Name: ______________________________ Date:_________

PCH: Ms. Loughran

Do Now:

1. Find the complete solution of the system using matrices.

$$-3x-5y+36z=10$$
$$-x+7z=5$$
$$x+y-10z=-4$$

$7z-5=x$

$$\left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ -1 & 0 & 7 & 5 \\ -3 & -5 & 36 & 10 \end{array}\right] \quad R_1+R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ -3 & -5 & 36 & 10 \end{array}\right]$$

$3R_1+R_3$

$$\begin{array}{cccc} 3 & 3 & -30 & -12 \\ -3 & -5 & 36 & 10 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & -2 & 6 & -2 \end{array}\right]$$

$R_2+\frac{1}{2}R_3$

$$\begin{array}{cccc} 0 & 1 & -3 & 1 \\ 0 & -1 & 3 & -1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$y-3z=1$

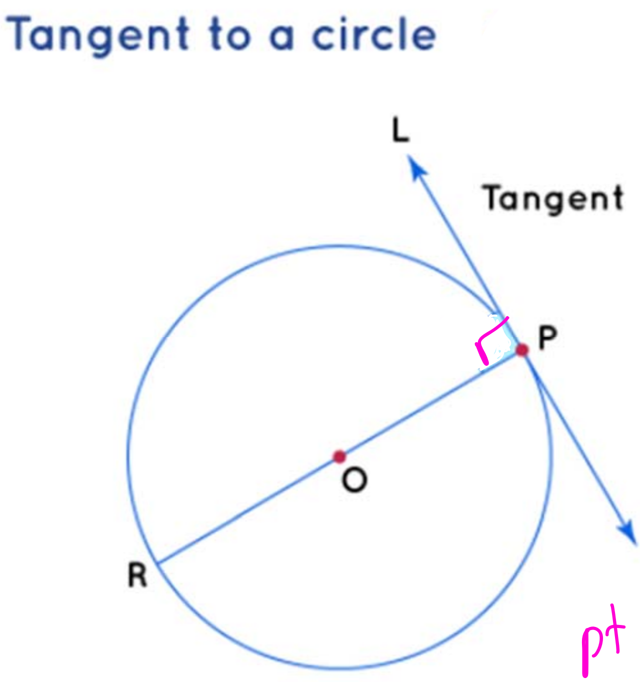
$y=3z+1$

infinite # of solutions

$(7z-5, 3z+1, z)$

Name: ____ Date:________

PCH: More with Circles Ms. Loughran



the tangent line to a circle is perpendicular ($\perp$) to the radius at the point of tangency

$\perp$ lines have slopes that are negative reciprocals

pt slope: $y - y_1 = m(x - x_1)$

1. Write an equation of the line tangent to the circle $x^2 + y^2 = 25$ at the point in the fourth quadrant where $x = 3$.

center: $(0,0)$

① need the entire pt, well $x = 3$

$3^2 + y^2 = 25$

$y^2 = 16$

$y = \pm 4$ the pt is in QIV

$(3,-4)$

② $m_{radius} = \frac{-4-0}{3-0} = \frac{-4}{3}$

$m_{tan} = \frac{3}{4}$

$y + 4 = \frac{3}{4}(x - 3)$

2. Write an equation of the line tangent to the circle $x^2 + 14x + y^2 + 18y = 39$ at the point in the second quadrant where $x = -2$.

$x^2 + 14x + 49 + y^2 + 18y + 81 = 39 + 49 + 81$

$(x+7)^2 + (y+9)^2 = 169$

$c: (-7, -9)$

$m_{radius} = \dfrac{3-(-9)}{-2-(-7)} = \dfrac{12}{5}$

$m_{tan} = \dfrac{-5}{12}$

$y - 3 = \frac{-5}{12}(x+2)$

need the rest of the point

$(-2+7)^2 + (y+9)^2 = 169$

$25 + (y+9)^2 = 169$

$(y+9)^2 = 144$

$y + 9 = \pm 12$

$y = -9 \pm 12 \begin{cases} 3 & (-2, 3) \text{ QII} \\ -21 & \cancel{(-2, -21)} \end{cases}$

