Name: ____ Date: ___________
PCH: Constructing Rational Functions Given Characteristics Ms. Loughran

Do Now:

1. Without a calculator, sketch the graph of $y = \dfrac{x-2}{x^2-3x-4}$

$(x-4)(x+1)$

no holes

VA: $x = -1, 4$

HA: $y = 0$

cross? $(2,0)$

$\dfrac{x-2}{(x-4)(x+1)} = 0$

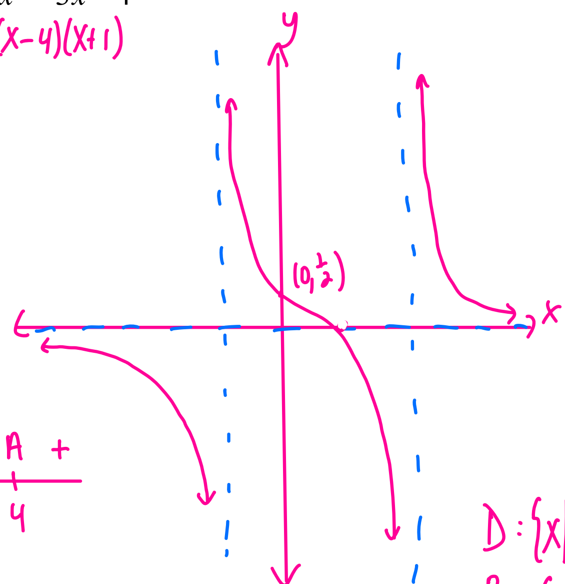
$x - 2 = 0$

$x = 2$

x-int: $(2,0)$

y-int: $(0, \frac{1}{2})$

y: $-$ VA $+$ 0 $-$ VA $+$ at -1, 2, 4



D: $\{x \mid x \neq -1, 4\}$

R $(-\infty, \infty)$

Classwork:

1. Construct a rational function with the following characteristics:

Horizontal Asymptote: $y = 0$

Vertical Asymptote: $x = -3$

Hole at $(2,5)$

degree of num < degree of denom.
in the denom. of RF you have $(x+3)$
$(x-2)$ is divided out of original function
and when $x = 2$ is plugged into the RF
you get $y = 5$

$$y = \frac{a(x-2)}{(x+3)(x-2)} = \frac{25(x-2)}{(x+3)(x-2)}$$

RF: $(2,5)$

$y = \dfrac{a}{x+3}$

$5 = \dfrac{a}{2+3}$

$a = 25$

$$y = \frac{25x - 50}{x^2 + x - 6}$$

2. Construct a rational function with the following characteristics:

Horizontal Asymptote: $y = 0$
Vertical Asymptote: $x = 5$
Hole at $(2,3)$

degree of num < degree of den.
(X-5) is a factor of RF's denom.
divide (X-2) out of original and when you plug X=2 into RF you get y=3

$$y = \frac{a(x-2)}{(x-5)(x-2)}$$

RF: (2,3)

$$y = \frac{a}{x-5}$$

$$3 = \frac{a}{-3}$$

$$a = -9$$

$$y = \frac{-9(x-2)}{x^2-7x+10} = \frac{-9x+18}{x^2-7x+10}$$

3. Construct a rational function with the following characteristics:

Oblique Asymptote: $y = x - 2$
Vertical Asymptote: $x = -1$

degree of num. > degree of denom.
in the den. of RF (X+1)

think of

$$2\tfrac{1}{2} = \frac{2(2)+1}{2}$$

$$y = x - 2 + \frac{1}{x+1}$$

← would be any # here except for 0
If a = 0 then you would have a hole at X = -1 instead of a VA
let's say you did:

