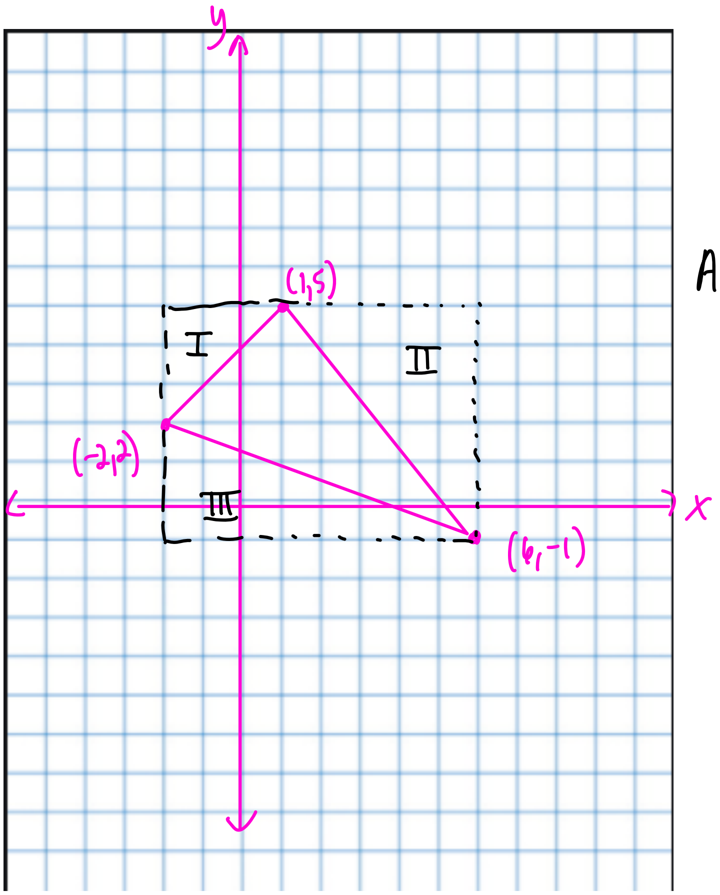
Name:__________________________________ Date:________

PCH: Applications of Matrices and Determinants Ms. Loughran

Do Now:

1. Find the area of the triangle whose vertices are $(1,5)$, $(-2,2)$, and $(6,-1)$. Feel free to use the graph paper to plot the triangle.



$A_{rec} = 8 \cdot 6 = 48$

$A_{\Delta I} = 15$

$A_{\Delta II} = \frac{9}{2}$

$A_{\Delta III} = 12$

$48 - (15 + \frac{9}{2} + 12) = 16.5$

2. Set up a 3 x 3 matrix where the values of x from question 1 make up column 1, their corresponding y values make up column 2, and the 3rd column is made up of all 1's. Find the determinant of this matrix.

$12 \quad -1 \quad -10$

$$\begin{bmatrix} 1 & 5 & 1 \\ -2 & 2 & 1 \\ 6 & -1 & 1 \end{bmatrix} \begin{matrix} 1 & 5 \\ -2 & 2 \\ 6 & -1 \end{matrix}$$

$2 + 30 + 2$

$$\det = 34 - (1) = 33$$

3. Can you come up with a rule using the determinant of a matrix to find the area of a triangle when given its vertices? (Make sure to account for what will happen if your determinant is negative.)

$$\text{Area}_{\triangle} = \pm \frac{1}{2} \det$$

4. Try to adjust the rule from question 3 to find the area of a parallelogram with vertices $(-3,4)$, $(-1,-2)$, $(1,3)$, and $(3,-3)$. (Hint – Imagine drawing in one of its diagonals to see the relationship between triangles and parallelograms.)

$$\text{Area}_{\square} = 2 \cdot \pm \frac{1}{2} \det$$

$$= \pm \det$$

$$\text{Area}_{\square} = 22$$

The **area of a triangle** with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\text{Area} = \pm\,\frac{1}{2}\,\text{determinant of} \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive area.