3. Write an equation of a line tangent to the circle $x^2 + y^2 - 10x - 14y = 95$ at a point where $x = 10$.

$x^2 - 10x + 25 + y^2 - 14y + 49 = 95 + 25 + 49$

$(x-5)^2 + (y-7)^2 = 169$

$c: (5, 7)$

need rest of the point

$(10-5)^2 + (y-7)^2 = 169$

$(y-7)^2 = 144$

$y - 7 = \pm 12$

$y = 7 \pm 12 \begin{cases} 19 & (10, 19) \\ -5 & (10, -5) \end{cases}$

@ $(10, 19)$

$m_r = \dfrac{19-7}{10-5} = \dfrac{12}{5}$

$m_t = -\dfrac{5}{12}$

$y - 19 = \frac{-5}{12}(x - 10)$

@ $(10, -5)$

$m_r = \dfrac{7-(-5)}{5-10} = \dfrac{12}{-5}$

$m_t = \dfrac{5}{12}$

$y + 5 = \frac{5}{12}(x - 10)$

4. Find the equation of the line that has a positive slope and is tangent to the circle $(x-1)^2+(y-1)^2=4$ at one of its y-intercepts.

$c: (1,1)$

need to find y-int. (let $x=0$)

$(0-1)^2+(y-1)^2=4$

$1+(y-1)^2=4$

$(y-1)^2=3$

$y-1=\pm\sqrt{3}$

$y=1\pm\sqrt{3}$

$(0, 1+\sqrt{3})$

$(0, 1-\sqrt{3})$

$(0, 1+\sqrt{3})$

$m_r=\dfrac{1+\sqrt{3}-1}{0-1}=\dfrac{\sqrt{3}}{-1}$

$m_t=\dfrac{1}{\sqrt{3}}$

$y-(1+\sqrt{3})=\dfrac{1}{\sqrt{3}}x$

$(0, 1-\sqrt{3})$

$m_r=\dfrac{1-\sqrt{3}-1}{0-1}=\dfrac{-\sqrt{3}}{-1}=\sqrt{3}$

$m_t=-\dfrac{1}{\sqrt{3}}$ ← negative slope

5. Write the equation of a circle that passes through (2, 8), (5, 7) and (6, 6).

Method 1 use the center-radius form of the circle $(x-h)^2+(y-k)^2=r^2$

$(2-h)^2+(8-k)^2=r^2$

$(5-h)^2+(7-k)^2=r^2$

$(6-h)^2+(6-k)^2=r^2$

$(2-h)^2+(8-k)^2=(5-h)^2+(7-k)^2$

$\cancel{h^2}-4h+4+\cancel{k^2}-16k+64=\cancel{h^2}-10h+25+\cancel{k^2}-14k+49$ ($+68$, $+74$)

$6h-2k=6$

$3h-k=3$

$(5-h)^2+(7-k)^2=(6-h)^2+(6-k)^2$

$\cancel{h^2}-10h+25+\cancel{k^2}-14k+49=\cancel{h^2}-12h+36+\cancel{k^2}-12k+36$ ($+74$, 72)

$2h-2k=-2$

$-1(h-k=-1)$

$3h-k=3$

$-h+k=1$

$2h=4$

$h=2$

$-2+k=1$

$k=3$

center: $(2,3)$

$(x-2)^2+(y-3)^2=r^2$ $(2,8)$

$(\cancel{2-2})^2+(8-3)^2=r^2$

$5=r$

$(x-2)^2+(y-3)^2=25$

Classwork 02-16

Name:______________________ Date:__________

PCH: Circles Ms. Loughran

Do Now: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

1. Find the length of the line segment determined by points $A(x, y)$ and $C(h, k)$.

$d = \sqrt{(x-h)^2+(y-k)^2}$

$d^2 = (x-h)^2+(y-k)^2$

$(x-h)^2+(y-k)^2 = r^2$ if r: radius

An equation of the circle with center (h, k) and radius r is

This is called the ~~standard form~~ circle-radius form for the equation of the circle. If the center of the circle is the origin, then the equation is

(0,0)

