\frac{11}{6}x + \frac{10}{3}\right)$

$6y = -11x + 20$

standard form: $11x + 6y = 20$

(7) (a) $7x - 2$

$g(x) = 7x$

$f(x) = x - 2$

$f(g(x))$

(b) $\frac{18}{\sqrt{x^2-3}}$

$g(x) = x^2 - 3$

$f(x) = \frac{18}{\sqrt{x}}$

$f(g(x))$

(8) $f(g(x)) = g(f(x)) = x$

$f\left(\frac{1}{2}x^2 + \frac{3}{2}\right)$ $\quad$ $g\left(\sqrt{2x-3}\right)$

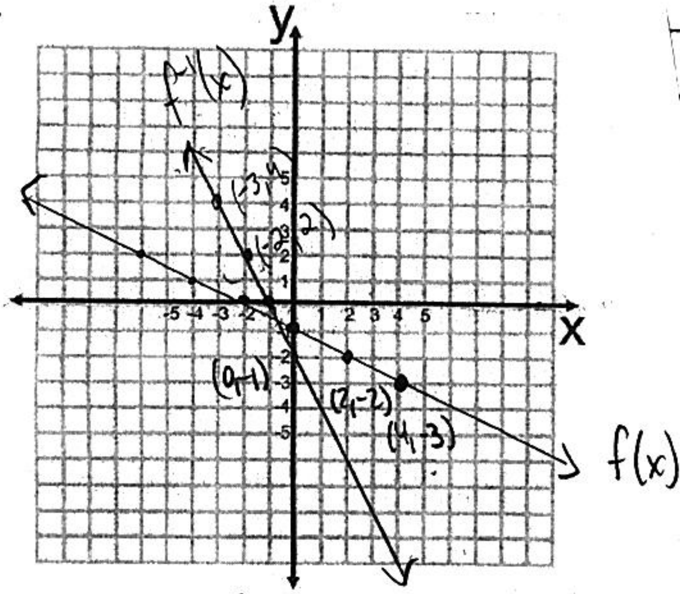
$\sqrt{2\left(\frac{1}{2}x^2 + \frac{3}{2}\right) - 3}$ $\quad$ $\frac{1}{2}\left(\sqrt{2x-3}\right)^2 + 3/2$

$\sqrt{x^2 + 3 - 3}$ $\quad$ $\frac{1}{2}(2x-3) + 3/2$

$\sqrt{x^2}$ $\quad$ $x - 3/2 + \frac{3}{2}$

(4)

(9)



(10) $\dfrac{5k^2-20k}{12+5k-2k^2} = \dfrac{5k\cancel{(k-4)}}{-(2k+3)\cancel{(k-4)}}$

$-2k^2+5k+12$

$-(2k^2-5k-12)$ $ac=24$, $b=-5$

$-(2k^2-8k+3k-12)$

$-(2k(k-4)\ 3(k-4))$

$-(2k+3)(k-4)$

$\dfrac{5k}{-(2k+3)}$ $k \neq 4, -\frac{3}{2}$

(b) $\dfrac{5h^{\cancel{3}\,2}}{h^2-h} = \dfrac{5h^2}{h-1}$ $h \neq 0, 1$

$\cancel{h}(h-1)$

(11) (a) $|x+3| = 5$

x's distance from -3 $=5$

-8 -3 2

$\{-8, 2\}$

b) $|5-4x| \leq 6$

$|4x-5| \leq 6$

$4\left|x-\frac{5}{4}\right| \leq 6$

$\left|x-\frac{5}{4}\right| \leq \frac{6}{4}$

x's distance from $\frac{5}{4}$ $\leq \frac{6}{4}$

$-\frac{1}{4}$ $\frac{5}{4}$ $\frac{11}{4}$

$[-\frac{1}{4}, \frac{11}{4}]$

12 (a) $(x-1)(x+1)$ $\frac{1 - \frac{1}{\cancel{1-x}}}{4 + \frac{3}{\cancel{x^2-1}}}$ $\cancel{(x-1)}(x+1)$ $(x-1)(x+1)$

