Name: _____

PCH Midterm Review Packet

This review packet is a sampling of questions related to topics we have covered during Q1 and Q2. You should also be studying your prior tests, assignments, and notes from both quarters to prepare for the midterm.

1. Factor completely:

(a) $5(x-7)^4 (x+1)^2 + 2(x+1)(x-7)^5$

(b) $5x^{\frac{1}{2}} + 33x^{-\frac{1}{2}} - 14x^{-\frac{3}{2}}$

(c) $x^{\frac{3}{5}} + 3x^{\frac{2}{5}} - 25x^{\frac{1}{5}} - 75$

(d) $x^4 + 14x^2 + 81$ using advanced completing the square method

2. Express in simplest form:

(a) $\dfrac{(2x-1)^2 \cdot 4(3x+5)^3 \cdot 3 - 2(2x-1) \cdot 2(3x+5)^4}{(2x-1)^4}$

(b) $\dfrac{49y^{-2} - x^{-2}}{x^{-2} - 12(xy)^{-1} + 35y^{-2}}$

(c) $\dfrac{\dfrac{5(x+h)}{(x+h)-4} - \dfrac{5x}{x-4}}{h}$

(d) $\dfrac{x^2 - y^2 - 16x - 16y}{x^2 - y^2 + 16x + 16y}$

(e) $\dfrac{125x^3 - 8y^3}{16x^2 - 24xy + 9y^2} \div \dfrac{25x^2 - 4y^2}{6y^2 + 7xy - 20x^2}$

(f) $\dfrac{5}{x^2-4xy+4y^2} - \dfrac{1}{2y^2-xy}$

(g) $\dfrac{x-100}{10-\sqrt{x}}$ Express with a rationalized denominator.

3. If $f(x)=7x-\dfrac{2}{5}$, find the value of $\dfrac{f(b)-f(a)}{b-a}$.

4. Find the inverse of $y=-\left(4-\dfrac{3}{x+1}\right)$

5. Show by composition whether $g(x)$ and $h(x)$ are inverses of each other:

$$g(x)=\dfrac{(x-4)^3}{2}+9,\ \ h(x)=4+\sqrt[3]{2x-18}$$

6. (a) Create 3 functions, $f(x), g(x),$ and $h(x)$, such that

$$(f\,og\,oh)(x)=\dfrac{1}{\sqrt[4]{(7x+3)}}.$$

(Note: You may not use the identity function.)

(b) Create 4 functions, $f(x), g(x), h(x)$ and $k(x)$, such that

$$(f\,og\,oh\,ok)(x)=\dfrac{1}{\sqrt[4]{(7x+3)}}.$$

(Note: You may not use the identity function.)

7. If $f(x)=\dfrac{3}{x-3}$ and $g(x)=\dfrac{1}{x}-3$, then find

(a) $f(g(x))$

(b) $g(f(x))$

(c) $f(f(x))$

(d) $g(g(x))$

8. Rewrite as a piecewise function and then sketch the graph:

(a) $y = |5x + 1|$

(b) $y = \dfrac{|2 - x|}{x - 2}$

9. Solve for x using the geometric definition of absolute value.

(a) $|8 - 5x| - 2 \geq 6$ (Express your answer in interval notation.)

(b) $3|2 - 7x| - 5 = 16$

10. Solve for x using the geometric definition of absolute value.

(a) $\left|\dfrac{3}{4}x + 2\right| - 3 = 15$.

(b) $\left|\dfrac{6x + 5}{7}\right| - 9 < 1$ (Express your answer in set builder notation.)

11. Solve for x and express your answer on a number line.

(a) $\dfrac{1}{x - 3} \leq \dfrac{3}{2x + 1}$ (Express your answer in interval notation.)

(b) $25x^3 + 3x > 2x(1 - 5x)$ (Express your answer in set builder notation.)

12. Find $\dfrac{f(x + h) - f(x)}{h}$ for each given function.

(a) $f(x) = \sqrt{6x - 1}$
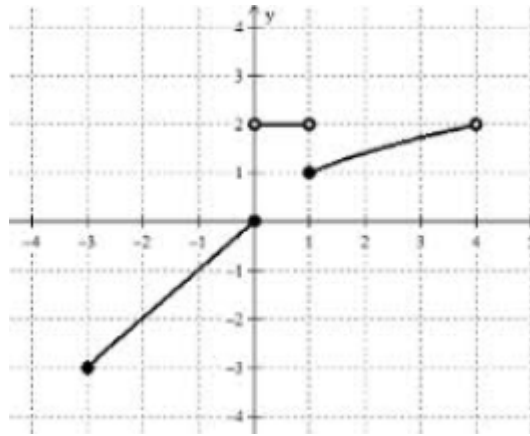
(b) $f(x) = 4x^2 - 11x + 5$