$(x-0)^2+(y-0)^2=r^2$

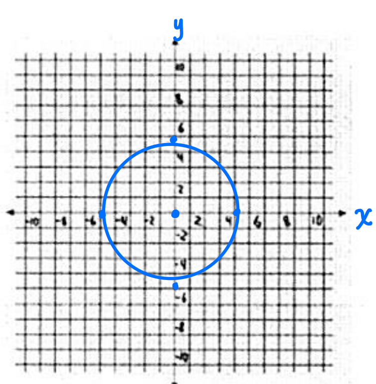
$x^2+y^2=r^2$

1. Graph each equation.

(a) $x^2 + y^2 = 25$

C: (0,0)

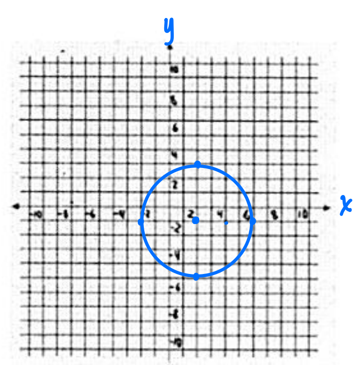
r = 5



(b) $(x-2)^2 + (y+1)^2 = 16$

C: (2,-1)

r = 4



2. Find an equation of the circle with radius 3 and center $(-1,4)$.

$$(x+1)^2+(y-4)^2=9$$

3. Find the center and radius of the circle whose equation is $(x+2)^2+(y-3)^2=10$.

$c:\ (-2,3)$

$r=\sqrt{10}$

4. Write an equation of the circle whose diameter has endpoints (0,0) and (6,8).

midpoint: $\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$

$\left(\frac{0+6}{2},\ \frac{0+8}{2}\right)=(3,4)$

distance: $\sqrt{(3-0)^2+(4-0)^2}=\sqrt{25}=5$

radius

$$(x-3)^2+(y-4)^2=25$$

or

$(x-3)^2+(y-4)^2=r^2$

$(0-3)^2+(0-4)^2=r^2$

$9+16=r^2$

$25=r^2$

5. Points P(1,-5) and Q(-3,3) are the endpoints of a diameter of a circle. Find the center, radius, and equation of the circle.

Center: $\left(\frac{1+(-3)}{2}, \frac{-5+3}{2}\right) = (-1,-1)$

center/radius

$(x-h)^2+(y-k)^2=r^2$

$(x+1)^2+(y+1)^2=r^2$

$(1+1)^2+(-5+1)^2=r^2$

$4+16=r^2$

$r^2=20$

$(x+1)^2+(y+1)^2=20$

c: $(-1,-1)$

$r=\sqrt{20}$ or $2\sqrt{5}$

6. Find the center and radius of the circle $x^2+y^2+4x-6y-12=0$.

$x^2+4x+4 \quad +y^2-6y+9 \quad = 12+4+9$

$(x+2)^2+(y-3)^2=25$

C: $(-2,3)$

$r=5$

7. Find the center and radius of the circle whose equation is $x^2+y^2+2x-6y+7=0$.

$x^2+2x+1 \quad +y^2-6y+9 \quad = -7+1+9$

$(x+1)^2+(y-3)^2=3$

C: $(-1,3)$

$r=\sqrt{3}$

8. Find the center and radius of the circle whose equation is. $x^2+y^2+6y+2=0$

$x^2+y^2+6y+9 \quad = -2+9$

$x^2+(y+3)^2=7$

c: $(0,-3)$

$r=\sqrt{7}$

9. Find the center and radius of the circle whose equation is $x^2+y^2-4x+10y+13=0$.

$x^2-4x+4+y^2+10y+25 \quad = -13+4+25$

$(x-2)^2+(y+5)^2=16$

c: $(2,-5)$

$r=4$

$\frac{\cancel{4}^{2}}{3}\cdot\frac{1}{\cancel{2}}$

10. Find the center and radius of the circle whose equation is $9x^2+12x+9y^2-77=0$.

$\underbrace{x^2+\frac{4}{3}x+\frac{4}{9}}+y^2 \quad = \frac{77}{9}+\frac{4}{9}$

$(x+\frac{2}{3})^2+y^2 \quad = 9$

c: $(-\frac{2}{3},0)$

$r= 3$

Practice

Problems 1-3: Find the center and radius of each circle below.

