1. Factor completely:

(a) $5(x-7)^4(x+1)^2+2(x+1)(x-7)^5$

$(x-7)^4(x+1)\left(5(x+1)+2(x-7)\right)$

$(x-7)^4(x+1)(5x+5+2x-14)$

$(x-7)^4(x+1)(7x-9)$

(b) $5x^{\frac{1}{2}}+33x^{-\frac{1}{2}}-14x^{-\frac{3}{2}}$

$x^{-3/2}(5x^2+33x-14)$

$x^{-3/2}(5x-2)(x+7)$, $x>0$

(c) $x^{\frac{3}{5}}+3x^{\frac{2}{5}}-25x^{\frac{1}{5}}-75$

$x^{2/5}(x^{\frac{1}{5}}+3)-25(x+3)$

$(x^{2/5}-25)(x^{\frac{1}{5}}+3)$

$(x^{\frac{1}{5}}-5)(x^{\frac{1}{5}}+5)(x^{\frac{1}{5}}+3)$

(d) x^4+14x^2+81 using advanced completing the square method

$x^4+14x^2+81+4x^2-4x^2$

$x^4+18x^2+81-4x^2$

$(x^2+9)^2-4x^2$

$(x^2-2x+9)(x^2+2x+9)$

2. Express in simplest form:

(a) $$\frac{(2x-1)^2\bullet 4(3x+5)^3\bullet 3-2(2x-1)\bullet 2(3x+5)^4}{(2x-1)^4}$$

$$\frac{12(2x-1)^2(3x+5)^3-4(2x-1)(3x+5)^4}{(2x-1)^4}$$

$$\frac{4(2x-1)(3x+5)^3\left(3(2x-1)-(3x+5)\right)}{(2x-1)^4}$$

$$\frac{4(2x-1)(3x+5)^3(6x-3-3x-5)}{(2x-1)^4}$$

$$\frac{4\cancel{(2x-1)}(3x+5)^3(3x-8)}{(2x-1)^{\cancel{4}\ 3}}$$

$$\frac{4(3x+5)^3(3x-8)}{(2x-1)^3}$$ $x\neq\frac{1}{2}$

(b) $$\frac{49y^{-2}-x^{-2}}{x^{-2}-12(xy)^{-1}+35y^{-2}}$$

$$\frac{x^2y^2\,\frac{49}{y^2}-\frac{1}{x^2}\,x^2y^2}{x^2y^2\,\frac{1}{x^2}-\frac{12}{xy}\,x^2y^2+\frac{35}{y^2}\,x^2y^2}$$

$x,y\neq 0$

$y\neq 7x, 5x$

$$\frac{49x^2-y^2}{y^2-12xy+35x^2}$$

$$\frac{\overset{-1}{\cancel{(7x-y)}}(7x+y)}{\cancel{(y-7x)}(y-5x)}=-\frac{(7x+y)}{y-5x}$$

(c) $$\frac{(x+h-4)(x-4)\ \frac{5(x+h)}{(x+h)-4}-\frac{5x}{x-4}\ (x+h-4)(x-4)}{h((x+h)-4)(x-4)}$$

$$\frac{5(x+h)(x-4)-5x(x+h-4)}{h(x+h-4)(x-4)}$$

$$\frac{5(x^2+hx-4x-4h)-5x^2-5xh+20x}{h(x+h-4)(x-4)}$$

$$\frac{\cancel{5x^2}+\cancel{5hx}-\cancel{20x}-20h-\cancel{5x^2}-\cancel{5xh}+\cancel{20x}}{h(x+h-4)(x-4)}$$

$h\neq 0$

$x\neq 4, 4-h$

$$\frac{-20\cancel{h}}{\cancel{h}(x+h-4)(x-4)}=\frac{-20}{(x+h-4)(x-4)}$$

(d) $$\frac{x^2-y^2-16x-16y}{x^2-y^2+16x+16y}$$

$$\frac{(x-y)(x+y)-16(x+y)}{(x-y)(x+y)+16(x+y)}$$

take out GCF on top & bottom

$$\frac{\cancel{(x+y)}(x-y-16)}{\cancel{(x+y)}(x-y+16)}$$ $x\neq -y, y-16$

$$\frac{x-y-16}{x-y+16}$$

(e) $$\frac{125x^3-8y^3}{16x^2-24xy+9y^2} \div \frac{25x^2-4y^2}{6y^2+7xy-20x^2}$$

$$x \neq \frac{3}{4}y \,;\, \pm\frac{2}{5}y$$

$$\frac{(\cancel{5x-2y})(25x^2+10xy+4y^2)}{(4x-3y)^2} \cdot \frac{\overset{-1}{(\cancel{3y-4x})}(2y+\cancel{5x})}{(\cancel{5x-2y})(\cancel{5x+2y})}$$

$$-\frac{(25x^2+10xy+4y^2)}{4x-3y}$$

(f) $$\frac{\overset{y}{}\,5}{\underset{y\;(x-2y)^2}{x^2-4xy+4y^2}} - \frac{1\;\;(-1)(x-2y)}{\underset{y(2y-x)(-1)(x-2y)}{2y^2-xy}}$$

$$\frac{5y+x-2y}{y(x-2y)^2}$$

$$\frac{3y+x}{y(x-2y)^2} \qquad y \neq 0, \frac{x}{2}$$

(g) $$\frac{x-100}{10-\sqrt{x}}$$ Express with a rationalized denominator.

$$\frac{\overset{-1}{(\cancel{\sqrt{x}-10})}(\sqrt{x}+10)}{\cancel{10-\sqrt{x}}}$$

$$x \neq 100$$

$$-(\sqrt{x}+10)$$

3. If $f(x)=7x-\frac{2}{5}$, find the value of $\frac{f(b)-f(a)}{b-a}$.

f(x) is a linear function, so the average rate of change is the slope, which is 7.