$$y = \frac{\overbrace{(x-2)(x+1)}^{x^2-x-2}+1}{x+1}$$

$$y = \frac{x^2-x-1}{x+1}$$

$$y = x - 2 + \frac{5}{x+1}$$

$$y = \frac{(x-2)(x+1)+5}{x+1}$$

$$y = \frac{x^2-x+3}{x+1}$$

4. Construct a rational function with the following characteristics:

Horizontal asymptote: *none*
Vertical Asymptotes: $x = 0,\ x = 2$

degree of num > degree of denom.
in RF there are factors of $x(x-2)$

$$y = \frac{x^3 + 3}{x(x-2)}$$

← create a numerator that doesn't create a hole

or

$$y = \frac{x^4 + 1}{x(x-2)}$$

5. Construct a rational function with the following characteristics:

Horizontal asymptote: $y = 0$
Vertical Asymptote: $x = 1$
Hole at $x = -3$

degree of num < degree of den.
$x-1$ is a factor of RF
divide out $(x+3)$ in original

$$y = \frac{a(x+3)}{(x+3)(x-1)}$$

where a can be anything

choose an a bc they are asking for one rational functions

$$y = \frac{2(x+3)}{(x+3)(x-1)} = \frac{2x+6}{x^2+2x-3}$$

or if $a=1$ $\quad y = \dfrac{x+3}{x^2+2x-3}$

6. Construct a rational function with the following characteristics:

Horizontal asymptote: $y = 0$ — d of n < d of den.

Vertical Asymptotes: $x = 0$ and $x = 2$ — $x(x-2)$ in den. of RF

Hole at $(3, 7)$ — $(x-3)$ divides out and when we plug $x=3$ into RF you get $y=7$

$$y = \frac{a(x-3)}{x(x-2)(x-3)}$$

RF:

$$7 = \frac{a}{3(3-2)}$$

$$a = 21$$

$$y = \frac{21(x-3)}{x(x-2)(x-3)} = \frac{21x-63}{x^3-5x^2+6x}$$

7. Construct a rational function with the following characteristics:

Oblique Asymptote: $y = x - 2$ — degree of num > degree of den.

Vertical Asymptote: $x = -1$ — factor of RF den. $(x+1)$

x-intercepts: $(-2, 0)$ and $(3, 0)$

$$y = x - 2 + \frac{a}{x+1}$$

$$y = \frac{(x+1)(x-2)+a}{x+1}$$

$$y = \frac{x^2-x-2+a}{x+1}$$

when $x^2-x-2+a=0$

$(x+2)(x-3)=0$

$x^2-x-6=0$

$-2+a=-6$

$a=-4$

$$y = \frac{x^2-x-6}{x+1}$$

8. Construct a rational function with the following characteristics:

Horizontal asymptote: $y = 4$ degree of num = degree of den.

Vertical Asymptote: $x = 0$ and $x = 2$ RF den. $x(x-2)$

$$y = \frac{4x^2+1}{x(x-2)} = \frac{4x^2+1}{x^2-2x}$$

or

$$y = \frac{4x^2-3}{x(x-2)} = \frac{4x^2-3}{x^2-2x}$$

Homework 01-05

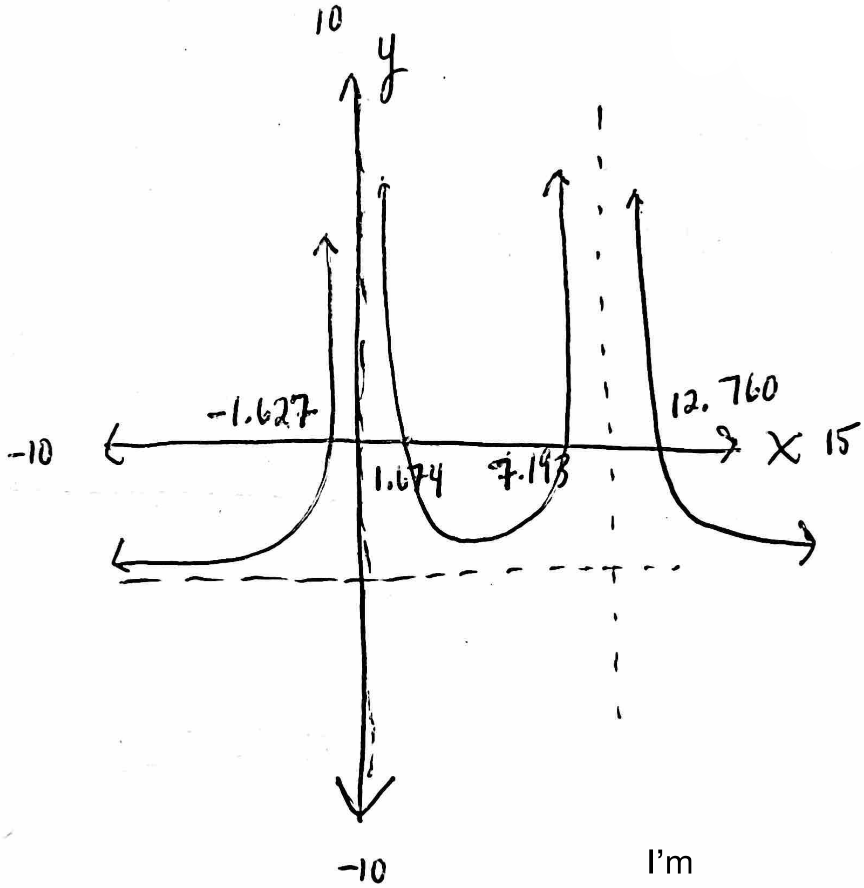
1. Solve the following equation graphically by doing each of the following:
   (a) Draw a complete graph of the function showing all intercepts and asymptotes.
   (b) Write the window settings you use on your graph.
   (c) Find the solution set

