Name:____ Date:______________

PCH: Operations on Matrices Ms. Loughran

Two matrices are equal if they have the same order $m \times n$ and their corresponding entries are equal.

entry in row 2 column 1

1. Solve for a_{11}, a_{12}, a_{21}, and a_{22} in the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

entry in row 1, column 1

$a_{11} = 2$
$a_{12} = -1$
$a_{21} = -3$
$a_{22} = 0$

Matrix Addition

You can add two matrices (of the same order) by adding their corresponding entries. The sum of two matrices of different orders is undefined.

2. $\underbrace{\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}}_{2\times2} + \underbrace{\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}}_{2\times2} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5. The sum of $A = \underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix}}_{3\times3}$ and $B \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}}_{3\times2}$ is undefined

Scalar Multiplication

In work with matrices, numbers are usually referred to as scalars. For our purposes, scalars will always be real numbers. You can multiply a matrix A by a scalar c by multiplying each entry in A by c.

The symbol $-A$ represents the scalar product $(-1)A$. Moreover, if A and B are of the same order, $A-B$ represents the sum of A and $(-1)B$. That is,

$$A-B=A+(-1)B \qquad \text{(Subtraction of matrices)}$$

6. For the following matrices, find (a) $3A$
   (b) $-B$
   (c) $3A-B$

$$A=\begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } B=\begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

a) $3A=\begin{bmatrix} 6 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix}$

b) $-B=\begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix}$

c) $3A-B=\begin{bmatrix} 4 & 6 & 12 \\ -10 & 4 & -6 \\ 7 & 0 & 4 \end{bmatrix}$

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ and let c and d be scalars.

1. $A + B = B + A$ (commutative prop of matrix addition)

2. $A + (B + C) = (A + B) + C$ (associative prop of matrix addition)

3. $(cd)A = c(dA)$ (associative prop. of matrix multiplication)

4. $IA = A$ (scalar identity)

5. $c(A + B) = cA + cB$ (distributive prop.)

6. $(c + d)A = cA + dA$ (distributive prop.)

7. $$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

8. Solve for X in the equation $3X + A = B$, where

$$-A \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}.$$

$3X = B - A$

$X = \frac{1}{3}(B - A)$

$$B - A = \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix} \qquad X = \begin{bmatrix} -\frac{4}{3} & 2 \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

$\frac{1}{3}(B - A)$

Matrix Multiplication

To find the entries of the product, multiply each row of A by each column of B. Note that the number of columns of A must be equal to the of rows of B.

$3\times 2 \qquad 2\times 2$

3×2

AB

9. Find the product AB where

$$A=\begin{bmatrix}-1 & 3\\ 4 & -2\\ 5 & 0\end{bmatrix} \text{ and } B=\begin{bmatrix}-3 & 2\\ -4 & 1\end{bmatrix}.$$

$$\begin{bmatrix}-1(-3)+3(-4) & -1(2)+3(1)\\ 4(-3)+(-2)(-4) & 4(2)+-2(1)\\ 5(-3)+0(-4) & 5(2)+0(1)\end{bmatrix}=\begin{bmatrix}-9 & 1\\ -4 & 6\\ -15 & 10\end{bmatrix}$$

$2\times 3 \qquad 3\times 3$

2×3

10. $\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & -2\end{bmatrix}\begin{bmatrix}-2 & 4 & 2\\ 1 & 0 & 0\\ -1 & 1 & -1\end{bmatrix}=$

$$\begin{bmatrix}1(-2)+0(1)+3(-1) & 1(4)+0(0)+3(1) & 1(2)+0(0)+3(-1)\\ 2(-2)+(-1)(1)+(-2)(-1) & 2(4)+(-1)(0)+(-2)(1) & 2(2)+(-1)(0)+(-2)(-1)\end{bmatrix}$$

$$\begin{bmatrix}-5 & 7 & -1\\ -3 & 6 & 6\end{bmatrix}$$

11. $\begin{bmatrix}3 & 4\\ -2 & 5\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3+0 & 0+4\\ -2+0 & 0+5\end{bmatrix}=\begin{bmatrix}3 & 4\\ -2 & 5\end{bmatrix}$

12. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$ $\begin{bmatrix} -1+2 & 2-2 \\ -1+1 & 2-1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

13. $\underset{1\times 3}{\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}}\underset{3\times 1}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}} =$ $\underset{1\times 1}{\begin{bmatrix} 2+2-3 \end{bmatrix}} = \begin{bmatrix} 1 \end{bmatrix}$

14. $\underset{3\times 1}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}}\underset{1\times 3}{\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}} =$

$$\underset{3\times 3}{\begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}}$$

3×2 3×4

15. Find the product of AB. If $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$.

undefined

Practice

Perform the indicated operation when possible.