4. Find the inverse of $y=-\left(4-\frac{3}{x+1}\right)$

$$x = -\left(4-\frac{3}{y+1}\right)$$

$$x = -4 + \frac{3}{y+1}$$

$$x+4 = \frac{3}{y+1}$$

$$(x+4)(y+1) = 3$$

$$y+1 = \frac{3}{x+4}$$

$$y = \frac{3}{x+4} - 1 \;,\; x \neq -4$$

5. Show by composition whether $g(x)$ and $h(x)$ are inverses of each other:

$$g(x)=\frac{(x-4)^3}{2}+9, \; h(x)=4+\sqrt[3]{2x-18}$$

Yes they are since $f(g(x))=g(f(x))=x$

$$g(h(x))$$

$$g\left(4+\sqrt[3]{2x-18}\right) = \frac{(\cancel{4}+\sqrt[3]{2x-18}-\cancel{4})^3}{2}+9 = \frac{(\sqrt[3]{2x-18})^{\cancel{3}}}{2}+9$$

$$= \frac{2x-18}{2}+9$$

$$x-9+9 = x$$

$$h(g(x))$$

$$h\left(\frac{(x-4)^3}{2}+9\right) = 4+\sqrt[3]{2\left(\frac{(x-4)^3}{2}+9\right)-18}$$

$$= 4+\sqrt[3]{(x-4)^3+18-18}$$

$$= 4+\sqrt[3]{(x-4)^3}$$

$$= 4+x-4 = x$$

6. (a) Create 3 functions, $f(x), g(x)$, and $h(x)$, such that $(f \circ g \circ h)(x)=\frac{1}{\sqrt[4]{(7x+3)}}$.

(Note: You may not use the identity function.)

$$f(x) = \frac{1}{x}$$

$$g(x) = \sqrt[4]{x}$$

$$h(x) = 7x+3$$

(b) Create 4 functions, $f(x), g(x), h(x)$ and $k(x)$, such that $(f \circ g \circ h \circ k)(x) = \frac{1}{\sqrt[4]{(7x+3)}}$.

(Note: You may not use the identity function.)

$f(x) = \frac{1}{x}$

$g(x) = \sqrt[4]{x}$

$h(x) = x+3$

$k(x) = 7x$

7. If $f(x) = \frac{3}{x-3}$ and $g(x) = \frac{1}{x} - 3$, then find

$x \neq 0, \frac{1}{6}$

(a) $f(g(x))$

$f\left(\frac{1}{x}-3\right) = \frac{3}{\frac{1}{x}-3-3} = \frac{3}{\frac{1}{x}-6}$ or $\frac{3x}{1-6x}$

(b) $g(f(x))$

$g\left(\frac{3}{x-3}\right) = \frac{1}{\frac{3}{x-3}} - 3 = \frac{x-3}{3} - 3$ $x \neq 3$

$\frac{x-3-9}{3}$ $\frac{x-12}{3}$

(c) $f(f(x))$

$f\left(\frac{3}{x-3}\right) = \frac{3}{\frac{3}{x-3}-3} = \frac{3x-9}{3-3(x-3)}$ $x \neq 3, 4$

$\frac{3x-9}{3-3x+9} = \frac{3x-9}{12-3x}$

or $\frac{\cancel{3}(x-3)}{\cancel{3}(4-x)}$

(d) $g(g(x))$

$g\left(\frac{1}{x}-3\right) = \frac{1}{\frac{1}{x}-3} - 3 = \frac{x}{1-3x} - 3$

$\frac{x-3(1-3x)}{1-3x}$

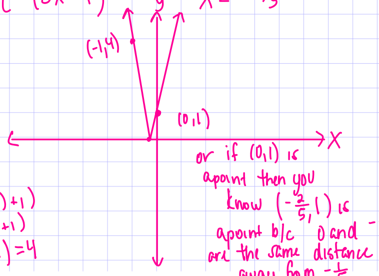
$\frac{x-3+9x}{1-3x}$

$\frac{10x-3}{1-3x}$ $x \neq \frac{1}{3}, 0$

8. Rewrite as a piecewise function and then sketch the graph:

(a) $y = |5x+1|$

$y = \begin{cases} 5x+1 & \text{if } 5x+1 \geq 0,\ x \geq -\frac{1}{5} \\ -(5x+1) & x < -1/5 \end{cases}$



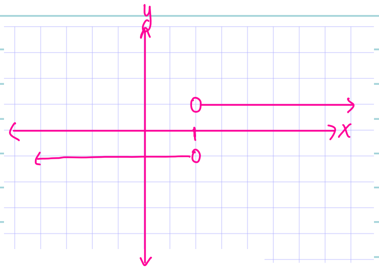
$x=0$
$y=1$

$x=-1$
$y=-(5(-1)+1)$
$y=-(-5+1)$
$y=-(-4)=4$

or if (0,1) is apoint then you know $(-\frac{2}{5}, 1)$ is apoint b/c 0 and $-2/5$ are the same distance away from $-\frac{1}{5}$

