Name: ______________________________ Date: _________

PCH Ms. Loughran

Do Now:

1. Find the values of x and y.

$$\begin{bmatrix} -2 & -6 \\ -4 & 3 \\ 5 & y \\ 4 & -6 \end{bmatrix} \cdot \begin{bmatrix} 2 & x & 2 \\ -2 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 14 \\ -14 & 8 & -17 \\ 10 & -10 & 10 \\ 20 & -8 & 26 \end{bmatrix}$$

$10 - 2y = 10$

$-2y = 0$

$y = 0$

$-2x + 0 = 4$

$-2x = 4$

$x = -2$

Homework 02-12

51. (a) $AB = \begin{bmatrix} 6 & 10 & 14 & 28 \end{bmatrix} \begin{bmatrix} 2000 & 2500 \\ 3000 & 1500 \\ 2500 & 1000 \\ 1000 & 500 \end{bmatrix} = \begin{bmatrix} 105{,}000 & 58{,}000 \end{bmatrix}$

(b) That day they canned 105,000 ounces of tomato sauce and 58,000 ounces of tomato paste.

52. (a) $AC = \begin{bmatrix} 120 & 50 & 60 \\ 40 & 25 & 30 \\ 60 & 30 & 20 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 97.00 \\ 46.50 \\ 41.00 \end{bmatrix}$ Amy's stand sold \$97 worth of produce on Saturday, Beth's stand sold \$46.50 worth, and Chad's stand sold \$41 worth.

(b) $BC = \begin{bmatrix} 100 & 60 & 30 \\ 35 & 20 & 20 \\ 60 & 25 & 30 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 70.00 \\ 33.50 \\ 48.50 \end{bmatrix}$ Amy's stand sold \$70 worth of produce on Sunday, Beth's stand sold \$33.50 worth, and Chad's stand sold \$48.50 worth.

(c) $A + B = \begin{bmatrix} 120 & 50 & 60 \\ 40 & 25 & 30 \\ 60 & 30 & 20 \end{bmatrix} + \begin{bmatrix} 100 & 60 & 30 \\ 35 & 20 & 20 \\ 60 & 25 & 30 \end{bmatrix} = \begin{bmatrix} 220 & 110 & 90 \\ 75 & 45 & 50 \\ 120 & 55 & 50 \end{bmatrix}$ This represents the melons, squash, and tomatoes they sold during the weekend.

(d) $(A + B)C = \left(\begin{bmatrix} 120 & 50 & 60 \\ 40 & 25 & 30 \\ 60 & 30 & 20 \end{bmatrix} + \begin{bmatrix} 100 & 60 & 30 \\ 35 & 20 & 20 \\ 60 & 25 & 30 \end{bmatrix} \right) \begin{bmatrix} 0.10 \\ 0.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 220 & 110 & 90 \\ 75 & 45 & 50 \\ 120 & 55 & 50 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 167.00 \\ 80.00 \\ 89.50 \end{bmatrix}$

During the weekend, Amy's stand sold \$167 worth, Beth's stand sold \$80 worth, and Chad's stand sold \$89.50 worth of produce. Notice that $(A + B)C = AC + BC = \begin{bmatrix} 97.00 \\ 46.50 \\ 41.00 \end{bmatrix} + \begin{bmatrix} 70.00 \\ 33.50 \\ 48.50 \end{bmatrix} = \begin{bmatrix} 167.00 \\ 80.00 \\ 89.50 \end{bmatrix}$.

Practice Questions

① Using matrices find the area of the parallelogram whose vertices are (0,0), (7,2), (5,9) and (12,11)

$$10 + 0 + 0$$

$$\left[\begin{array}{ccc} 0 & 0 & 1 \\ 7 & 2 & 1 \\ 5 & 9 & 1 \end{array}\right]\begin{array}{cc} 0 & 0 \\ 7 & 2 \\ 5 & 9 \end{array}$$

$$0 + 0 + 63$$

$\det = 63 - 10 = 53$

$\text{Area}_{\square} = 53$

* $A_{\Delta} = \pm \frac{1}{2} \det$

② Using matrices determine if the points (7,0), (-1,6) and (-5,9) are collinear. If they are find the equation of the line in slope intercept form that passes through those points.