1) $-4\begin{bmatrix} 5 & 1 \\ 6 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 6 & -5 & 3 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$

3) $-4w\begin{bmatrix} -w & -4+u & 0 \\ v & 5v & 3wv \end{bmatrix}$

4) $\begin{bmatrix} -4 & -5 & 5 \\ 1 & 6 & 3 \\ -2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -6 & -1 & -6 \\ 6 & -3 & -2 \\ 4 & -1 & -3 \end{bmatrix}$

5) $\begin{bmatrix} -5wu \\ 6 \\ v-1 \end{bmatrix} - \left(\begin{bmatrix} -5v \\ 6v \\ 5u+6 \end{bmatrix} - \begin{bmatrix} -3v \\ -5 \\ 3vu \end{bmatrix} \right)$

6) $\begin{bmatrix} -3y & 3x \\ -2 & -4x+2 \\ y^2 & 2x \end{bmatrix} - \begin{bmatrix} x & x-2 \\ 4 & y \\ x-1 & xy \end{bmatrix}$

7) $\begin{bmatrix} -4b \\ 2b \\ 6b \end{bmatrix} + 2\begin{bmatrix} 3a \\ ab \\ a+4 \end{bmatrix}$

8) $-5\left(\begin{bmatrix} 1 & 0 \\ -2 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} -5 & 4 \\ -6 & 0 \\ 4 & 4 \end{bmatrix} \right)$

9) $\begin{bmatrix} 3 & -3 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 & 1 \\ 6 & -5 & 4 \end{bmatrix}$

10) $\begin{bmatrix} 3 & 1 \\ -3 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -4 & -1 \end{bmatrix}$

11) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \\ 3 & 4 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} -2 & 6 \\ 0 & 0 \end{bmatrix}$

12) $\begin{bmatrix} 4 & -2 \\ -3 & 6 \end{bmatrix} \cdot \left(\begin{bmatrix} 4 & -5 & 4 & 0 \\ 6 & 3 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ -6 & -1 \end{bmatrix} \right)$

Practice

① $\begin{bmatrix} -20 & -4 \\ -24 & 0 \end{bmatrix}$

② $\begin{bmatrix} 7 & -5 & 4 & -5 \end{bmatrix}$

③ $\begin{bmatrix} 4w^2 & 16w-4uw & 0 \\ -4vw & -20vw & -12vw^2 \end{bmatrix}$

④ $\begin{bmatrix} 2 & -4 & 11 \\ -5 & 9 & 5 \\ -6 & 3 & 4 \end{bmatrix}$

⑤ $\begin{bmatrix} -5wu \\ 6 \\ v-1 \end{bmatrix} - \begin{bmatrix} -2v \\ 6v+5 \\ 5u+6-3vu \end{bmatrix} = \begin{bmatrix} -5wu+2v \\ 1-6v \\ v-7-5u+3vu \end{bmatrix}$

⑥ $\begin{bmatrix} -3y-x & 2x+2 \\ -6 & -4x+2-y \\ y^2-x+1 & 2x-xy \end{bmatrix}$

⑦ $\begin{bmatrix} -4b \\ 2b \\ 6b \end{bmatrix} + \begin{bmatrix} 6a \\ 2ab \\ 2a+8 \end{bmatrix} = \begin{bmatrix} -4b+6a \\ 2b+2ab \\ 6b+2a+8 \end{bmatrix}$

⑧ $-5 \begin{bmatrix} -4 & 4 \\ -8 & -3 \\ 10 & -2 \end{bmatrix} = \begin{bmatrix} 20 & -20 \\ 40 & 15 \\ -50 & 10 \end{bmatrix}$

