Name: _____ Date:_____
PCH: Ms. Loughran

Do Now:

1. Find the complete solution of the system using matrices.

$$x+4y-2z=-3$$
$$2x-y+5z=12$$
$$8x+5y+11z=30$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 2 & -1 & 5 & 12 \\ 8 & 5 & 11 & 30 \end{array}\right] \quad -2R_1+R_2 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & -9 & 9 & 18 \\ 8 & 5 & 11 & 30 \end{array}\right]$$

$$-\frac{1}{9}R_2 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 8 & 5 & 11 & 30 \end{array}\right] \quad -8R_1+R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & -27 & 27 & 54 \end{array}\right]$$

$$27R_2+R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$y-z=-2$
$y=z-2$ $(5-2z, z-2, z)$

$x+4y-2z=-3$
$x+4(z-2)-2z=-3$
$x+4z-8-2z=-3$
$x=-2z+5$

Continuing in yesterday's packet…

5. Write the equation of a circle that passes through (2, 8), (5, 7) and (6, 6).

Method 2: Use the general form of a circle

$$x^2 + y^2 + Bx + Cy + D = 0$$

For (2,8)

$$2^2 + 8^2 + 2B + 8C + D = 0$$

① $2B + 8C + D = -68$

For (5,7)

$$5^2 + 7^2 + 5B + 7C + D = 0$$

② $5B + 7C + D = -74$

For (6,6)

$$6^2 + 6^2 + 6B + 6C + D = 0$$

③ $6B + 6C + D = -72$

$-1 + 2$ to eliminate D

$$-2B - 8C - D = 68$$
$$5B + 7C + D = -74$$
$$3B - C = -6$$

$-2 + 3$ to eliminate D

$$-5B - 7C - D = 74$$
$$6B + 6C + D = -72$$
$$B - C = 2$$

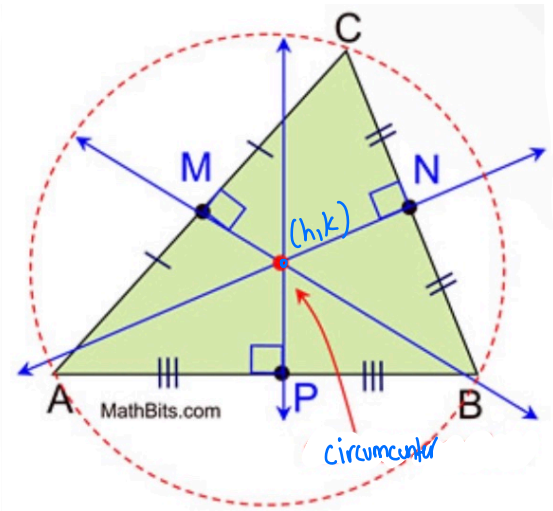
$$3B - C = -6$$
$$-B + C = -2$$

$$2B = -8$$
$$B = -4$$

$$B - C = 2$$
$$-4 - C = 2$$
$$-C = 6$$
$$C = -6$$

$$6(-4) + 6(-6) + D = -72$$
$$-24 - 36 + D = -72$$
$$-60 + D = -72$$
$$D = -12$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$



Method 3:
A Geometric Approach

The 3 ⊥ bisectors of a Δ come together at a point called the circumcenter

the circumcenter is equally distance from the 3 vertices and the common difference is the radius of a circle that passes through the 3 vertices. (circumcircle)