(b) $y = \frac{|2-x|}{x-2}$

$y = \begin{cases} \frac{2-x}{x-2} = -1 & \text{if } 2-x>0,\ 2>x,\ x<2 \\ \frac{-(2-x)}{x-2} = 1 & x>2 \end{cases}$



9. Solve for x using the geometric definition of absolute value.

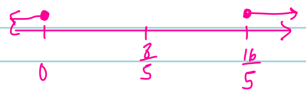
(a) $|8-5x| - 2 \geq 6$ (Express your answer in interval notation.)

$|5x-8| \geq 8$

$5|x - 8/5| \geq 8$

$|x - 8/5| \geq 8/5$

x's distance from $8/5 \geq 8/5$



$(-\infty, 0] \cup [\frac{16}{5}, \infty)$

(b) $3|2-7x| - 5 = 16$

$3|7x-2| - 5 = 16$

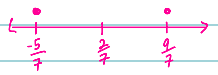
$3|7x-2| = 21$

$|7x-2| = 7$

$7|x - \frac{2}{7}| = 7$

$|x - \frac{2}{7}| = 1$

x's distance from $\frac{2}{7} = 1$



$\{-\frac{5}{7}, \frac{9}{7}\}$

10. Solve for x using the geometric definition of absolute value.

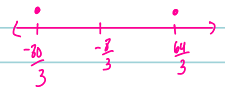
(a) $\left|\frac{3}{4}x+2\right|-3=15$.

$$\left|\frac{3}{4}x+2\right| = 18$$

$$\frac{3}{4}\left|x+\frac{8}{3}\right| = 18$$

$$\left|x+\frac{8}{3}\right| = 24$$

x's distance from $-8/3 = 24$



$$\left\{-\frac{80}{3}, \frac{64}{3}\right\}$$

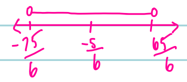
(b) $\left|\frac{6x+5}{7}\right|-9<1$ (Express your answer in set builder notation.)

$$\left|\frac{6x+5}{7}\right| < 10$$

$$\frac{6}{7}\left|x+\frac{5}{6}\right| < 10$$

$$\left|x+\frac{5}{6}\right| < \frac{70}{6}$$

x's distance from $-\frac{5}{6} < \frac{70}{6}$



$$\left\{x \,\middle|\, -\frac{75}{6} < x < \frac{65}{6}\right\}$$

11. Solve for x and express your answer on a number line.

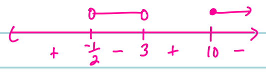
(a) $\frac{1}{x-3} \le \frac{3}{2x+1}$ (Express your answer in interval notation.)

$$\frac{1}{x-3} - \frac{3}{2x+1} \le 0$$

$$\frac{2x+1-3(x-3)}{(x-3)(2x+1)} \le 0$$

$$\frac{2x+1-3x+9}{(x-3)(2x+1)} \le 0$$

$$\frac{10-x}{(x-3)(2x+1)} \le 0$$



$$\left(-\frac{1}{2}, 3\right) \cup [10, \infty)$$

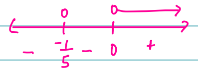
(b) $25x^3+3x>2x(1-5x)$ (Express your answer in set builder notation.)

$$25x^3+3x > 2x-10x^2$$

$$25x^3+10x^2+x > 0$$

$$x(25x^2+10x+1) > 0$$

$$x(5x+1)^2 > 0$$



$$\{x \mid x>0\}$$

12. Find $\frac{f(x+h)-f(x)}{h}$ for each given function.

(a) $f(x)=\sqrt{6x-1}$

$$\frac{\left(\sqrt{6x+6h-1}-\sqrt{6x-1}\right)}{h} \cdot \frac{\left(\sqrt{6x+6h-1}+\sqrt{6x-1}\right)}{\left(\sqrt{6x+6h-1}+\sqrt{6x-1}\right)}$$

$$\frac{6x+6h-1-(6x-1)}{h\left(\sqrt{6x+6h-1}+\sqrt{6x+1}\right)} = \frac{6\cancel{h}}{\cancel{h}\left(\sqrt{6x+6h-1}+\sqrt{6x-1}\right)}$$

$h \neq 0$

$x > \frac{1}{6}$

$x > \frac{1-6h}{6}$

$\sqrt{6x+6h-1}+\sqrt{6x-1} \neq 0$

(b) $f(x)=4x^2-11x+5$

$$\frac{4(x+h)^2-11(x+h)+5-(4x^2-11x+5)}{h}$$

$$\frac{\cancel{4x^2}+8xh+4h^2\cancel{-11x}-11h\cancel{+5}\cancel{-4x^2}\cancel{+11x}\cancel{-5}}{h}$$

$$\frac{\cancel{h}(8x+4h-11)}{\cancel{h}}$$

$8x+4h-11 \qquad h \neq 0$

(c) $f(x)=\frac{7}{x-9}$

$$\frac{\frac{7}{x+h-9} - \frac{7}{x-9}}{h}$$

$$\frac{\overset{7x-63}{7(x-9)} - \overset{-7x-7h+63}{7(x+h-9)}}{h(x+h-9)(x-9)}$$

