Name:_____ Date:_____

PCH: Cramer's Rule Ms. Loughran

Do Now:

1. Find $\begin{vmatrix} c & a & t \\ d & o & g \\ e & m & u \end{vmatrix}$ c a / d o / e m

det = cou + age + tdm − toe − cgm − adu

2. $\underset{2\times3}{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}} \cdot \underset{3\times2}{\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}} = \begin{bmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{bmatrix}$

3. Solve the systems of equations algebraically:

A $-4x-6z=-12$

B $-6x-4y-2z=6$

C $-x+2y+z=9$

B + 2C to eliminate y

$-6x-4y-2z=6$

$-2x+4y+2z=18$

$-8x=24$

$x=-3$

Plug x = -3 into A

$12-6z=-12$

$-6z=-24$

$z=4$

Plug z = 4 and x = -3 into C

$3+2y+4=9$

$2y=2$

$y=1$

(-3, 1, 4)

Let's take the coefficient matrix of the system of equations in Do Now 3.

$$D = \begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \begin{matrix} -4 & 0 \\ -6 & -4 \\ -1 & 2 \end{matrix}$$

$-24 + 16 + 0 \quad -8$

$16 + 0 + 72 \quad 88$

Find $|D|$.

$$|D| = 88 - (-8) = 96$$

Now, let's take the **augmented matrix** of the system of equations given to you in Do Now 3

$$\left[\begin{array}{ccc|c} -4 & 0 & -6 & -12 \\ -6 & -4 & -2 & 6 \\ -1 & 2 & 1 & 9 \end{array}\right]$$

Replace the **first** column of D with the last column of the augmented matrix given above.

$$D_x = \begin{bmatrix} -12 & 0 & -6 \\ 6 & -4 & -2 \\ 9 & 2 & 1 \end{bmatrix} \begin{matrix} -12 & 0 \\ 6 & -4 \\ 9 & 2 \end{matrix}$$

$216 + 48 + 0$

$48 + 0 - 72 \quad -24$

$|D_x| = -24 - 264 = -288$

Divide $|D_x|$ by $|D|$ and record your answer here. $\frac{-288}{96} = -3$

↑ x value

What do you notice? How can we find the other values of y and z?

$$D_y = \begin{bmatrix} -4 & -12 & -6 \\ -6 & 6 & -2 \\ -1 & 9 & 1 \end{bmatrix} \begin{matrix} -4 & -12 \\ -6 & 6 \\ -1 & 9 \end{matrix}$$

$36 + 72 + 72$

$-24 \quad -24 \quad +324$

276

$|D_y| = 276 - 180 = 96$

y-value: $\frac{96}{96} = 1$

$-48 \quad -48 \quad + 0 \quad -96$

$$D_z = \begin{bmatrix} -4 & 0 & -12 \\ -6 & -4 & 6 \\ -1 & 2 & 9 \end{bmatrix} \begin{matrix} -4 & 0 \\ -6 & -4 \\ -1 & 2 \end{matrix}$$

$144 + 0 + 144$

288

$|D_z| = 288 - (-96)$

384

z-value $= \frac{|D_z|}{|D|} = \frac{384}{96}$

$= 4$

Cramer's rule to solve systems of linear equations

Steps:

1. Set up a coefficient matrix.
2. Find the determinant of the coefficient matrix. If the determinant $\neq 0$ you can use Cramer's Rule.
3. To find x value, replace first column (x column) with the answer column and find determinant. Now divide this determinant by the original matrix's determinant, this quotient is your x value.
4. To solve for y value, replace second column (y column) with the answer column and find the determinant. Now divide this determinant by the original matrix's determinant, this quotient is your y value.

Practice

Solve each of the following systems using Cramer's Rule, if possible.

1. $5x + 4y = 2$
   $-x + y = -22$

$D = \begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix}$ $\quad |D| = 5-(-4) = 9$

$D_x = \begin{bmatrix} 2 & 4 \\ -22 & 1 \end{bmatrix}$ $\quad |D_x| = 2-(-88) = 90$

$x\text{-value} = \frac{90}{9} = 10$

$D_y = \begin{bmatrix} 5 & 2 \\ -1 & -22 \end{bmatrix}$ $\quad |D_y| = -110-(-2) = -108$

$y\text{-value} = \frac{-108}{9} = -12$

$(10, -12)$

2. $2x-5y=2$
$3x-7y=1$

$D=\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}$ $\quad |D| = 1$ $\quad D_y=\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

$|D_y| = -4$

$D_x=\begin{bmatrix} 2 & -5 \\ 1 & -7 \end{bmatrix}$ $\quad |D_x| = -9$

$x\text{-value} = \frac{-9}{1} = -9$ $\quad y\text{-value} = \frac{-4}{1} = -4$

3. A $-2x+8y=1$
B $x-4y=5$

$D:\begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix}$ $\quad |D| = 0$

you can't use Cramer's Rule
(what that means when det=0, is that the system has no solution or infinitely many solutions)

A+2B

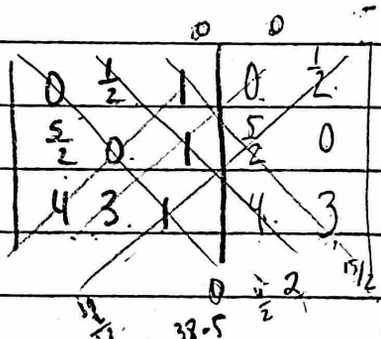
$-2x+8y=1$
$2x-8y=10$
$0 \neq 11$

no solution

Homework 02-05

(5) $\frac{33}{8}$ (7) 10 (27) $3x-5y=0$ (29) $x+3y-5=0$ (23) not collinear (25) collinear