5. Write the equation of a circle that passes through A (2, 8), B (5, 7) and C (6, 6).

⊥ bisector of AB

midpoint: $\left(\frac{7}{2}, \frac{15}{2}\right)$

$m_{AB} = \frac{8-7}{2-5} = \frac{1}{-3}$

$m_{\perp \text{ bisector of } AB} = 3$

$y - \frac{15}{2} = 3\left(x - \frac{7}{2}\right)$

$y = 3\left(x - \frac{7}{2}\right) + \frac{15}{2}$

$y = 3x - \frac{21}{2} + \frac{15}{2}$

$y = 3x - 3$

⊥ bisector of BC

midpoint BC $= \left(\frac{11}{2}, \frac{13}{2}\right)$

$m_{BC} = \frac{7-6}{5-6} = \frac{1}{-1} = -1$

$m_{\perp \text{ bisector of } BC} = 1$

$y - \frac{13}{2} = 1\left(x - \frac{11}{2}\right)$

$y = x - \frac{11}{2} + \frac{13}{2}$

$y = x + 1$

$3x-3=x+1$

$2x=4$

$x=2$

$y=2+1=3$

center of circle $(2,3)$

$$(x-2)^2+(y-3)^2=r^2$$

plug in $(2,8)$

$$(2-2)^2+(8-3)^2=r^2$$

$$(8-3)^2=r^2$$

$$5=r$$

$$(x-2)^2+(y-3)^2=25$$

6. Write the equation of a circle that passes through $(-3,-3),(-1,11),$ and $(5,13)$.

$(-3-h)^2+(-3-k)^2=(-1-h)^2+(11-k)^2$

$h^2+6h+9+k^2+6k+9=h^2+2h+1+k^2-22k+121$

$6h+6k+18=2h-22k+122$

$4h+28k=104$

$h+7k=26$

$(-1-h)^2+(11-k)^2=(5-h)^2+(13-k)^2$

$h^2+2h+1+k^2-22k+121=h^2-10h+25+k^2-26k+169$

$2h-22k+122=-10h-26k+194$

$12h+4k=72$

$3h+k=18$

$-3h-21k=-78$

$3h+k=18$

$-20k=-60$

$k=3$

$h+7(3)=26$

$h+21=26$

$h=5$

center $(5,3)$

$(x-5)^2+(y-3)^2=r^2$

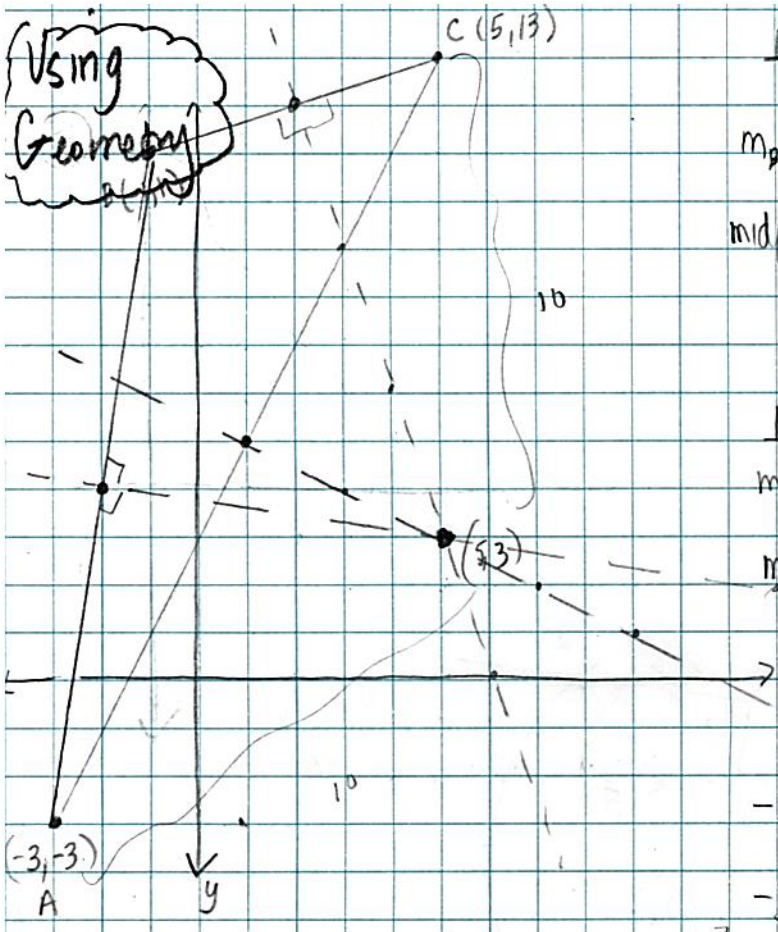
$(-3,-3)$ $(-3-5)^2+(-3-3)^2=r^2$

$64+36=r^2$

$100=r^2$

$(x-5)^2+(y-3)^2=100$

Using Geometry



⊥ bisector of BC

$m_{BC} = \frac{13-11}{5-(-1)} = \frac{2}{6} = \frac{1}{3}$ ⊥ bisector $m = -3$

$\text{midpt}_{BC}\left(\frac{-1+5}{2}, \frac{11+13}{2}\right) \Rightarrow (2, 12)$

$y - 12 = -3(x-2) \Rightarrow y = -3x + 18$

⊥ bisector of BA

$m_{BA} = \frac{-3-11}{-3-(-1)} = \frac{-14}{-2} = +7$, ⊥ $m = -\frac{1}{7}$

$\text{midpt}_{BA}\left(\frac{-1+(-3)}{2}, \frac{11+(-3)}{2}\right) \Rightarrow (-2, 4)$

