1. Factor completely:

(a) $5(x-7)^4(x+1)^2+2(x+1)(x-7)^5$

$(x-7)^4(x+1)\big(5(x+1)+2(x-7)\big)$

$(x-7)^4(x+1)(5x+5+2x-14)$

$(x-7)^4(x+1)(7x-9)$

(b) $5x^{\frac{1}{2}}+33x^{-\frac{1}{2}}-14x^{-\frac{3}{2}}$

$x^{-3/2}(5x^2+33x-14)$

$x^{-3/2}(5x-2)(x+7)$, $x>0$

(c) $x^{\frac{3}{5}}+3x^{\frac{2}{5}}-25x^{\frac{1}{5}}-75$

$x^{2/5}(x^{\frac{1}{5}}+3)-25(x+3)$

$(x^{2/5}-25)(x^{\frac{1}{5}}+3)$

$(x^{\frac{1}{5}}-5)(x^{\frac{1}{5}}+5)(x^{\frac{1}{5}}+3)$

(d) x^4+14x^2+81 using advanced completing the square method

$x^4+14x^2+81+4x^2-4x^2$

$x^4+18x^2+81-4x^2$

$(x^2+9)^2-4x^2$

$(x^2-2x+9)(x^2+2x+9)$

2. Express in simplest form:

(a) $\dfrac{(2x-1)^2\cdot4(3x+5)^3\cdot3-2(2x-1)\cdot2(3x+5)^4}{(2x-1)^4}$

$\dfrac{12(2x-1)^2(3x+5)^3-4(2x-1)(3x+5)^4}{(2x-1)^4}$

$\dfrac{4(2x-1)(3x+5)^3\big(3(2x-1)-(3x+5)\big)}{(2x-1)^4}$

$\dfrac{4(2x-1)(3x+5)^3(6x-3-3x-5)}{(2x-1)^4}$

$\dfrac{4\cancel{(2x-1)}(3x+5)^3(3x-8)}{(2x-1)^{\cancel{4}\,3}}$

$\dfrac{4(3x+5)^3(3x-8)}{(2x-1)^3}$ $x\neq\frac{1}{2}$

(b) $\dfrac{49y^{-2}-x^{-2}}{x^{-2}-12(xy)^{-1}+35y^{-2}}$

$\dfrac{x^2y^2\,\frac{49}{y^2}-\frac{1}{x^2}\,x^2y^2}{x^2y^2\,\frac{1}{x^2}-\frac{12}{xy}\,x^2y^2+\frac{35}{y^2}\,x^2y^2}$

$x,y\neq0$

$y\neq7x,5x$

$\dfrac{49x^2-y^2}{y^2-12xy+35x^2}$

$\dfrac{\overset{-1}{\cancel{(7x-y)}}(7x+y)}{\cancel{(y-7x)}(y-5x)}=\dfrac{-(7x+y)}{y-5x}$

(c) $\dfrac{\frac{5(x+h)}{(x+h)-4}-\frac{5x}{x-4}}{h}$

$(x+h-4)(x-4)$ $(x+h-4)(x-4)$ $h((x+h)-4)(x-4)$

$\dfrac{5(x+h)(x-4)-5x(x+h-4)}{h(x+h-4)(x-4)}$

$\dfrac{5(x^2+hx-4x-4h)-5x^2-5xh+20x}{h(x+h-4)(x-4)}$

$\dfrac{\cancel{5x^2}+\cancel{5hx}-\cancel{20x}-20h-\cancel{5x^2}-\cancel{5xh}+\cancel{20x}}{h(x+h-4)(x-4)}$

$h\neq0$

$x\neq4,\ 4-h$

$\dfrac{-20\cancel{h}}{\cancel{h}(x+h-4)(x-4)}=\dfrac{-20}{(x+h-4)(x-4)}$

(d) $\dfrac{x^2-y^2-16x-16y}{x^2-y^2+16x+16y}$

$\dfrac{(x-y)(x+y)-16(x+y)}{(x-y)(x+y)+16(x+y)}$

take out GCF on top & bottom

$\dfrac{\cancel{(x+y)}(x-y-16)}{\cancel{(x+y)}(x-y+16)}$ $x\neq-y,\ y-16$

$\dfrac{x-y-16}{x-y+16}$

(e) $\dfrac{125x^3-8y^3}{16x^2-24xy+9y^2} \div \dfrac{25x^2-4y^2}{6y^2+7xy-20x^2}$