(5) $(0,\frac{1}{2}),(\frac{5}{2},0),(4,3)$

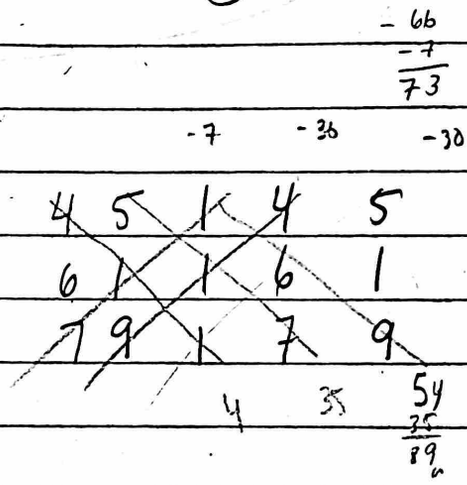


$A=\pm\frac{1}{2}$

$\pm\frac{1}{2}\left(\frac{19}{2}+\left(\frac{-5}{4}\right)\right)$

$\pm\frac{1}{2}\left(\frac{33}{4}\right)=\frac{33}{8}$

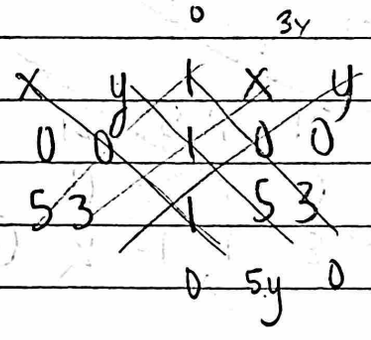
(7)



$93+(-73)=20$

$\frac{1}{2}(20)=10$

(27)



$5y+0$

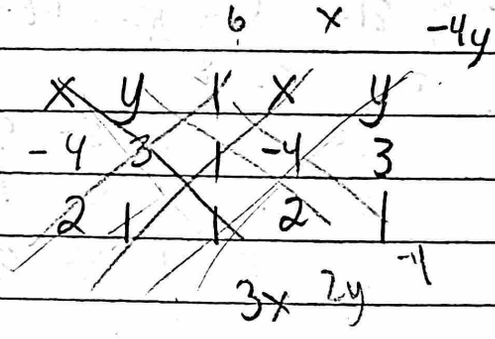
$-3x+0+0$

$-3x+5y=0$

or

$3x-5y=0$

(29)



$3x+2y-4$

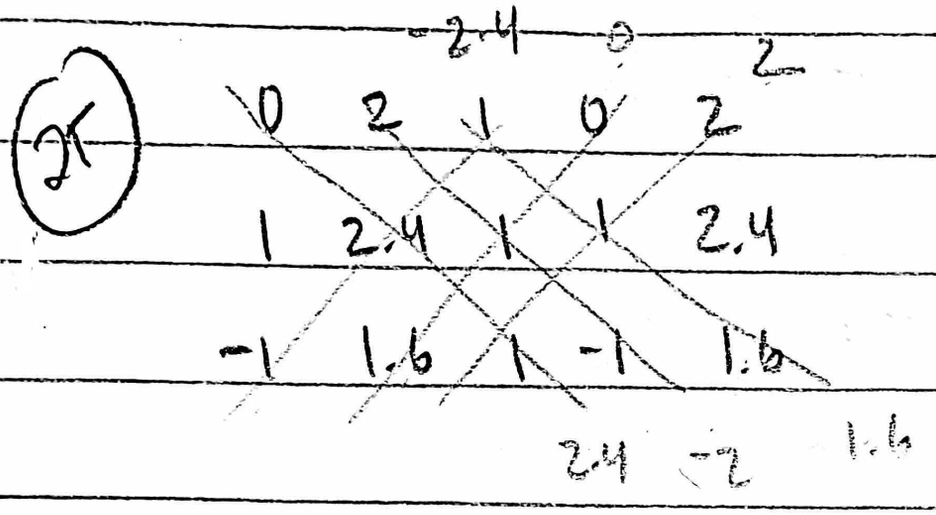
$-x+4y-6$

$2x+6y-10=0$

or

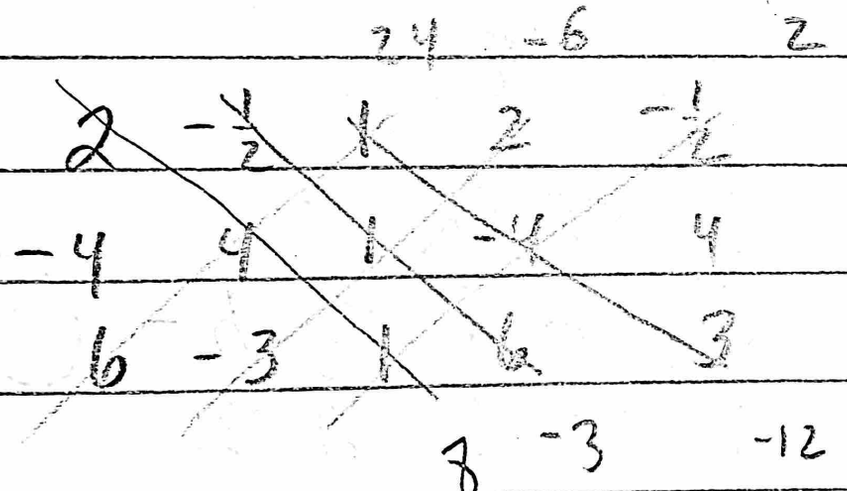
$x+3y-5=0$

(0, 2) (1, 2.4) (−1, 1.6)



$2 + (2) = 0$ collinear

(23) $(2, -\frac{1}{2})$ $(-4, 4)$ $(6, -3)$



$-7 + (-24 + 6 - 2) = 13$ not collinear