$$\frac{-7\cancel{h}}{\cancel{h}(x+h-9)(x-9)} = \frac{-7}{(x+h-9)(x-9)}$$

$x \neq 9, 9-h$

$h \neq 0$

(d) $f(x) = \dfrac{x}{x-10}$

$$\frac{\frac{x+h}{x+h-10} - \frac{x}{x-10}}{h}$$

$$\frac{(x+h)(x-10) - x(x+h-10)}{h(x+h-10)(x-10)}$$

$$\frac{\cancel{x^2} + \cancel{xh} - \cancel{10x} - 10h - \cancel{x^2} - \cancel{xh} + \cancel{10x}}{h(x+h-10)(x-10)}$$

$$\frac{-10\cancel{h}}{\cancel{h}(x+h-10)(x-10)}$$

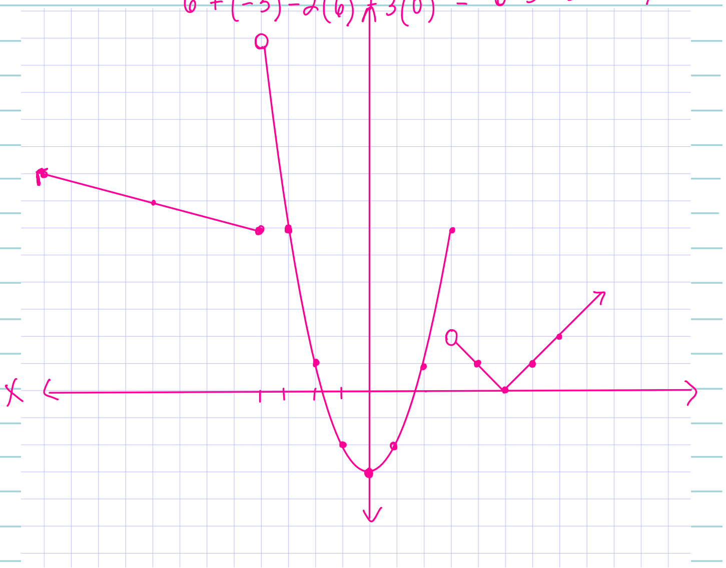
$x \neq 10,\ 10-h$

$h \neq 0$

13. (a) Sketch the piecewise function: $f(x) = \begin{cases} 5 - \frac{1}{4}x & \text{if} \quad x \le -4 \\ x^2 - 3 & \text{if} \quad -4 < x \le 3 \\ |x-5| & \text{if} \quad x > 3 \end{cases}$

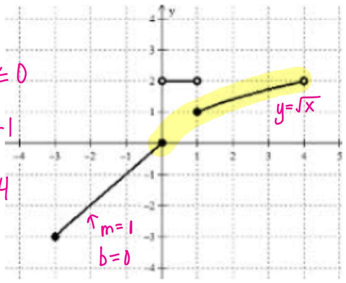
(b) Find the value of $f(3) + f(0) - 2f(-4) + 3f(5)$

$6 + (-3) - 2(6) + 3(0) = 6 - 3 - 12 = -9$



14. Write a piecewise function for the graph shown below:

$f(x) = \begin{cases} x & \text{if } -3 \le x \le 0 \\ 2 & 0 < x < 1 \\ \sqrt{x} & 1 \le x < 4 \end{cases}$



15. In the function $f(x) = \frac{5}{2}x + b$, b is a constant. If $f(8) = 3$, what is the value of $f(-4)$?

$3 = \frac{5}{\cancel{2}}(\cancel{8}^{4}) + b$

$3 = 20 + b$

$-17 = b$

$f(x) = \frac{5}{2}x - 17$

$f(-4) = \frac{5}{2}(-4) - 17 = -10 - 17 = -27$

16. Write the equation of a line in point-slope form that is parallel to $6x - 7y = 3$ and passes through $(-1,5)$.

$6x - 7y = 3$

$6x - 3 = 7y$

$\frac{6}{7}x - 3/7 = y$

$m = 6/7$

$m_{\parallel} = \frac{6}{7}$

$y - 5 = \frac{6}{7}(x+1)$

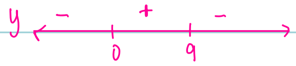
17. Sketch each graph. Label all x and y intercepts with their coordinates.

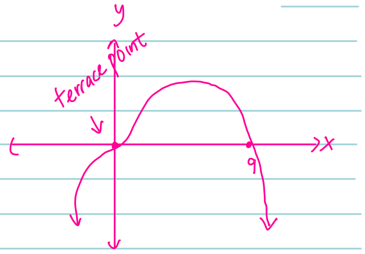
(a) $y = 9x^3 - x^4$

$y = x^3(9 - x)$

zeros: 0 (triple), 9

y-int: (0,0)





(b) $y = x^3 - 7x^2 - 4x + 28$

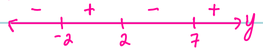
$y = x^2(x-7) - 4(x-7)$

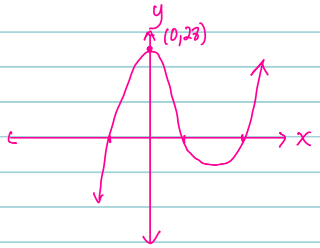
$y = (x^2-4)(x-7)$

$y = (x-2)(x+2)(x-7)$

zeros: $\pm 2, 7$

y-int: (0, 28)