$x \neq \frac{3}{4}y; \pm\frac{2}{5}y$

$\dfrac{(\cancel{5x-2y})(25x^2+10xy+4y^2)}{(4x-3y)^2} \cdot \dfrac{(\cancel{3y-4x})(2y+\cancel{5x})}{(\cancel{5x-2y})(5x+\cancel{2y})}$

$-\dfrac{(25x^2+10xy+4y^2)}{4x-3y}$

(f) $\dfrac{5}{x^2-4xy+4y^2} - \dfrac{1}{2y^2-xy}$

$\dfrac{y}{y}\;\dfrac{5}{(x-2y)^2} \qquad \dfrac{(-1)(x-2y)}{y(2y-x)(-1)(x-2y)}$

$\dfrac{5y+x-2y}{y(x-2y)^2}$

$\dfrac{3y+x}{y(x-2y)^2} \qquad y \neq 0, \frac{x}{2}$

(g) $\dfrac{x-100}{10-\sqrt{x}}$ Express with a rationalized denominator.

$\dfrac{(\cancel{\sqrt{x}-10})(\sqrt{x}+10)}{\cancel{10-\sqrt{x}}}$

$-(\sqrt{x}+10) \qquad x \neq 100$

3. If $f(x)=7x-\dfrac{2}{5}$, find the value of $\dfrac{f(b)-f(a)}{b-a}$.

f(x) is a linear function, so the average rate of change is the slope, which is 7.

4. Find the inverse of $y=-\left(4-\dfrac{3}{x+1}\right)$

$x = -\left(4-\dfrac{3}{y+1}\right)$

$x = -4+\dfrac{3}{y+1}$

$x+4 = \dfrac{3}{y+1}$

$(x+4)(y+1) = 3$

$y+1 = \dfrac{3}{x+4}$

$y = \dfrac{3}{x+4}-1, \quad x \neq -4$

5. Show by composition whether $g(x)$ and $h(x)$ are inverses of each other:

$g(x)=\dfrac{(x-4)^3}{2}+9, \; h(x)=4+\sqrt[3]{2x-18}$

Yes they are since $h(g(x)) = g(h(x)) = x$

$g(h(x))$

$g\left(4+\sqrt[3]{2x-18}\right) = \dfrac{(\cancel{4}+\sqrt[3]{2x-18}-\cancel{4})^3}{2}+9 = \dfrac{(\sqrt[3]{2x-18})^3}{2}+9$

$= \dfrac{2x-18}{2}+9$

$x-9+9 = x$

$h(g(x))$

$h\left(\dfrac{(x-4)^3}{2}+9\right) = 4+\sqrt[3]{2\left(\dfrac{(x-4)^3}{2}+9\right)-18}$

$= 4+\sqrt[3]{(x-4)^3+18-18}$

$= 4+\sqrt[3]{(x-4)^3}$

$= 4+x-4 = x$

6. (a) Create 3 functions, $f(x), g(x)$, and $h(x)$, such that $(f \circ g \circ h)(x)=\dfrac{1}{\sqrt[4]{(7x+3)}}$.

(Note: You may not use the identity function.)

$f(x) = \dfrac{1}{x}$

$g(x) = \sqrt[4]{x}$

$h(x) = 7x+3$

(b) Create 4 functions, $f(x), g(x), h(x)$ and $k(x)$, such that $(f \circ g \circ h \circ k)(x) = \frac{1}{\sqrt[4]{(7x+3)}}$.

(Note: You may not use the identity function.)

$f(x) = \frac{1}{x}$
$g(x) = \sqrt[4]{x}$
$h(x) = x+3$
$k(x) = 7x$

7. If $f(x) = \frac{3}{x-3}$ and $g(x) = \frac{1}{x} - 3$, then find

$x \neq 0, \frac{1}{6}$

(a) $f(g(x))$

$f\left(\frac{1}{x}-3\right) = \frac{3}{\frac{1}{x}-3-3} = \frac{3}{\frac{1}{x}-6}$ or $\frac{3x}{1-6x}$

(b) $g(f(x))$

$g\left(\frac{3}{x-3}\right) = \frac{1}{\frac{3}{x-3}} - 3 = \frac{x-3}{3} - 3$ $\quad x \neq 3$