$y - 4 = -\frac{1}{7}(x+2)$

$-3(x-2) + 12 = -\frac{1}{7}(x+2) + 4$

$-3x + 6 + 12 = -\frac{1}{7}x - \frac{2}{7} + 4$

$\left(-3x + 18 = -\frac{1}{7}x + \frac{26}{7}\right)$

$-21x + 126 = -x + 26$

$100 = 20x$

$x = 5$

$y = -3(5) + 18$

$y = -15 + 18$

$y = 3$

(5,3) center

$(x-5)^2 + (y-3)^2 = r^2$

(-3,-3)

$(-3-5)^2 + (-3-3)^2 = r^2$

$64 + 36 = r^2$

$r^2 = 100$

$(x-5)^2 + (y-3)^2 = 100$

Homework 02-26

Name: _____ Date: _____

PCH: Circle Practice Ms. Loughran

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1. Find the center and radius of the circle $(x+2)^2+(y-3)^2=10$.

   center: $(-2,3)$ $r=\sqrt{10}$

2. Write an equation of the circle with a center at (4,0) and a radius of 3.

   $(x-4)^2+y^2=9$

3. Write an equation of the circle whose diameter has endpoints (0,0) and (6,8).

   C: $(3,4)$ $r=5$ $(x-3)^2+(y-4)^2=25$

4. Find the center and radius of a circle $x^2+y^2+4x-6y-12=0$.

   $x^2+4x+4+y^2-6y+9=12+4+9$ C: $(-2,3)$

   $(x+2)^2+(y-3)^2=25$ $r=5$

5. Write an equation of the line tangent to the circle $x^2+y^2=80$ at the point in the 1st quadrant where x = 4.

   $4^2+y^2=80$ $y^2=64$ $y=\pm 8$ QI (4,8) C: (0,0)

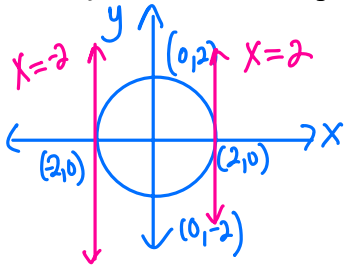
   $m_{rad}=\frac{8-0}{4-0}=\frac{8}{4}=2$ $m_{tan}=-\frac{1}{2}$ $y-8=-\frac{1}{2}(x-4)$

6. Write an equation of the line(s) tangent to the circle $x^2+y^2-8x+12y+42=0$ at the points where x = 5.

7. Write an equation of the tangent line to the circle $2x^2+4x+2y^2+8y-3=0$ at the point $\left(-\frac{1}{2},\frac{1}{2}\right)$.

8. Write the equations of the tangents to the circle $x^2+y^2=4$ whose slopes are undefined.

   C: (0,0) $r=2$ $x=\pm 2$



4 vertical lines

(6) $x^2-8x+y^2+12y+42=0$

$x^2-8x+16+y^2+12y+36=-42+16+36$

$(x-4)^2+(y+6)^2=10$ C: (4,-6)

When $x=5$

$(5-4)^2+(y+6)^2=10$

$1+(y+6)^2=10$

$(y+6)^2=9$

$y+6=\pm 3$ → $y=-6\pm 3$ < -9, -3

@ (5,-9)

$m_{rad}=\frac{-9-(-6)}{5-4}=-3$ $m_{tan}=\frac{1}{3}$ $y+9=\frac{1}{3}(x-5)$

@ (5,-3)

$m_{rad}=\frac{-3-(-6)}{5-4}=\frac{3}{1}=3$ $m_{tan}=-\frac{1}{3}$ $y+3=-\frac{1}{3}(x-5)$

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7. Write an equation of the tangent line to the circle $2x^2 + 4x + 2y^2 + 8y - 3 = 0$ at the point $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

$x^2 + 2x + y^2 + 4y = \frac{3}{2}$

$x^2 + 2x + 1 + y^2 + 4y + 4 = \frac{3}{2} + 1 + 4$

$(x+1)^2 + (y+2)^2 = \frac{13}{2}$

$C: (-1, -2)$

$m_{rad} = \frac{\frac{1}{2} - (-2)}{-\frac{1}{2} - (-1)} = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$

$m_{tan} = -\frac{1}{5}$

$y - \frac{1}{2} = -\frac{1}{5}\left(x + \frac{1}{2}\right)$