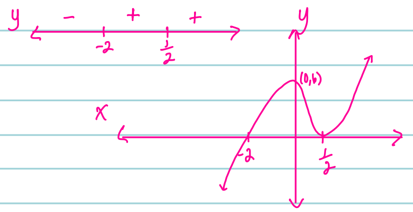




(c) $y = 3(2x-1)^2(x+2)$

zeros: $\frac{1}{2}$ (double), -2

y-int: (0, 6)



18. The table below shows a selection of corresponding t and v(t) values for a polynomial v(t). State an interval in which v(t) is guaranteed to have a root.

t (minutes)	0	12	20	24	40
v(t) (meters per minute)	0	200	240	–220	150

(20, 24) by the IVT if v(20) > 0 and v(24) < 0 then v(t) = 0 at some point in the interval.

or

(24, 40) by the IVT if v(24) < 0 and v(40) > 0 then v(t) = 0 at some point in the interval.

19. Given $P(x) = 3x^3 - 5x^2 + 25x + 9$, find all of the roots of $P(x)$.

prz: $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 3} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{3}$

$P(1) \neq 0$

$P(-1) \neq 0$

$$\begin{array}{r|rrrr} 3 & 3 & -5 & 25 & 9 \\ & & 9 & 12 & \\ \hline & 3 & 4 & 37 & \end{array}$$

$$\begin{array}{r|rrrr} -3 & 3 & -5 & 25 & 9 \\ & & -9 & 42 & \\ \hline & 3 & -14 & & \end{array}$$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -5 & 25 & 9 \\ & & 1 & & \\ \hline & 3 & -4 & & \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -5 & 25 & 9 \\ & & -1 & 2 & -9 \\ \hline & \underbrace{3 \quad -6 \quad 27}_{\div 3} & & & 0 \end{array}$$

$(3x+1)(x^2-2x+9) = 0$

$x = -\frac{1}{3}$

$x^2 - 2x + 9 = 0$

$x^2 - 2x + 1 = -9 + 1$

$(x-1)^2 = -8$

$x - 1 = \pm\sqrt{-8}$

$x - 1 = \pm 2i\sqrt{2}$

$x = 1 \pm 2i\sqrt{2}$

$\left\{-\frac{1}{3}, 1 \pm 2i\sqrt{2}\right\}$

20. Is there *guaranteed* to be a real root of $P(x) = x^3 - 7x^2 + 5x - 8$ between 1 and 5? Justify your work with an explanation.

$P(1) = 1 - 7 + 5 - 8 < 0$

$P(5) = 125 - 175 + 125 - 8 < 0$

Since $P(1) < 0$ and $P(5) < 0$ there is not guaranteed to be a root between 1 and 5 (no sign change)

21. Sketch each function. Label any asymptotes with their equations and holes with their coordinates. State the coordinates of all x and y intercepts and state the domain.

(a) $y = \frac{6-4x}{2x+3} = \frac{2(3-2x)}{2x+3}$

holes: none

VA: $x = -3/2$

HA: $y = -2$

cross? no

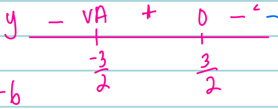
$\frac{6-4x}{2x+3} = -2$

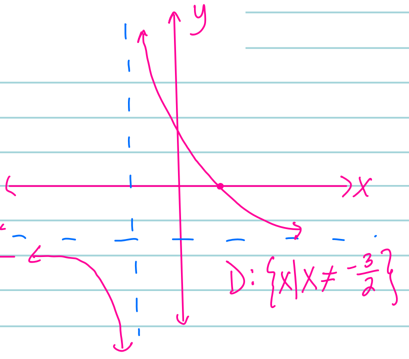
$6 - 4x = -6x - 6$

$12 = -2x \quad x = -6$

x-int: $\left(\frac{3}{2}, 0\right)$

y-int: (0, 2)