-1

$x \neq \pm 1, \pm \frac{1}{2}$

$$\frac{x^2-1+x+1}{4x^2-4+3} = \frac{x^2+x}{4x^2-1} = \frac{x(x+1)}{(2x-1)(2x+1)}$$

(b)

$$\frac{9-x^{-2}}{3x^{-1}-x^{-2}} = \frac{x^2 \cdot 9 - \frac{1}{x^2} \cdot x^2}{x^2 \cdot \frac{3}{x} - \frac{1}{x^2} \cdot x^2} = \frac{9x^2-1}{3x-1} = \frac{\cancel{(3x-1)}(3x+1)}{\cancel{3x-1}}$$

$= 3x+1 \quad x \neq 0, \frac{1}{3}$

(c)

$$3(y-4) \cdot \frac{1}{3y} + \frac{9(y^2+1)}{y^2-4y} + \frac{y-2}{36-9y}(-1)(y)$$

$3(y-4)\,3y$ $\quad (9)\,y(y-4)$ $\quad 9(4-y)(-1)y$

$$\frac{3y-12+9y^2+9-y^2+2y}{9y(y-4)}$$

* $\frac{8y^2+5y-3}{9y(y-4)}$ or $\frac{(8y-3)(y+1)}{9y(y-4)}$ $\quad y \neq 0, 4$

$8y^2+5y-3$ mult -24 add 5

$8y^2+8y-3y-3$

$8y(y+1)-3(y+1)$

$(8y-3)(y+1)$

* $2y^2-6y+5y-15 \to (2y+5)(y-3)$

2y(y-3)+5(y-3)

d) $$\frac{a^2+2ab+b^2}{a^2-b^2} \div \frac{2a^2-ab-b^2}{a^2-ab-2b^2}$$

e) $$\frac{2y^2-y-15}{3y^2-y-10} \div \frac{y^2-10y+21}{9y^2-25}$$ (6)

$3y^2-6y+5y-10$
$3y(y-2)+5(y-2)$
$(3y+5)(y-2)$

$$\frac{(\cancel{a+b})(a+b)}{(\cancel{a+b})(a-b)} \cdot \frac{(a-2b)(a+b)}{(2a+b)(a-b)}$$

$$\frac{(a+b)(a+b)(a-2b)}{(a-b)(2a+b)(a-b)}$$ $a \neq \pm b, \pm\frac{b}{2}$

$$\frac{(2y+5)\cancel{(y-3)}}{\cancel{(3y+5)}(y-2)} \cdot \frac{(3y-5)\cancel{(3y+5)}}{(y-7)\cancel{(y-3)}}$$

