Do Now: Perform the indicated operation when possible.

1) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

2) $\underset{3\times 2}{\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} g & h \\ k & m \end{bmatrix}} = \begin{bmatrix} ag+bk & ah+bm \\ cg+dk & ch+dm \\ eg+fk & eh+fm \end{bmatrix}$

Name:____ Date:_____________

PCH: Determinants of Square Matrices Ms. Loughran

Do Now:

1. Given: $A = \begin{bmatrix} 5 & 7 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$

   Find: (a) AB
   (b) BA

a) $AB = \begin{bmatrix} 68 & 3 \\ -3 & -6 \end{bmatrix}$ b) $BA = \begin{bmatrix} -1 & 7 \\ 48 & 63 \end{bmatrix}$

Determinants help us to see if a matrix is invertible. If $d \neq 0$ then the matrix is invertible (has an inverse).

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A det (A) = $|A|$ is $ad - bc$.

Find the determinant of each matrix.

1. $A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ det (A) or $|A| = 2(2) - (-3)(1) = 7$

2. $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ det (B) or $|B| = 4 - 4 = 0$

3. $C = \begin{bmatrix} 0 & \frac{3}{2} \\ 2 & 4 \end{bmatrix}$ det (C) or $|C| = 0 - 3 = -3$

The determinant of a matrix of order 1×1 is defined simply as the entry of the matrix.

4. $A = [-2]$ $\det(A) = |A| = -2$

Finding the determinant of a 3×3 matrix

1. Expand matrix by rewriting the matrix with first and second column repeated at the end.

2. Multiply along the diagonals running left to right, and add up numbers.

3. Multiply along the diagonals running right to left, and add up numbers.

4. Subtract what you got in step 3 from what you got in step 2.

$$-4 + 0 + 6$$

5. $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & 0 \end{matrix}$ $\det(A) = |A| = 16 - 2$

$= 14$

$$0 + 16 + 0$$

$$0 + 0 + 0$$

6. $B = \begin{bmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{bmatrix} \begin{matrix} 6 & 3 \\ 0 & 0 \\ 4 & -6 \end{matrix}$ $\det(B) = 0 - 0$

$= 0$

$$0 + 0 + 0$$

$0 + 90 + 0$

7. $C = \begin{bmatrix} 5 & -1 & 2 \\ 4 & 0 & 6 \\ -2 & 3 & 0 \end{bmatrix} \begin{matrix} 5 & -1 \\ 4 & 0 \\ -2 & 3 \end{matrix}$

$0 + 12 + 24$

$|C| = 36 - 90 = -54$

$-16 - 30 + 0$

8. $D = \begin{bmatrix} -3 & 8 & 4 \\ 0 & 1 & 2 \\ -4 & 5 & 2 \end{bmatrix} \begin{matrix} -3 & 8 \\ 0 & 1 \\ -4 & 5 \end{matrix}$

$-6 - 64 + 0$

$|D| = -70 - (-46)$

$|D| = -24$

$0 + 0 + 0$

9. $E = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 5 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 5 \\ 0 & 0 \end{matrix}$

$-10 + 0 + 0$

$|E| = -10 - 0$

$|E| = -10$

10. $F = \begin{bmatrix} 1 & a & b \\ 0 & c & d \\ 2 & x & y \end{bmatrix} \begin{matrix} 1 & a \\ 0 & c \\ 2 & x \end{matrix}$

$\det(F) = cy + 2ad - (2bc + dx)$
$= cy + 2ad - 2bc - dx$

11. What value of x makes the determinant – 4?

$0 + 0 + 0$

$\begin{bmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{bmatrix} \begin{matrix} -2 & 0 \\ -6 & x \\ -4 & 0 \end{matrix}$

$\det = 2x - (0) = 2x$

$2x = -4$

$x = -2$

$\begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix}$ ← means to find the determinant

$5 - 12 = -7$

Homework 02-01

Name: Key Date: ____________

PCH: Matrix Multiplication Practice

Find each product or state that it is undefined.

1) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$ $\begin{bmatrix} 6 & 0 \\ -27 & 12 \end{bmatrix}$

2) $\overset{2\times1}{\begin{bmatrix} 6 \\ -3 \end{bmatrix}} \cdot \overset{1\times2}{\begin{bmatrix} -5 & 4 \end{bmatrix}}$ $\begin{bmatrix} -30 & 24 \\ 15 & -12 \end{bmatrix}$

3) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$ $\begin{bmatrix} -5 & -10 \\ 8 & 13 \end{bmatrix}$

4) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$ $\begin{bmatrix} -13 & -19 \\ -11 & -1 \end{bmatrix}$

5) $\overset{3\times2}{\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix}} \cdot \overset{2\times2}{\begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}}$ $\begin{bmatrix} -10 & -20 \\ 10 & -16 \\ 18 & -24 \end{bmatrix}$

6) $\overset{2\times3}{\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix}} \cdot \overset{3\times2}{\begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}}$ $\begin{bmatrix} -14 & -3 \\ -19 & 22 \end{bmatrix}$

7) $\overset{3\times1}{\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix}} \cdot \overset{1\times2}{\begin{bmatrix} 3 & -1 \end{bmatrix}}$ $\begin{bmatrix} -15 & 5 \\ 18 & -6 \\ 0 & 0 \end{bmatrix}$

8) $\overset{2\times3}{\begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix}} \cdot \overset{2\times3}{\begin{bmatrix} 4 & 5 & -5 \\ 5 & -1 & 6 \end{bmatrix}}$ undefined

9) $\overset{3\times2}{\begin{bmatrix} 3 & -1 \\ -3 & 6 \\ -6 & -6 \end{bmatrix}} \cdot \overset{2\times2}{\begin{bmatrix} -1 & 6 \\ 5 & 4 \end{bmatrix}}$ $\begin{bmatrix} -8 & 14 \\ 33 & 6 \\ -24 & -60 \end{bmatrix}$

10) $\overset{2\times2}{\begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix}} \cdot \overset{2\times1}{\begin{bmatrix} -4 \\ 3 \end{bmatrix}}$ $\begin{bmatrix} -8 \\ -11 \end{bmatrix}$

11) $\overset{4\times3}{\begin{bmatrix} -1 & 1 & -1 \\ 5 & 2 & -5 \\ 6 & -5 & 1 \\ -5 & 6 & 0 \end{bmatrix}} \cdot \overset{3\times2}{\begin{bmatrix} 6 & 5 \\ 5 & -6 \\ 6 & 0 \end{bmatrix}}$ $\begin{bmatrix} -7 & -11 \\ 10 & 13 \\ 17 & 60 \\ 0 & -61 \end{bmatrix}$

-25
-36
-61

12) $\overset{4\times2}{\begin{bmatrix} -2 & -6 \\ -4 & 3 \\ 5 & 0 \\ 4 & -6 \end{bmatrix}} \cdot \overset{2\times3}{\begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -3 \end{bmatrix}}$ $\begin{bmatrix} 8 & 4 & 14 \\ -14 & 8 & -17 \\ 10 & -10 & 10 \\ 20 & -8 & 26 \end{bmatrix}$

13) $\overset{1\times2}{\begin{bmatrix} 2 & -5v \end{bmatrix}} \cdot \overset{2\times2}{\begin{bmatrix} -5u & -v \\ 0 & 6 \end{bmatrix}}$ 1×2

$\begin{bmatrix} -10u & -32v \end{bmatrix}$

14) $\overset{2\times2}{\begin{bmatrix} -4 & -y \\ -2x & -4 \end{bmatrix}} \cdot \overset{2\times2}{\begin{bmatrix} -4x & 0 \\ 2y & -5 \end{bmatrix}}$

$\begin{bmatrix} 16x-2y^2 & 5y \\ 8x^2-8y & 20 \end{bmatrix}$