D: $\{x \mid x \neq -\frac{3}{2}\}$

(b) $$y=\frac{x^2-3x-10}{x-1}$$

$y=\frac{(x-5)(x+2)}{(x-1)}$

holes: none

VA: $x=1$

HA: none

OA: $y=x-2$

x-int: $(-2,0)$, $(5,0)$

y-int: $(0,10)$

$1|\ \ 1\ \ -3\ \ \cancel{-10}$

$1\ \ \cancel{-2}$

$1\ \ -2$

x-int $(2,0)$

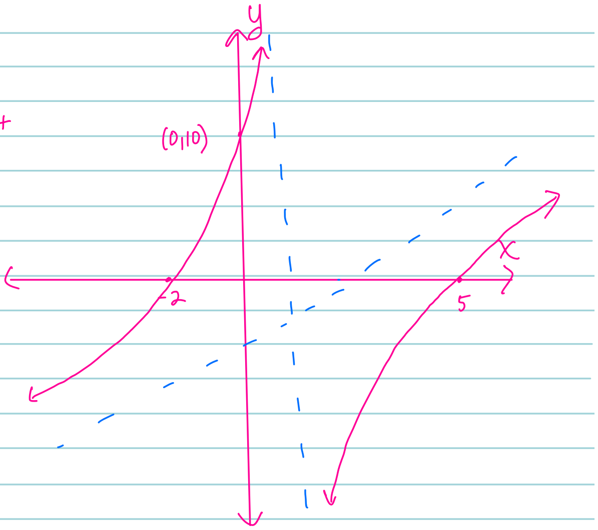
Cross? no

$x-2 = \frac{x^2-3x-10}{x-1}$

$\cancel{x^2}-\cancel{3x}+2 \neq \cancel{x^2}-\cancel{3x}-10$

– 0 + VA – 0 +

-2 1 5



D: $\{x \mid x \neq 1\}$

(c) $$y=\frac{x^2-3x+2}{x-1}$$

$y=\frac{(x-2)\cancel{(x-1)}}{\cancel{x-1}}$

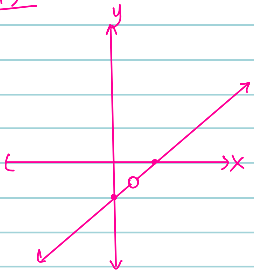
RF: $y=x-2$

hole: $(1,-1)$

no asymptotes

x-int: $(2,0)$

y-int: $(0,-2)$



D: $\{x \mid x \neq 1\}$

(d) $$y=\frac{x-1}{x^2-3x+2}$$

$y=\frac{\cancel{x-1}^{\,1}}{\cancel{(x-1)}(x-2)}$

RF $y=\frac{1}{x-2}$

hole: $(1,-1)$

VA: $x=2$

HA: $y=0$

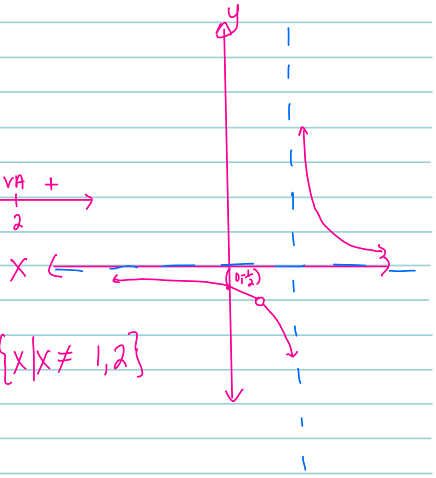
cross? no

x-int: none

y-int: $(0,-\frac{1}{2})$

y – VA +

2



D: $\{x \mid x \neq 1,2\}$

(e) $$y=\frac{3x^2}{x^3+x}$$

$y=\frac{3x^{\cancel{2}}}{\cancel{x}(x^2+1)}$

RF: $y=\frac{3x}{x^2+1}$

hole: $(0,0)$

VA: none

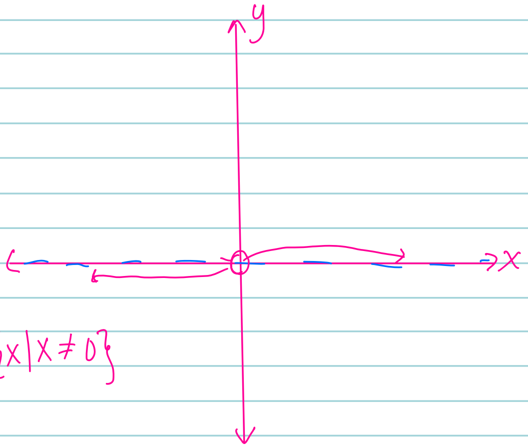
HA: $y=0$

cross? $(0,0)$

y – hole +

0

x-int: none } b/c of hole
y-int: none



D: $\{x \mid x \neq 0\}$

(f) $y=\frac{x^2-9x}{x^2-1}$

$y=\frac{x(x-9)}{(x-1)(x+1)}$

hole: none

VA: $x=\pm 1$ x-int: $(0,0),(9,0)$

HA: $y=1$ y-int: $(0,0)$

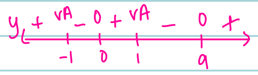
cross? $(\frac{1}{9},1)$

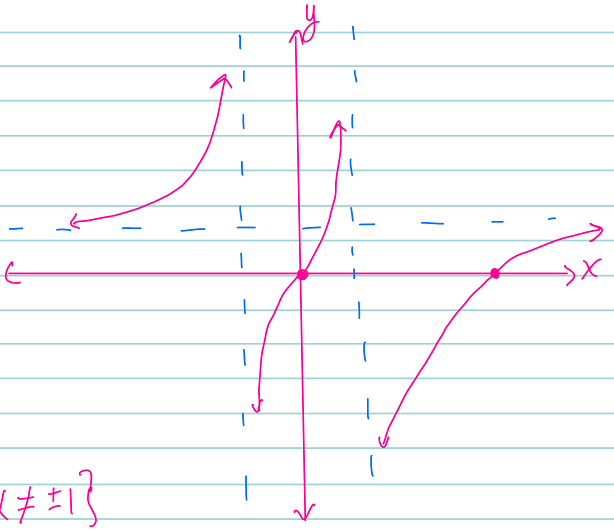
$\frac{x^2-9x}{x^2-1}=1$

$x^2-9x=x^2-1$

$-9x=-1$

$x=\frac{1}{9}$





D: $\{x \mid x \neq \pm 1\}$

22. Determine algebraically if each function is even, odd, or neither.

(a) $f(x)=x^3-4x^2+2x-3$ $f(-x)=-x^3-4x^2-2x-3$ neither

(b) $f(x)=\frac{2-x^2}{x^3}$ $f(-x)=\frac{2-x^2}{-x^3}$ odd

(c) $f(x)=\frac{x^3+x}{x^5}$ $f(-x)=\frac{-x^3-x}{-x^5}$ even

(d) $f(x)=x^2\sqrt{4-x^6}$ $f(-x)=x^2\sqrt{4-x^6}$ even

23. State the coordinates of the point(s) where each crosses its end behavior asymptote.

