Name:________________________________ Date:________

PCH: Inverses of Matrices Ms. Loughran

Do Now:

1. Find AB and BA, if possible.

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A and B are inverses

The identity matrix of a square matrix has entries of 1 on its main diagonal and 0's as all other entries.

I_2 means the identity matrix of a 2×2 matrix, I_3 means the identity matrix of a 3×3 matrix and so on.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let A be an $n\times n$ matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, A^{-1} is called the **inverse** of A.

1. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

Plan: $AB = BA = I_2$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

Plan: $AB = BA = I_3$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find the inverse of a 2×2 matrix we are going to use the determinant.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A is $ad - bc$, and $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3. Find A^{-1} and verify that $AA^{-1} = A^{-1}A = I_2$

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$$

$\det(A) = 12 - 10 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} = A^{-1} = \begin{bmatrix} \frac{3}{2} & \frac{-5}{2} \\ -1 & 2 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Find the inverse of A.

$$A = \begin{bmatrix} 7 & -4 \\ 8 & 0 \end{bmatrix}$$

$\det(A) = 0 - (-32) = 32$

$$A^{-1} = \frac{1}{32}\begin{bmatrix} 0 & 4 \\ -8 & 7 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{8} \\ -\frac{1}{4} & \frac{7}{32} \end{bmatrix}$$

5. Find the inverse of B, if it exists.

$$B = \begin{bmatrix} 8 & 4 \\ -4 & -2 \end{bmatrix}$$

$\det(B) = -16 - (-16) = 0$

B's inverse does not exist

B is not invertible

We can use inverses to solve systems of linear equations.

If A is an invertible matrix (if A has an inverse), the system of linear equations represented by $AX = B$ has a unique solution:

$$AX = B$$

$$A^{-1} \cdot AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

can we do BA^{-1}?

no, matrix mult. is not commutative

6. Solve the system using the inverse, if possible.

$$2x - 5y = 15$$
$$3x - 6y = 36$$

$$\underset{A}{\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 15 \\ 36 \end{bmatrix}}$$

Plan: $X = A^{-1} \cdot B$

① find A^{-1}

$\det(A) = -12 - (-15) = 3$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

$x = 30$
$y = 9$

$\frac{6}{11} + \frac{16}{11}$

7. Solve the system using the inverse, if possible.

$$\begin{aligned} 3x + 4y &= -2 \\ 5x + 3y &= 4 \end{aligned}$$

Plan: $X = A^{-1} \cdot B$

$$\overset{A}{\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} \overset{B}{\begin{bmatrix} -2 \\ 4 \end{bmatrix}}$$

$\det(A) = 9 - 20 = -11$

$$A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -\frac{3}{11} & \frac{4}{11} \\ \frac{5}{11} & -\frac{3}{11} \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$x = 2$
$y = -2$

For 8 and 9, verify if B is the inverse of A.

8. $A = \begin{bmatrix} 3 & -2 \\ -4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$

Yes

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9.
$$A = \begin{bmatrix} 1 & 1 & -2 \\ -3 & -2 & 5 \\ -6 & 4 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -14 & -6 & \frac{1}{2} \\ -9 & 4 & \frac{1}{2} \\ -12 & -5 & \frac{1}{2} \end{bmatrix}$$

should be 0 if they were inverses

$$AB = \begin{bmatrix} 1 & 8 & \\ & & \\ & & \end{bmatrix}$$

not inverses

10.

For what value(s) of x does the matrix M have an inverse?

$$M = \begin{bmatrix} x & 1 \\ 2 & x+1 \end{bmatrix}$$

$\{x \mid x \neq -2, 1\}$

$\det(m) = x^2 + x - 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

Let's find the inverse

$$m^{-1} = \frac{1}{x^2+x-2}\begin{bmatrix} x+1 & -1 \\ -2 & x \end{bmatrix}$$

$$m^{-1} = \begin{bmatrix} \frac{x+1}{x^2+x-2} & \frac{-1}{x^2+x-2} \\ \frac{-2}{x^2+x-2} & \frac{x}{x^2+x-2} \end{bmatrix}$$

Homework 02-06

1. $x = \frac{141}{31}$

   $y = \frac{-9}{62}$

2. $x = -.2$

   $y = .7$

3. $x = -1$

   $y = 14$

   $z = 11$

4. Cramer's Rule is not possible since $\det = 0$