$$\frac{(2y+5)(3y-5)}{(y-2)(y-7)}$$ $y \neq \pm\frac{5}{3}, 2, 7, 3$

(13) (a) $x^2+9x+14<0$

$(x+7)(x+2)<0$

		-7		-2	
x+7	−		−		+
x+2	−		+		+
P	+		−		+

(a) $\{x \mid -7<x<-2\}$
(b) $(-7, -2)$

(b) $$\frac{3x}{4} \leq \frac{3x-6}{8}$$

$$\frac{3x^{(2)}}{4(2)} - \frac{3x-6}{8} \leq 0$$

$$\frac{6x-3x+6}{8} \leq 0$$

$$\frac{3x+6}{8} \leq 0$$

		-2	
3x+6	−		+
8	+		+
Q	−		+

(a) $\{x \mid x \leq -2\}$
(b) $(-\infty, -2]$

(c) $$\frac{x^2-3x-10}{(x-1)^2} > 0$$

$(x-5)(x+2)$

		-2		1		5	
(x-5)(x+2)	+		−		−		+
$(x-1)^2$	+		+		+		+
Q	+		−		−		+

(a) $\{x \mid x<-2 \lor x>5\}$
(b) $(-\infty, -2) \cup (5, \infty)$

14 (a) $28x^3 - 49x^2 + 21x$

$7x(4x^2 - 7x + 3)$ $ac = 12$, $b = -7$

$7x(4x^2 - 4x - 3x + 3)$

$7x(4x(x-1) - 3(x-1))$

$7x(4x-3)(x-1)$

(b) $2x^3 + 3x^2 - 2x - 3$

$x^2(2x+3) - 1(2x+3)$

$(x^2-1)(2x+3)$

$(x-1)(x+1)(2x+3)$

(c) $75x^2 - 3$

$3(25x^2 - 1)$

$3(5x-1)(5x+1)$

(d) $8x^3 + 27$

$(2x+3)(4x^2 - 6x + 9)$

(e) $x^6 - 64y^3$

$(x^2 - 4y)(x^4 + 4x^2y + 16y^2)$

(f) $x^4 - 6x^2 - 27$

$(x^2 - 9)(x^2 + 3)$

$(x-3)(x+3)(x^2+3)$

15 a) $f(x) = -x^2 - 2x + 8$ $\overbrace{-9}$

$f(x) = -(x^2 + 2x + 1 - 1 - 8)$

$f(x) = -(x+1)^2 + 9$

v: $(-1, 9)$ ← maximum D: $(-\infty, \infty)$

a of sym: $x = -1$ R: $(-\infty, 9]$

x-int

$0 = -(x+1)^2 + 9$

$-9 = -(x+1)^2$

$9 = (x+1)^2$

$\pm\sqrt{9} = x+1$

$\pm 3 = x+1$

$-3 = x+1$ $3 = x+1$

$-4 = x$ $2 = x$

$(-4, 0)$, $(2, 0)$

y-int

$y = -(0+1)^2 + 9$

$y = -1 + 9$

$y = 8$

$(0, 8)$

b) $f(x) = 2(x-1)^2 - 2$

vertex: $(1, -2)$ ← minimum D: $(-\infty, \infty)$

a of sym. $x = 1$ R: $[-2, \infty)$

x-int:

$0 = 2(x-1)^2 - 2$

$2 = 2(x-1)^2$

$1 = (x-1)^2$

$\pm\sqrt{1} = x-1$

$\pm 1 = x-1$

$-1 = x-1$ $1 = x-1$

$0 = x$ $2 = x$

$(0, 0)$, $(2, 0)$

y-int. $(0, 0)$

c) $f(x) = -x^2 + 5x + 6$

$f(x) = -\left(x^2 - 5x + \frac{25}{4} \overbrace{- \frac{25}{4} - 6}^{-\frac{49}{4}}\right)$

$f(x) = -\left(x - \frac{5}{2}\right)^2 - \left(-\frac{49}{4}\right)$

$f(x) = -\left(x - \frac{5}{2}\right)^2 + \frac{49}{4}$

vertex: $\left(\frac{5}{2}, \frac{49}{4}\right)$ maximum   D: $(-\infty, \infty)$

a of sym: $x = \frac{5}{2}$ R: $\left(-\infty, \frac{49}{4}\right)$

x-int

$0 = -\left(x - \frac{5}{2}\right)^2 + \frac{49}{4}$

$-\frac{49}{4} = -\left(x - \frac{5}{2}\right)^2$

$\frac{49}{4} = \left(x - \frac{5}{2}\right)^2$

$\pm\sqrt{\frac{49}{4}} = x - \frac{5}{2}$

$\pm\frac{7}{2} = x - \frac{5}{2}$

$-\frac{7}{2} = x - \frac{5}{2}$ $\frac{7}{2} = x - \frac{5}{2}$

$-1 = x$ $6 = x$

$(-1, 0), (6, 0)$

y-int

$y = -\left(0 - \frac{5}{2}\right)^2 + \frac{49}{4}$

$y = -\left(\frac{25}{4}\right) + \frac{49}{4}$

$y = \frac{24}{4}$

$y = 6$

$(0, 6)$

d) $f(x) = 4x^2 + 12x + \underbrace{6}_{}$ $-3/4$

$f(x) = 4\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4} + \frac{6}{4}\right)$