↑ horizontal or oblique

(a) $y=\frac{x^2+1}{x^2+x-2}$

HA: $y=1$

$1=\frac{x^2+1}{x^2+x-2}$

$x^2+x-2=x^2+1$

$x-2=1$

$x=3$ $(3,1)$

(b) $y=\frac{(x+1)(x-1)^2}{x^2}=\frac{(x+1)(x^2-2x+1)}{x^2}$

$=\frac{x^3-2x^2+x+x^2-2x+1}{x^2}$

HA: none

OA: $y=x-1$

$y=\frac{x^3-x^2-x+1}{x^2}$

$$\begin{array}{r} x-1 \\ x^2\overline{)\,x^3-x^2-x+1} \\ \underline{x^3} \\ -x^2 \\ \underline{-x^2} \\ -x \end{array}$$

$x-1=\frac{x^3-x^2-x+1}{x^2}$

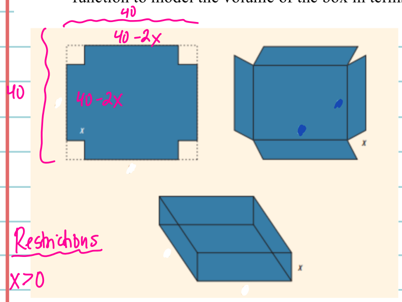
$x^3-x^2=x^3-x^2-x+1$

$0=-x+1$ $(1,0)$

$x=1$

↑ plug into $y=x-1$

24. A square piece of cardboard measures 40 cm on each side. Congruent squares of length x cm are cut from the four corners and the sides are folded up to form a rectangular open top box. Find a function to model the volume of the box in terms of x.



$V = lwh$

$V(x) = (40-2x)(40-2x)\,x \qquad 0 < x < 20$

Restrictions

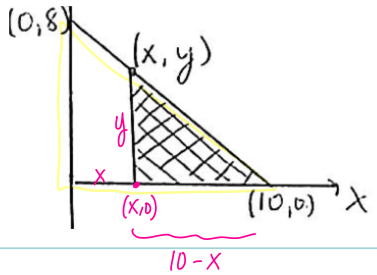
$x > 0$

$40 - 2x > 0$

$40 > 2x$

$20 > x$

25. Find the area of the shaded triangle as a function of x:



$A = \frac{1}{2}bh$

$A(x) = \frac{1}{2}(10-x)\,y \leftarrow$ need y in terms of x

$A(x) = \frac{1}{2}(10-x)(8 - \frac{4}{5}x), \quad 0 < x < 10$

$\frac{8}{10} = \frac{y}{10-x}$

$80 - 8x = 10y$

$8 - \frac{8}{10}x = y$

$8 - \frac{4}{5}x = y$

Restrictions

$x > 0$

$x < 10$

26. The vertices of a rectangle are at $(0,0),(x,0),(0,y)$ and (x,y), with (x,y) as a point on the graph of $y = 8 - x^3$ in the first quadrant. Express the area of the rectangle as a function of x.

$\uparrow \; -x^3 + 8$

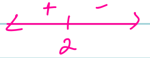
x^3 reflected over x-axis ↑8

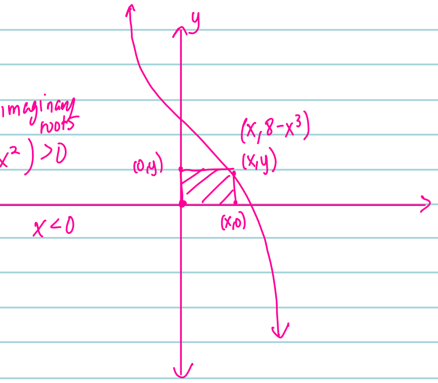
Restrictions

$x > 0$

$8 - x^3 > 0 \quad \swarrow$ imaginary roots

$(2-x)(4+2x+x^2) > 0$

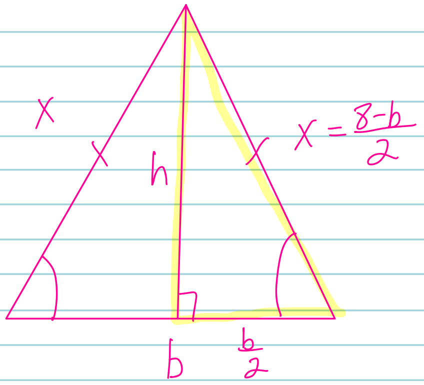




$A = lw$

$A(x) = x(8-x^3), \quad 0 < x < 2$

27. An isosceles triangle has a perimeter of 8 in. Find a function that models its area in terms of the length of its base.