$$\frac{10}{x^2}+\frac{30}{(10-x)^2}=4$$

$$\frac{10}{x^2}+\frac{30}{(10-x^2)}-4=0$$

PVA : $x = 0, 10$

EB : $y = 0+0-4 = -4$



I'm

$\{-1.627, 1.674, 7.193, 12.760\}$

2. Solve the following rational inequality below graphically by doing the following:

(a) Draw a complete graph of the function showing all intercepts and asymptotes.

(b) Write the window settings you use on your graph.

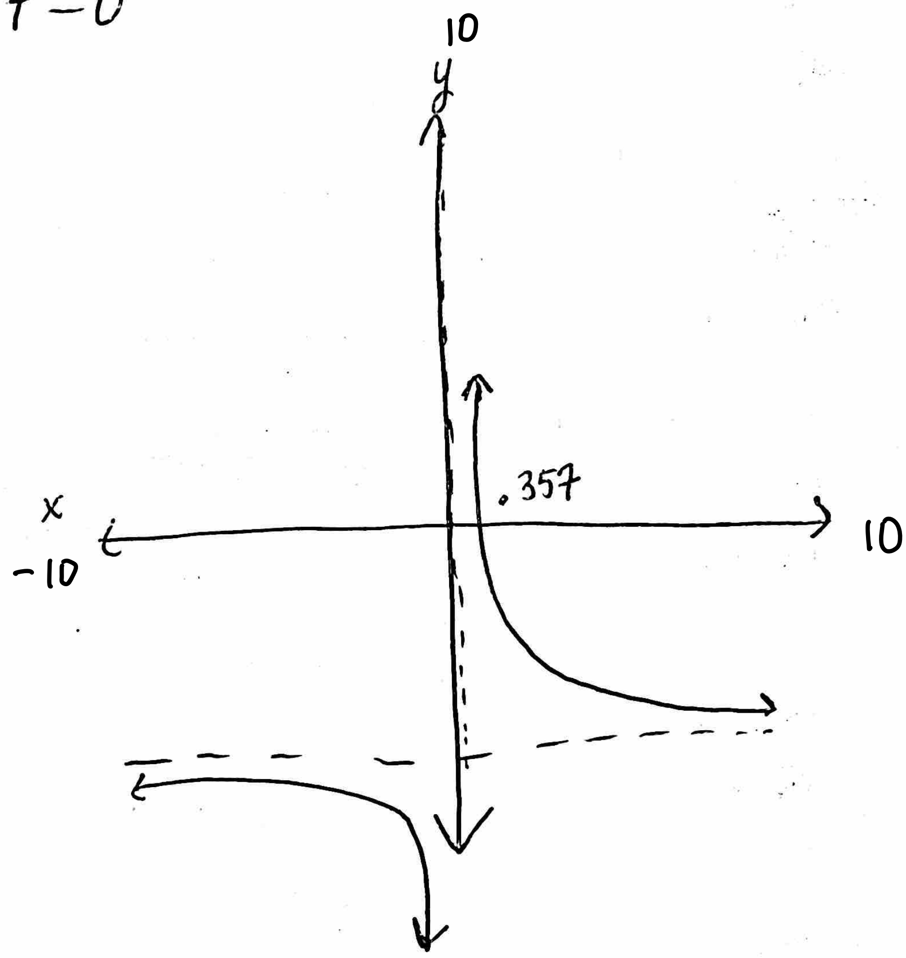
(c) State the solution set using both set builder notation and interval notation.