$$-30 + 63 + 0$$

$$\left[\begin{array}{ccc} 7 & 0 & 1 \\ -1 & 6 & 1 \\ -5 & 9 & 1 \end{array}\right]\begin{array}{cc} 7 & 0 \\ -1 & 6 \\ -5 & 9 \end{array}$$

$$42 + 0 + (-9)$$

$\det = 33 - 33 = 0$

$\therefore$ these points are collinear

$$0 + 6x + 7y$$

$$\left[\begin{array}{ccc} x & y & 1 \\ 7 & 0 & 1 \\ -1 & 6 & 1 \end{array}\right]\begin{array}{cc} x & y \\ 7 & 0 \\ -1 & 6 \end{array}$$

$$0 - y + 42$$

$\det = -y + 42 - (6x + 7y)$

$\det = -y + 42 - 6x - 7y$

$\det = -8y - 6x + 42$

eq. of line: $0 = -8y - 6x + 42$

$8y = -6x + 42$

$y = -\frac{3}{4}x + \frac{42}{8}$

$y = -\frac{3}{4}x + \frac{21}{4}$

③ Solve for x:

$$\begin{vmatrix} 2x & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & x \end{vmatrix} = 8x^2 - 3x + 12$$

do now

④ Solve the following system using an inverse matrix

$x + y = 4$
$2x + 3y = 9$

$X = A^{-1} \cdot B$

$$\begin{array}{ccccc} A & & X & & B \\ \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} & \cdot & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} 4 \\ 9 \end{bmatrix} \end{array}$$

$X = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\det(A) = 3 - 2 = 1$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

$x = 3$
$y = 1$

⑤ Determine if A and B are inverses of each other.

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$$

Plan: $AB = BA = I_2 \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 9 & \\ & \end{bmatrix}$$

∴ they are not inverses

⑥ Find the value of y using Cramer's Rule

$$-4x - 2y - z = -11$$
$$-x - 2y = -6$$
$$x - y - 5z = 5$$

$2 + 0 - 10$

$$\begin{bmatrix} -4 & -2 & -1 \\ -1 & -2 & 0 \\ 1 & -1 & -5 \end{bmatrix} \begin{matrix} -4 & -2 \\ -1 & -2 \\ 1 & -1 \end{matrix}$$

$-40 + 0 - 1$

$\det = -41 - (-8) = -33$

$6 + 0 - 55$

$$\begin{bmatrix} -4 & -11 & -1 \\ -1 & -6 & 0 \\ 1 & 5 & -5 \end{bmatrix} \begin{matrix} -4 & -11 \\ -1 & -6 \\ 1 & -5 \end{matrix}$$

$-120 + 0 + 5$

$\det = -115 - (-49) = -66$

$y\text{-value} = \dfrac{-66}{-33} = 2$

⑦ Find x and y

$$\begin{bmatrix} -1 & 1 & -1 \\ 5 & 2 & -5 \\ 6 & -5 & 1 \\ x & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 5 \\ y & -6 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} -7 & -11 \\ 10 & 13 \\ 17 & 60 \\ 0 & -61 \end{bmatrix}$$

$5x - 36 + 0 = -61$

$5x - 36 = -61$

$5x = -25$

$x = -5$

$-6 + y - 6 = -7$

$y - 12 = -7$

$y = 5$

⑧ Express as a single matrix with entries in simplest form:

$$7\begin{bmatrix} y^3 & y+3 & 4 \\ -7 & 2y & y^2 \end{bmatrix} - 2\begin{bmatrix} y+1 & y & -2y \\ 8 & -y & 4 \end{bmatrix}$$

$$\begin{bmatrix} 7y^3 & 7y+21 & 28 \\ -49 & 14y & 7y^2 \end{bmatrix} + \begin{bmatrix} -2y-2 & -2y & 4y \\ -16 & 2y & -8 \end{bmatrix}$$

$$\begin{bmatrix} 7y^3-2y-2 & 5y+21 & 4y+28 \\ -65 & 16y & 7y^2-8 \end{bmatrix}$$