$f(x) = 4\left(x + \frac{3}{2}\right)^2 + 4\left(-\frac{3}{4}\right)$

$f(x) = 4\left(x + \frac{3}{2}\right)^2 - 3$

vertex: $\left(-\frac{3}{2}, -3\right)$ minimum   D: $(-\infty, \infty)$

a of s: $x = -\frac{3}{2}$ R: $[-3, \infty)$

x-int:

$0 = 4\left(x + \frac{3}{2}\right)^2 - 3$

$3 = 4\left(x + \frac{3}{2}\right)^2$

$\frac{3}{4} = \left(x + \frac{3}{2}\right)^2$

$\pm\sqrt{\frac{3}{4}} = x + \frac{3}{2}$

$\pm\frac{\sqrt{3}}{2} = x + \frac{3}{2}$

$-\frac{3}{2} \pm \frac{\sqrt{3}}{2} = x$

$\left(-\frac{3}{2} \pm \frac{\sqrt{3}}{2}, 0\right)$

y-int

$y = 4\left(0 + \frac{3}{2}\right)^2 - 3$

$y = 4\left(\frac{9}{4}\right) - 3$

$y = 9 - 3$

$y = 6$

$(0, 6)$

e) $f(x) = -3x^2 - 6x - 7$ $\frac{4}{3}$

$f(x) = -3\left(x^2 + 2x + 1 \underbrace{- 1 + \frac{7}{3}}_{}\right)$

$f(x) = -3(x+1)^2 - \cancel{3}\left(\frac{4}{\cancel{3}}\right)$

$f(x) = -3(x+1)^2 - 4$

vertex: $(-1, -4)$ maximum   D: $(-\infty, \infty)$

a of s: $x = -1$ R: $(-\infty, -4]$

x-int:

$0 = -3(x+1)^2 - 4$

$4 = -3(x+1)^2$

$-\frac{4}{3} = (x+1)^2$

$\pm\sqrt{\frac{-4}{3}} = x + 1$

imaginary, so <u>no</u> x-intercepts

y-int:

$y = -3(0+1)^2 - 4$

$y = -3(1) - 4$

$y = -3 - 4$

$y = -7$

$(0, -7)$

Extra Questions

$\sqrt[3]{x^9}$ $\sqrt[3]{y^{15}}$

① Factor

$27x^9 + 125y^{15}$ $(3x^3)^2$ $(5y^5)^2$

$(3x^3 + 5y^5)(9x^6 - 15x^3y^5 + 25y^{10})$

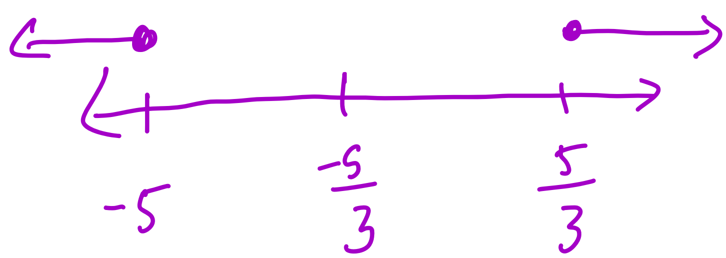
② Solve using the geometric definition and put solution set in interval notation

$|5+3x| \geq 10$

$|3x+5| \geq 10$

$3\left|x + \frac{5}{3}\right| \geq 10$

$\left|x + \frac{5}{3}\right| \geq \frac{10}{3}$



$(-\infty, -5] \cup [\frac{5}{3}, \infty)$

x's distance from $-\frac{5}{3} \geq \frac{10}{3}$