⑨ $\begin{bmatrix} -12 & 33 & -9 \\ 30 & 21 & 18 \end{bmatrix}$

⑩ $\begin{bmatrix} -1 & -10 \\ 13 & 13 \end{bmatrix}$

(11) $$\begin{array}{c} 4\times 2 \\ \begin{bmatrix} 3 & 2 \\ 2 & 1 \\ 3 & 4 \\ -1 & -1 \end{bmatrix} \end{array} \begin{array}{c} 2\times 2 \\ \begin{bmatrix} 0 & 0 \\ -4 & 12 \end{bmatrix} \end{array} = \begin{bmatrix} -8 & 24 \\ -4 & 12 \\ -16 & 48 \\ 4 & -12 \end{bmatrix}$$

(12) $$\begin{bmatrix} 4 & -2 \\ -3 & 6 \end{bmatrix} \cdot \text{undefined}$$

Order of SH: Mr. Incredible → Dash → Jack-Jack → Edna → Elastigirl → Voyd → Family → Frozone → Violet → Logo

Sample Solutions:

Frozone

$$\begin{cases} x-2y+z=1 \\ y+2z=5 \\ x+y+3z=8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 1 & 1 & 3 & 8 \end{array}\right]$$

$-1 \quad 2 \quad -1 \quad -1$

$$-R_1+R_3 \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 3 & 2 & 7 \end{array}\right]$$

$-0 \quad -3 \quad -6 \quad -15$

$$-3R_2+R_3 \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$$-\tfrac{1}{4}R_3 \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z=2$

$y+2z=5$
$y+4=5$
$y=1$

$x-2y+z=1$
$x\cancel{-2}\cancel{+2}=1$
$x=1$

$(1,1,2)$

Answer: (1, 1, -2)

Disney · PIXAR

INCREDIBLES 2

Logo

$$\begin{cases} x+y+z=2 \\ 2x-3y+2z=4 \\ 4x+y-3z=1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -3 & 1 \end{array}\right]$$

$-2 \quad -2 \quad -2 \qquad -4$

$$-2R_1+R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{array}\right] -\tfrac{1}{5}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{array}\right]$$

$-4 \quad -4 \quad -4 \quad -8$

$-4R_1+R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -7 & -7 \end{array}\right]$$

$0 \quad 3 \quad 0 \qquad 0$

$3R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -7 & -7 \end{array}\right]$$

$$-\tfrac{1}{7}R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z=1$

$y=0$

$x+y+z=2$

$x+0+1=2$

$x+1=2$

$x=1$

$(1, 0, 1)$

Answer: (-1, 4, 0)

$$\begin{cases} x+y+z=4 \\ -x+2y+3z=17 \\ 2x-y=-7 \end{cases}$$

Mr. Incredible

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ -1 & 2 & 3 & 17 \\ 2 & -1 & 0 & -7 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 2 & -1 & 0 & -7 \end{array}\right]$$

$-2 \quad -2 \quad -2 \quad -8$

$-2R_1+R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & -3 & -2 & -15 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$x+y+z=4$
$x+3+3=4$
$x=-2$

$(-2,+3,3)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & \frac{4}{3} & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z=3$

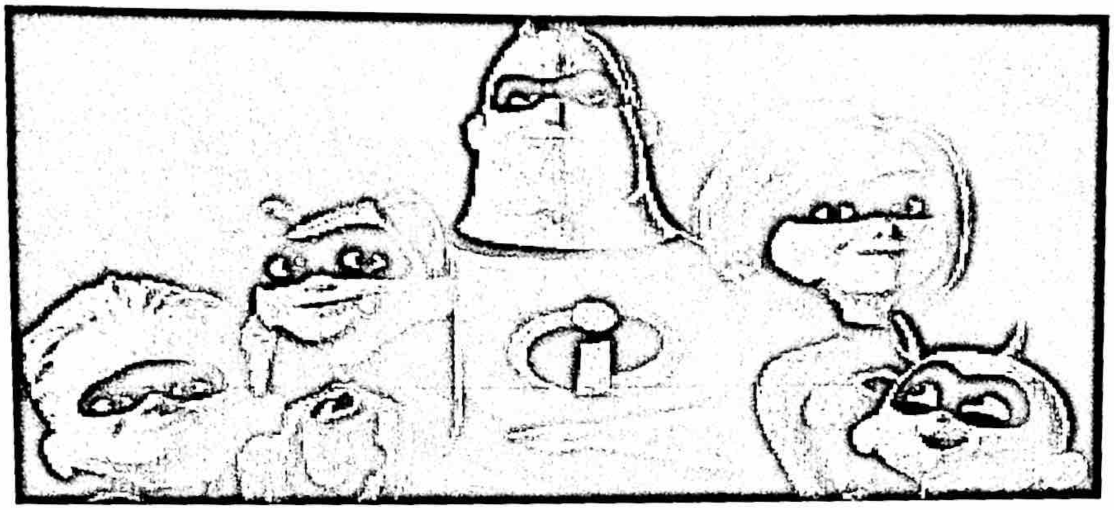
$y+\frac{4}{3}z=7$

Answer: (1, 0, 1)