Practice

1. Find the area of a triangle whose vertices are $(1,0), (2,2)$ and $(4,3)$.

$$8 + 3 + 0$$

$$\left[\begin{array}{ccc} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{array}\right] \begin{array}{cc} 1 & 0 \\ 2 & 2 \\ 4 & 3 \end{array}$$

$$2 + 0 + 6$$

$\det = 8 - 11 = -3$

$\text{Area} = -\frac{1}{2}(-3) = \frac{3}{2}$

2. Find the area of a triangle whose vertices are $(-3,5), (2,6)$ and $(3,-5)$.

$$\overbrace{18+15+10}^{43}$$

$$\left[\begin{array}{ccc} -3 & 5 & 1 \\ 2 & 6 & 1 \\ 3 & -5 & 1 \end{array}\right] \begin{array}{cc} -3 & 5 \\ 2 & 6 \\ 3 & -5 \end{array}$$

$$\underbrace{-18+15-10}_{-13}$$

$\det = -13 - 43 = -56$

$\text{Area} = -\frac{1}{2}(-56) = 28$

Going back to the Do Now questions, think about this If all of the points you were given for the vertices fell on the same line, would these points be able to be the vertices of a triangle? no

Think about what these would mean for what you would find as the area then? What would your determinant value be? 0

Test for Collinear Points: Three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear (lie on the same line) if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \text{(the determinant = 0)}$$

Practice

1. Determine whether the points $(-2,-2), (1,1)$ and $(7,5)$ lie on the same line.

$$7 \; -10 \; -2$$

$$\left[\begin{array}{ccc} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 7 & 5 & 1 \end{array}\right] \begin{array}{cc} -2 & -2 \\ 1 & 1 \\ 7 & 5 \end{array} \qquad \text{no}$$

$$-2 \; -14 + 5$$

$$\det = -11 - (-5) = -6 \neq 0$$

2. Determine whether the points $(3,-1), (0,-3)$ and $(12,5)$ are collinear.

$$-36 + 15 \quad 0$$

$$\left[\begin{array}{ccc} 3 & -1 & 1 \\ 0 & -3 & 1 \\ 12 & 5 & 1 \end{array}\right] \begin{array}{cc} 3 & -1 \\ 0 & -3 \\ 12 & 5 \end{array} \qquad \det = -21 - (-21) = 0$$

Yes

$$-9 \; -12 + 0$$

The test for collinear points can be adapted to another use. If you have two points on a rectangular coordinate system, you can find the equation of the line passing through the two points.

3. Find an equation of a line that passes through $(2,4)$ and $(-1,3)$ using the point slope formula.

$$m = \frac{4-3}{2-(-1)} = \frac{1}{3}$$

$$y-4 = \frac{1}{3}(x-2)$$ point slope form

$$y-4 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{10}{3}$$ slope intercept form

4. Build a matrix where column 1 is built from the variable x, and then the x values from your 2 points from above, column 2 is built from the variable y and then the y values corresponding to the x values you used in column 1, and then column 3 is all 1's. Find its determinant and set the determinant of this matrix = to 0 and write y in terms of x. What do you notice about this equation and the linear equation you wrote above?

$$-4 + 3x + 2y$$

$$\begin{bmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{bmatrix} \begin{matrix} x & y \\ 2 & 4 \\ -1 & 3 \end{matrix}$$

$$4x - y + 6$$

$$\det = 4x - y + 6 - (-4 + 3x + 2y)$$

$$\det = 4x - y + 6 + 4 - 3x - 2y$$

$$\det = x - 3y + 10$$

eq. of line: $0 = x - 3y + 10$

$$3y = x + 10$$

slope-intercept form $y = \frac{1}{3}x + \frac{10}{3}$

standard form: $x - 3y = -10$ or $3y - x = 10$

5. Find an equation of a line that passes through $(4,3)$ and $(2,2)$.

$$6 + 2x + 4y$$

$$\begin{bmatrix} x & y & 1 \\ 4 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{matrix} x & y \\ 4 & 3 \\ 2 & 2 \end{matrix}$$

$$3x + 2y + 8$$

$$\det = 3x + 2y + 8 - 6 - 2x - 4y$$

$$\det = x - 2y + 2$$

eq. of line: $0 = x - 2y + 2$

standard form: $x - 2y = -2$ or $2y - x = 2$

slope intercept form: $x + 2 = 2y$

$$\frac{1}{2}x + 1 = y$$

Homework 02-02

1. 5
3. 5
5. 27
7. -24
9. 6
11. 0
13. 0
15. -9
47. $x = 0, 1, 2$
49. $x = \pm 1$