⑨ Given $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$

Solve for X. $\quad 2X + 3A = B$

$$\frac{2X}{2} = \frac{B - 3A}{2}$$

$$X = \frac{1}{2}(B - 3A)$$

$$-3A = \begin{bmatrix} 6 & 3 \\ -3 & 0 \\ -9 & 12 \end{bmatrix} \qquad B - 3A = \begin{bmatrix} 6 & 6 \\ -1 & 0 \\ -13 & 11 \end{bmatrix}$$

$$\frac{1}{2}(B - 3A) = X = \begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ \frac{-13}{2} & \frac{11}{2} \end{bmatrix}$$

(10) Write the following system as an augmented matrix, then solve the system by putting the matrix in row echelon form

$$\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$$

a) $3x + 3y + 3z = 6 \Rightarrow x + y + z = 2$
$y - 3z = 1$
$2x + y + 5z = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 2 & 1 & 5 & 0 \end{array}\right]$$

$-2R_1 + R_3$

$-2 \quad -2 \quad -2 \quad -4$
$2 \quad 1 \quad 5 \quad 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & -1 & 3 & -4 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$\emptyset$

b) $2x + 14y - 4z = 2 \Rightarrow x + 7y - 2z = -1$

$-4x - 3y + z = 8$

$3x - 5y + 6z = 7$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ -4 & -3 & 1 & 8 \\ 3 & -5 & 6 & 7 \end{array}\right] \quad 4R_1 + R_2 \qquad \begin{array}{cccc} 4 & 28 & -8 & -4 \\ -4 & -3 & 1 & 8 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 25 & -7 & 4 \\ 3 & -5 & 6 & 7 \end{array}\right] \quad -3R_1 + R_3 \qquad \begin{array}{cccc} -3 & -21 & 6 & 3 \\ 3 & -5 & 6 & 7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 25 & -7 & 4 \\ 0 & -26 & 12 & 10 \end{array}\right] \quad R_3 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & -1 & 5 & 14 \\ 0 & -26 & 12 & 10 \end{array}\right]$$

$$-R_2 \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & -26 & 12 & 10 \end{array}\right] \quad 26R_2 + R_3 \qquad \begin{array}{cccc} 0 & 26 & -130 & -364 \\ 0 & -26 & 12 & 10 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & 0 & -118 & -354 \end{array}\right]$$

$$-\frac{1}{118}R_3 \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z = 3$

$y - 15 = -14$

$y = 1$

$x + 7 - 6 = -1$

$x + 1 = -1$

$x = -2$

$(-2, 1, 3)$

OR

b) $2x+14y-4z=2$ $\quad x+7y-2z=-1$
$-4x-3y+z=8$
$3x-5y+6z=7$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ -4 & -3 & 1 & 8 \\ 3 & -5 & 6 & 7 \end{array}\right]$$

$4R_1+R_2$
$$\begin{array}{cccc} 4 & 28 & -8 & -4 \\ -4 & -3 & 1 & 8 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 25 & -7 & 4 \\ 3 & -5 & 6 & 7 \end{array}\right]$$

$-3R_1+R_3$
$$\begin{array}{cccc} -3 & -21 & 6 & 3 \\ 3 & -5 & 6 & 7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 25 & -7 & 4 \\ 0 & -26 & 12 & 10 \end{array}\right] \quad R_3+R_2 \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & -1 & 5 & 14 \\ 0 & -26 & 12 & 10 \end{array}\right]$$

$$-R_2 \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & -26 & 12 & 10 \end{array}\right] \quad \frac{1}{2}R_3 \left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & -13 & 6 & 5 \end{array}\right]$$

$13R_2 + R_3$

$0 \quad 13 \quad -65 \quad -182$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & 0 & -59 & -177 \end{array}\right]$$

$-\frac{1}{59}R_3$

$$\left[\begin{array}{ccc|c} 1 & 7 & -2 & -1 \\ 0 & 1 & -5 & -14 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z = 3$

$y - 5z = -14$

$y - 5(3) = -14$

$y - 15 = -14$

$y = 1$

$x + 7y - 2z = -1$

$x + 7(1) - 2(3) = -1$

$x + 7 - 6 = -1$

$x + 1 = -1$

$x = -2$