1. $(x-3)^2+(y-2)^2=16$ — C: (3,2) r=4

2. $(x-1)^2+(y+3)^2=4$ — C(1,-3) r=2

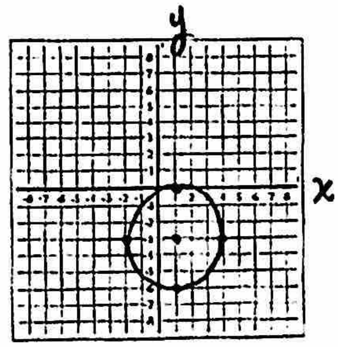
3. $(x+2)^2+(y-5)^2=1$ — c(-2, 5) r=1

Problems 4-5: Graph the following.

4. $(x-1)^2+(y+3)^2=9$

C(1,-3)

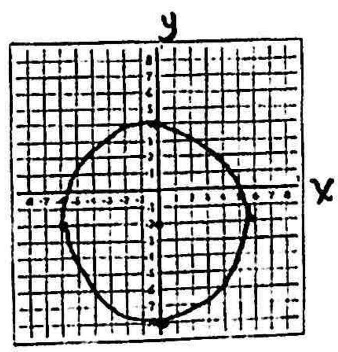
r=3



5. $x^2+(y+2)^2=36$

c(0,-2)

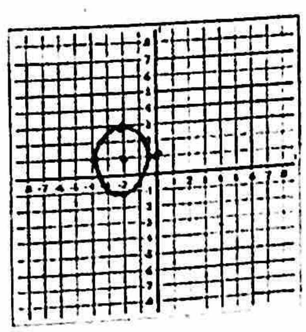
r=6



6. $(x+2)^2+(y-1)^2=4$

c(-2,1)

r=2



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7. Write the equation of a circle in standard form that has a radius of 5 and a center at $(3, -2)$.

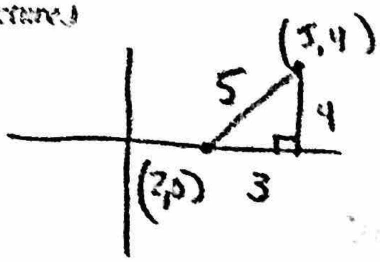
$(x-3)^2+(y+2)^2=25$

8. Write the equation of a circle in standard form that has a radius of 2 and a center at $(-1, -4)$.

$(x+1)^2+(y+4)^2=4$

9. Write the equation of a circle in standard form that passes through the point $(5, 4)$ and has a center at $(2, 0)$.
(Draw a picture.)

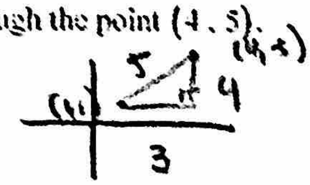
$(x-2)^2+y^2=25$



$r=5$

10. Write the equation of a circle whose center is at $(1, 1)$ that passes through the point $(4, 5)$.

$(x-1)^2+(y-1)^2=25$



$r=5$
$c(1,1)$

11. Find the radius of a circle with equation:

$$x^2 - 6x + y^2 + 10y = 2$$

$r=6$

$$x^2-6x+9+y^2+10y+25=2+9+25$$
$$(x-3)^2+(y+5)^2=36$$

12. Write the equation of the circle in standard form:

$$x^2 - 10x + y^2 - 8y = -32$$

$(x-5)^2+(y-4)^2=9$

$$x^2-10x+25+y^2-8y+16=-32+25+16$$
$$(x-5)^2+(y-4)^2=9$$

13. Write the equation of the circle in standard form:

$x^2 + 4x + y^2 + 6y = 0$

$\underline{(x+2)^2 + (y+3)^2 = 13}$

$x^2 + 4x + 4 + y^2 + 6y + 9 = 4 + 9$

$(x+2)^2 + (y+3)^2 = 13$

14. Write the equation of the circle in standard form:

$x^2 - 2x + y^2 - 4y - 3 = 0$

$\underline{(x-1)^2 + (y-2)^2 = 8}$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 3 + 1 + 4$

$(x-1)^2 + (y-2)^2 = 8$