$$2x+b=8$$

$$2x=8-b$$

$$x=\frac{8-b}{2}$$

$$A=\frac{1}{2}bh$$

$$A(b)=\frac{1}{2}b\left(\sqrt{16-4b}\right),$$

$$0<b<4$$

$$\left(\frac{b}{2}\right)^2+h^2=\left(\frac{8-b}{2}\right)^2$$

$$\frac{b^2}{4}+h^2=\frac{64-16b+b^2}{4}$$

$$h^2=\frac{64-16b+\cancel{b^2}}{4}-\frac{\cancel{b^2}}{4}$$

$$h^2=\frac{64-16b}{4}$$

$$h^2=16-4b$$

$$h=\pm\sqrt{16-4b}$$

Restrictions

$b>0$

$16-4b>0$

$-4b>-16$

$b<4$

$\frac{8-b}{2}>0$

$8-b>0$

$-b>-8$

$b<8$

A graphing calculator is required for Questions 28 – 29.

28. Solve for x graphically: $x-4+\frac{2x}{x^2-1}=2$. Draw a complete graph and label the window.

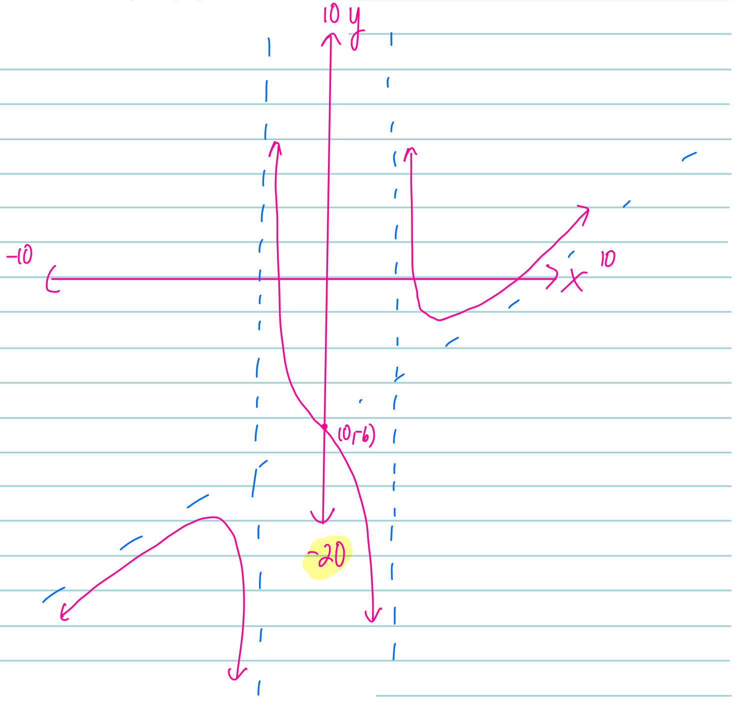
$x-4+\frac{2x}{x^2-1}-2=0$

$y=0$

PVA: $x=\pm 1$

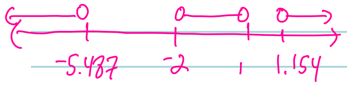
EB: $y=x-4+0-2=x-6$

$\{-.865, 1.231, 5.633\}$



29. Solve for x graphically: $\frac{3x-5}{x+2}+\frac{1}{x-1}<6$. Draw a complete graph and label the window. Show your solution on a number line and in interval notation.

↑ not necessary

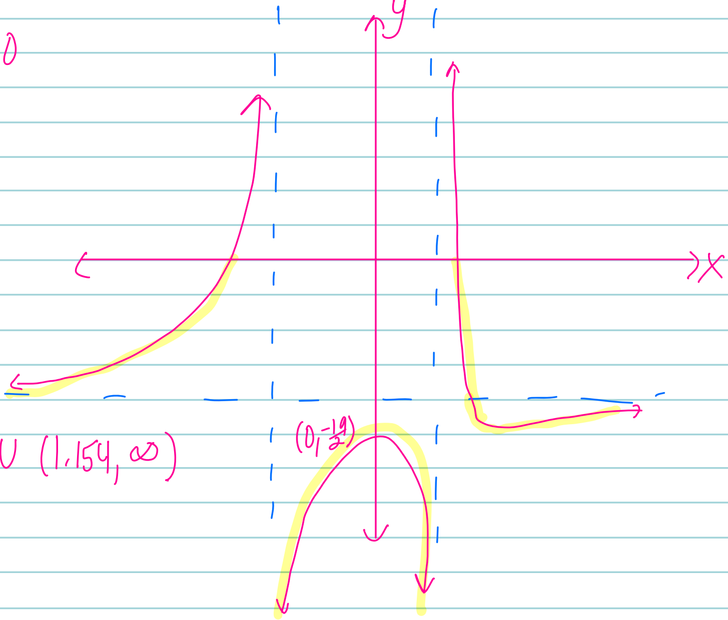


$\frac{3x-5}{x+2}+\frac{1}{x-1}-6<0$

$y<0$

PVA: $x=-2,1$

HA: $y=3+0-6=-3$



$(-\infty,-5.487)\cup(-2,1)\cup(1.154,\infty)$