(c) $f(x) = \dfrac{7}{x - 9}$

(d) $f(x) = \dfrac{x}{x - 10}$

13. (a) Sketch the piecewise function: $f(x) = \begin{cases} 5 - \dfrac{1}{4}x & \text{if} \quad x \le -4 \\ x^2 - 3 & \text{if} \quad -4 < x \le 3 \\ |x - 5| & \text{if} \quad x > 3 \end{cases}$

(b) Find the value of $f(3) + f(0) - 2f(-4) + 3f(5)$

14. Write a piecewise function for the graph shown below:

15. In the function $f(x) = \dfrac{5}{2}x + b$, b is a constant. If $f(8) = 3$, what is the value of $f(-4)$?

16. Write the equation of a line in point-slope form that is parallel to $6x - 7y = 3$ and passes through $(-1, 5)$.

17. Sketch each graph. Label all x and y intercepts with their coordinates.

(a) $y = 9x^3 - x^4$

(b) $y = x^3 - 7x^2 - 4x + 28$

(c) $y = 3(2x - 1)^2 (x + 2)$

18. The table below shows a selection of corresponding t and v(t) values for a polynomial v(t). State an interval in which v(t) is guaranteed to have a root.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

19. Given $P(x) = 3x^3 - 5x^2 + 25x + 9$, find all of the roots of $P(x)$.

20. Is there *guaranteed* to be a real root of $P(x) = x^3 - 7x^2 + 5x - 8$ between 1 and 5? Justify your work with an explanation.

21. Sketch each function. Label any asymptotes with their equations and holes with their coordinates. State the coordinates of all x and y intercepts and state the domain.

(a) $y = \dfrac{6 - 4x}{2x + 3}$

(b) $y = \dfrac{x^2 - 3x - 10}{x - 1}$

(c) $y = \dfrac{x^2 - 3x + 2}{x - 1}$

(d) $y = \dfrac{x - 1}{x^2 - 3x + 2}$

(e) $y = \dfrac{3x^2}{x^3 + x}$

(f) $y = \dfrac{x^2 - 9x}{x^2 - 1}$

22. Determine algebraically if each function is even, odd, or neither.

(a) $f(x) = x^3 - 4x^2 + 2x - 3$

(b) $f(x) = \dfrac{2 - x^2}{x^3}$

(c) $f(x) = \dfrac{x^3 + x}{x^5}$

(d) $f(x) = x^2 \sqrt{4 - x^6}$

23. State the coordinates of the point(s) where each crosses its end behavior asymptote.

(a) $y = \dfrac{x^2 + 1}{x^2 + x - 2}$

(b) $y = \dfrac{(x+1)(x-1)^2}{x^2}$

24. A square piece of cardboard measures 40 cm on each side. Congruent squares of length x cm are cut from the four corners and the sides are folded up to form a rectangular open top box. Find a function to model the volume of the box in terms of x.

25. Find the area of the shaded triangle as a function of x:
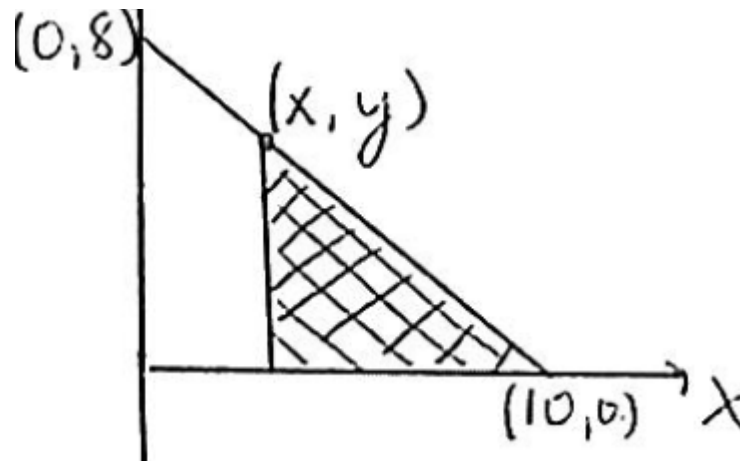
26. The vertices of a rectangle are at $(0,0), (x,0), (0,y)$ and (x, y), with (x, y) as a point on the graph of $y = 8 - x^3$ in the first quadrant. Express the area of the rectangle as a function of x.

27. An isosceles triangle has a perimeter of 8 in. Find a function that models its area in terms of the length of its base.

A graphing calculator is required for Questions 28 – 29.

28. Solve for x graphically: $x - 4 + \dfrac{2x}{x^2 - 1} = 2$. Draw a complete graph and label the window.

29. Solve for x graphically: $\dfrac{3x - 5}{x + 2} + \dfrac{1}{x - 1} < 6$. Draw a complete graph and label the window. Show your solution on a number line and in interval notation.