$$\frac{2}{x}+\frac{1}{2x}\leq 7$$

$$\frac{2}{X}+\frac{1}{2X}-7\leq 0$$

PVA: $X=0$

EB: $y=0+0-7=-7$



$(-\infty,0)\cup[.357,\infty)$

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PCH: Some Review Ms. Loughran

1. Solve using the geometric definition of absolute value and place your solution set in interval notation.

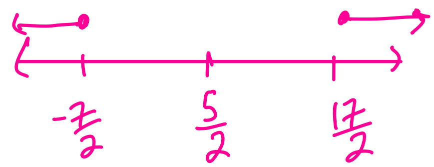
$$\left|\frac{5-2x}{4}\right| \geq 3$$

$\left|\frac{2x-5}{4}\right| \geq 3$

$\frac{1}{2}\left|x-\frac{5}{2}\right| \geq 3$

$\left|x-\frac{5}{2}\right| \geq 6$

x's distance from $\frac{5}{2} \geq 6$



$\left(-\infty, -\frac{7}{2}\right] \cup \left[\frac{17}{2}, \infty\right)$

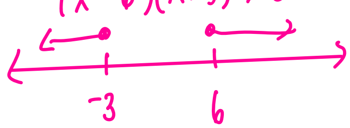
2. Given $f(x) = x^2 - 3x - 18$ (D: $\mathbb{R}$) and $g(x) = \sqrt{x}$, find $g(f(x))$ and its domain.

$g(f(x)) = \sqrt{x^2-3x-18}$

$D_{g(f(x))}$: $(-\infty, -3] \cup [6, \infty)$

$x^2-3x-18 \geq 0$

$(x-6)(x+3) \geq 0$



3. The volume of a cylindrical can with a top and a bottom is to be 16π in^3. Express the total surface area as a function of its radius, r.

$V = \pi r^2 h$

$16\cancel{\pi} = \cancel{\pi} r^2 h$

$\frac{16}{r^2} = h$

$r > 0$

$SA(r) = 2\pi r^2 + 2\pi \cancel{r}\left(\frac{16}{r^{\cancel{2}}}\right)$

$SA(r) = 2\pi r^2 + \frac{32\pi}{r}$, $r > 0$

4. Show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ given $f(x) = 2-(2x+1)^3$

$f(x) = 2-(2x+1)^3$

$x = 2-(2y+1)^3$

$x-2 = -(2y+1)^3$

$-x+2 = (2y+1)^3$

$\sqrt[3]{2-x} = 2y+1$

$\frac{\sqrt[3]{2-x}-1}{2} = y = f^{-1}(x)$

$f(f^{-1}(x)) = f^{-1}(f(x))$

$2-\left(\cancel{2}\left(\frac{\sqrt[3]{2-x}-1}{\cancel{2}}\right)+1\right)^3 = \frac{\sqrt[3]{2-(2-(2x+1)^3}-1}{2}$

$2-\left(\sqrt[3]{2-x}\cancel{-1}\cancel{+1}\right)^3$

$2-\left(\sqrt[3]{2-x}\right)^3$

$2-(2-x)$

x

$\frac{\sqrt[3]{\cancel{2}-\cancel{2}+(2x+1)^3}-1}{2}$

$\frac{\sqrt[3]{(2x+1)^3}-1}{2}$

$\frac{2x\cancel{+1}\cancel{-1}}{2}$

x

5. If $f(x) = \dfrac{2x-3}{x+1}$, find $f^{-1}(3)$.

$\frac{2x-3}{x+1} = 3$

$2x-3 = 3x+3$

$-6 = x$

$f^{-1}(3) = -6$