$y+\frac{4}{3}(3)=7$

$y+4=7$

$y=+3$



Incredible Family

$$\begin{cases} 10x+10y-20z=60 \\ 15x+20y+30z=-25 \\ -5x+30y-10z=45 \end{cases}$$

$x+y-2z=6$
$3x+4y+6z=-5$
$-x+6y-2z=9$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 6 \\ 3 & 4 & 6 & -5 \\ -1 & 6 & -2 & 9 \end{array}\right]$ R_1+R_3

$(1,1,-2)$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 6 \\ 3 & 4 & 6 & -5 \\ 0 & 7 & -4 & 15 \end{array}\right]$ $-3R_1+R_2$
$-3\ -3\ 6\ -18$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 7 & -4 & 15 \end{array}\right]$

12
7
4
-23
7
161

$x+1+4=6$
$x=1$

Answer: (3, 1, 1)

$-7R_2+R_3$
$0\ -7\ -84\ +161$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 0 & -88 & 176 \end{array}\right]$ $-\frac{1}{77}R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$z=-2$
$y+12(-2)=-23$
$y-24=-23$
$y=1$

Edna

$$\left[\begin{array}{ccc|c} 2 & -3 & -1 & 13 \\ -1 & 2 & -5 & 6 \\ 5 & -1 & -1 & 49 \end{array}\right]$$

$-R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & -6 \\ 2 & -3 & -1 & 13 \\ 5 & -1 & -1 & 49 \end{array}\right]$$

$-2 \quad 4 \quad -10 \quad 12$

$-2R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & -6 \\ 0 & 1 & -11 & 25 \\ 5 & -1 & -1 & 49 \end{array}\right]$$

$-5R_1 + R_3 \quad -5 \quad 10 \quad -25 \quad 30$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & -6 \\ 0 & 1 & -11 & 25 \\ 0 & 9 & -26 & 79 \end{array}\right]$$

$-9R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & -6 \\ 0 & 1 & -11 & 25 \\ 0 & 0 & 73 & -146 \end{array}\right]$$

$0 \quad -9 \quad 99 \quad -225$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & -6 \\ 0 & 1 & -11 & 25 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$z = -2$

$y - 11z = 25$

$y + 22 = 25$

$y = 3$

$x - 2y + 5z = -6$

$x - 6 - 10 = -6$

$x - 16 = -6$

$x = 10$

Violet —

$$\begin{cases} x+y+6z=3 \\ x+y+3z=3 \\ x+2y+4z=7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 2 & 4 & 7 \end{array}\right] \xrightarrow{-R_1+R_2} \begin{array}{c} -1\ -1\ -6\ -3 \\ \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & -3 & 0 \\ 1 & 2 & 4 & 7 \end{array}\right] \end{array} \quad -\tfrac{1}{3}R_2 \curvearrowright R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 1 & 2 & 4 & 7 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$z=0$

$y-2z=4$

$y-0=4$

$y=4$

$x+y+6z=3$

$x+y+0=3$

$x+4=3$

$x=-1$

Answer: (1, 1, 2)

$(-1,4,0)$

Baby Jack-Jack

$$\begin{cases} 2y+z=4 \\ x+y=4 \\ 3x+3y-z=10 \end{cases}$$

$-3 \quad -3 \quad 0 \quad -12$

$$\left[\begin{array}{ccc|c} 0 & 2 & 1 & 4 \\ 1 & 1 & 0 & 4 \\ 3 & 3 & -1 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 2 & 1 & 4 \\ 3 & 3 & -1 & 10 \end{array}\right] \xrightarrow{-3R_1+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & -1 & -2 \end{array}\right]$$

$\frac{1}{2}R_2$

$-R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z=2$

$y+\frac{1}{2}z=2$

$y+\frac{1}{2}(2)=2$

$y+1=2$

$y=1$

$x+y=4$

$x+1=4$

$x=3$

$(3, 1, 2)$

Answer: (-1, 0, 1)