$\frac{x-3-9}{3} \quad \frac{x-12}{3}$

(c) $f(f(x))$

$f\left(\frac{3}{x-3}\right) = \frac{3}{\frac{3}{x-3}-3} = \frac{3x-9}{3-3(x-3)}$ $\quad x \neq 3, 4$

$\frac{3x-9}{3-3x+9} = \frac{3x-9}{12-3x}$

(d) $g(g(x))$

$g\left(\frac{1}{x}-3\right) = \frac{1}{\frac{1}{x}-3} - 3 = \frac{x}{1-3x} - 3$

$\frac{x-3(1-3x)}{1-3x}$

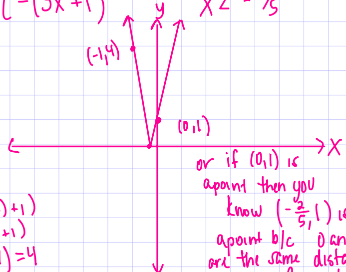
$\frac{x-3+9x}{1-3x}$

$\frac{10x-3}{1-3x}$ $\quad x \neq \frac{1}{3}, 0$

8. Rewrite as a piecewise function and then sketch the graph:

(a) $y = |5x+1|$

$y = \begin{cases} 5x+1 & \text{if } 5x+1 \geq 0,\ x \geq -\frac{1}{5} \\ -(5x+1) & x < -\frac{1}{5} \end{cases}$



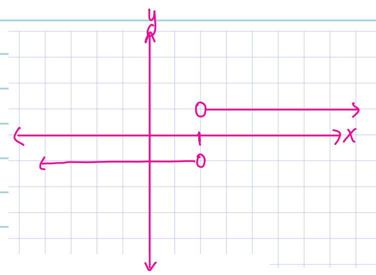
$x=0$
$y=1$

$x=-1$
$y=-(5(-1)+1)$
$y=-(-5+1)$
$y=-(-4)=4$

or if (0,1) is apoint then you know $\left(-\frac{2}{5}, 1\right)$ is apoint b/c 0 and $-\frac{2}{5}$ are the same distance away from $-\frac{1}{5}$

(b) $y = \frac{|2-x|}{x-2}$

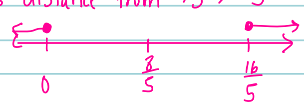
$y = \begin{cases} \frac{2-x}{x-2} = -1 & \text{if } 2-x>0,\ 2>x,\ x<2 \\ \frac{-(2-x)}{x-2} = 1 & x>2 \end{cases}$



9. Solve for x using the geometric definition of absolute value.

(a) $|8-5x| - 2 \geq 6$ (Express your answer in interval notation.)

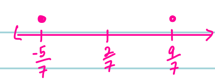
$|5x-8| \geq 8$
$5\left|x-\frac{8}{5}\right| \geq 8$
$\left|x-\frac{8}{5}\right| \geq \frac{8}{5}$
x's distance from $\frac{8}{5} \geq \frac{8}{5}$



$(-\infty, 0] \cup \left[\frac{16}{5}, \infty\right)$

(b) $3|2-7x| - 5 = 16$

$3|7x-2| - 5 = 16$
$3|7x-2| = 21$
$|7x-2| = 7$
$7\left|x-\frac{2}{7}\right| = 7$
$\left|x-\frac{2}{7}\right| = 1$
x's distance from $\frac{2}{7} = 1$



$\left\{-\frac{5}{7}, \frac{9}{7}\right\}$

10. Solve for x using the geometric definition of absolute value.

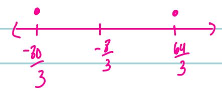
(a) $\left|\frac{3}{4}x+2\right|-3=15$.

$\left|\frac{3}{4}x+2\right| = 18$

$\frac{3}{4}\left|x+\frac{8}{3}\right| = 18$

$\left|x+\frac{8}{3}\right| = 24$

x's distance from $-8/3 = 24$



$\left\{-\frac{80}{3}, \frac{64}{3}\right\}$

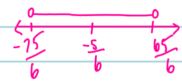
(b) $\left|\frac{6x+5}{7}\right|-9<1$ (Express your answer in set builder notation.)

$\left|\frac{6x+5}{7}\right| < 10$

$\frac{6}{7}\left|x+\frac{5}{6}\right| < 10$

$\left|x+\frac{5}{6}\right| < \frac{70}{6}$

x's distance from $-\frac{5}{6} < \frac{70}{6}$



$\left\{x \mid -\frac{75}{6} < x < \frac{65}{6}\right\}$