X

$2x - 0 = -2$

$2x = -2$

$x = -1$

$$\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$$

Elastigirl

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 2 & 0 & -1 & -2 \\ 3 & 5 & 2 & 22 \end{array}\right]$$

$-2R_1 + R_2$

$-2 \ -4 \ 2 \ -18$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & -4 & 1 & -20 \\ 3 & 5 & 2 & 22 \end{array}\right]$$

$-3R_1 + R_3$

$-3 \ -6 \ 3 \ -27$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & -4 & 1 & -20 \\ 0 & -1 & 5 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & -4 & 1 & -20 \\ 0 & 0 & -19 & 0 \end{array}\right]$$

$R_2 + 4R_3$

$-4R_3 \ [0 \ 4 \ -20 \ 20]$

$-\frac{1}{19}R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & -4 & 1 & -20 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$-\frac{1}{4}R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & 1 & -\frac{1}{4} & 5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Answer: (10, 3, -2)

$z = 0$

$y - \frac{1}{4}(0) = 5$

$y = 5$

$(-1, 5, 0)$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 7 \\ 2 & -1 & 1 & 6 \\ 3 & -2 & 4 & 11 \end{array}\right] \quad R_3 + -R_1 \quad \left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 2 & -1 & 1 & 6 \\ 3 & -2 & 4 & 11 \end{array}\right]$$

-2 -1 0 -7

Voyd

$$\begin{cases} 2x + y = 7 \\ 2x - y + z = 6 \\ 3x - 2y + 4z = 11 \end{cases}$$

-2 6 -8 -8

$-2R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 0 & 5 & -7 & -2 \\ 3 & -2 & 4 & 11 \end{array}\right]$$

-3 9 -12 -12

$-3R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 0 & 5 & -7 & -2 \\ 0 & 7 & -8 & -1 \end{array}\right]$$

$-7R_2 + 5R_3$

0 -35 49 14

0 35 -40 -5

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 0 & 5 & -7 & -2 \\ 0 & 0 & 9 & 9 \end{array}\right] \quad \tfrac{1}{9}R_3 \quad \left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 0 & 5 & -7 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

(3, 1, 1)

Answer: (-1, 5, 0)

$$\tfrac{1}{5}R_2 \left[\begin{array}{ccc|c} 1 & -3 & 4 & 4 \\ 0 & 1 & -\tfrac{7}{5} & -\tfrac{2}{5} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z = 1$

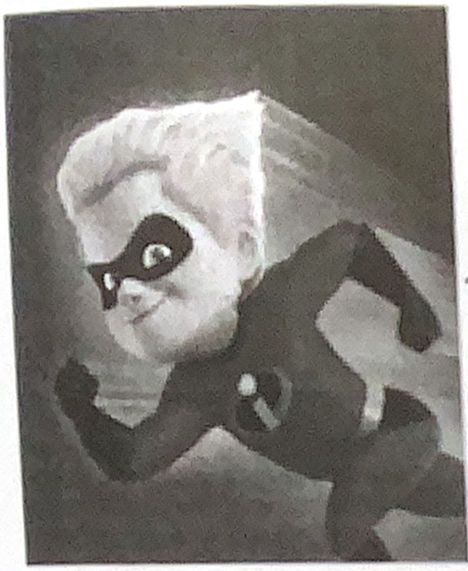
$y - \frac{7}{5} = -\frac{2}{5}$

$y = 1$

$x - 3 + 4 = 4$

$x + 1 = 4$

$x = 3$



Dash

$$\begin{cases} x+2y-z=-2 \\ x+z=0 \\ 2x-y-z=-3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{array}\right] \rightleftarrows \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 2 & -1 & -2 \\ 2 & -1 & -1 & -3 \end{array}\right]$$

$-1 \; 0 \; -1 \; 0$

$$-R_1+R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -2 \\ 2 & -1 & -1 & -3 \end{array}\right] \quad \tfrac{1}{2}R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 2 & -1 & -1 & -3 \end{array}\right]$$

$-2 \; 0 \; -2 \; 0$

$$-2R_1+R_3 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & -3 & -3 \end{array}\right] \quad R_2+R_3 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

Answer: (-2, 3, 3)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z=1 \quad y=0$

$y-z=-1$

$y-1=-1$

$x+y+z=0$

$x+0+1=0$

$(-1, 0, 1)$