Name:______________________________________ Date:____________

PCH: Ellipses Ms. Loughran

center: $(1, k)$

Do Now:

$h=1$

1. Find the equation of a circle with a center at $x=1$ and that passes through the points $(4,3)$, $(-2,-5)$, and $(5,2)$.

$$(4-1)^2+(3-k)^2=(-2-1)^2+(-5-k)^2$$

$$\cancel{9}+\cancel{k^2}-6k+9=\cancel{9}+\cancel{k^2}+10k+25$$

$$-16k=16$$

$$k=-1$$

$$(x-1)^2+(y+1)^2=r^2$$

plug in $(4,3)$

$$(4-1)^2+(3+1)^2=r^2$$

$$9+16=r^2$$

$$25=r^2$$

$$(x-1)^2+(y+1)^2=25$$

An **ellipse** is the locus of all points in a plane such that the sum of the distances from two given points in the plane, called foci, is constant.

The standard form of the equation of an ellipse with center at $(h,k)$, major axis of length $2a$ units and minor axis of length $2b$ units, where $c^2 = a^2 - b^2$, is as follows:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$$ when the major axis is parallel to the $x$-axis,

horizontal major axis (HMA) 

or

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1,$$ when the major axis is parallel to the $y$-axis.

vertical major axis (VMA) 

The foci are located on the major axis with formulas:

$(h+c,k)$ and $(h-c,k)$ if the major axis is parallel to the $x-$axis (HMA)
$(h,k+c)$ and $(h,k-c)$ if the major axis is parallel to the $\mathrm{y}-$axis (VMA)

In all ellipses, $a^2 > b^2$. You can use this information to determine the orientation of the major axis from the values given in the equation. If $a^2$ is the denominator of the $x$ term, the major axis is parallel to the $x$-axis. If $a^2$ is the denominator of the $y$ term, the major axis is parallel to the $y$-axis. The **vertices** of the ellipse are the endpoints of the major axis. The **covertices** are the endpoints of the minor axis.

1. Graph $\frac{x^2}{9} + \frac{y^2}{4} = 1$

center: $(0,0)$

HMA

$a = 3$ ⇄

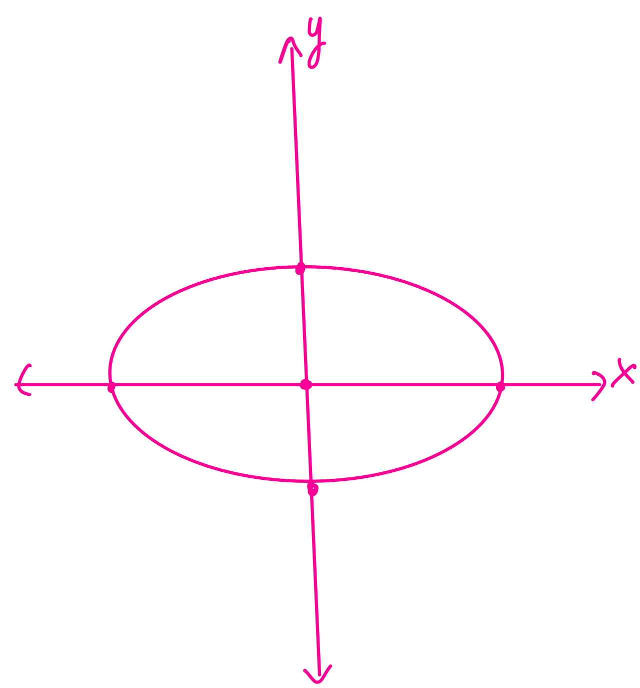
$b = 2$ ↑↓

$c = \sqrt{5}$ ⇄

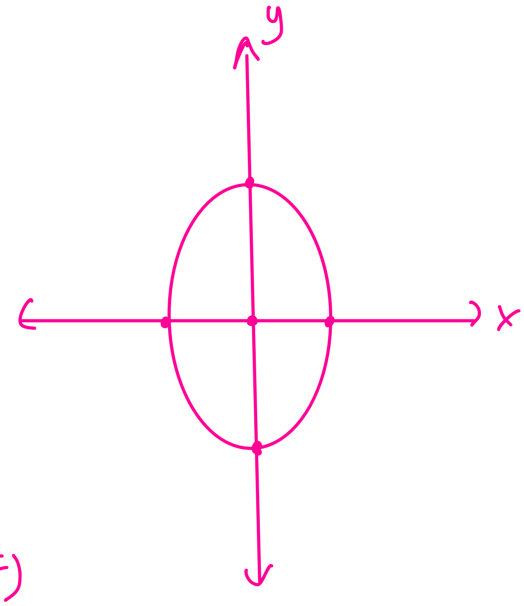
vertices: $(0 \pm 3, 0) \rightarrow (\pm 3, 0)$

covertices: $(0, 0 \pm 2) \rightarrow (0, \pm 2)$

foci: $(0 \pm \sqrt{5}, 0) \rightarrow (\pm\sqrt{5}, 0)$



2. Graph $\frac{y^2}{9}+\frac{x^2}{4}=1$




center: $(0,0)$

VMA

$a=3$ $\uparrow\downarrow$

$b=2$ $\rightleftarrows$

$c=\sqrt{5}$ $\uparrow\downarrow$

vertices: $(0, 0\pm3) \to (0,\pm3)$

covertices: $(0\pm2, 0) \to (\pm2, 0)$

foci: $(0, 0\pm\sqrt{5}) \to (0,\pm\sqrt{5})$

length of major axis: $6 = 2a = 2(3)$

length of minor: $4 = 2b = 2(2)$

3. Graph $\frac{(x-4)^2}{121}+\frac{(y+5)^2}{64}=1$

center: $(4,-5)$

HMA

$a=11$ $\rightleftarrows$

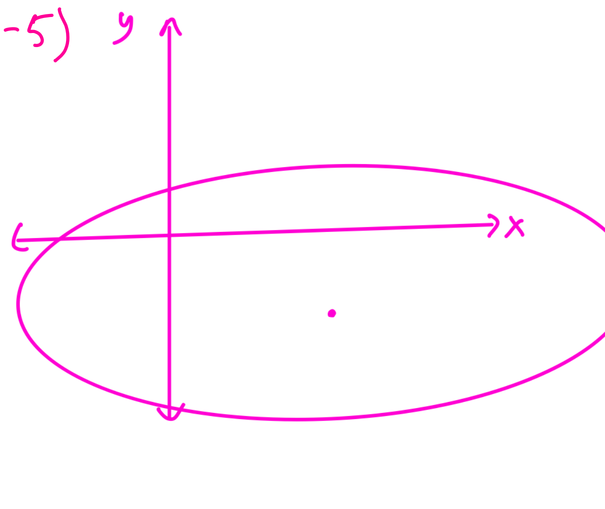
$b=8$ $\uparrow\downarrow$

$c=\sqrt{57}$ $\rightleftarrows$

vertices: $(4\pm11, -5)$ → $(15,-5)$, $(-7,-5)$

covertices: $(4, -5\pm8)$ → $(4,-13)$, $(4,3)$

foci: $(4\pm\sqrt{57}, -5)$



4. Graph $\frac{(y+2)^2}{25}+\frac{(x-3)^2}{16}=1$

center: $(3,-2)$

VMA

$a=5$ ↑↓

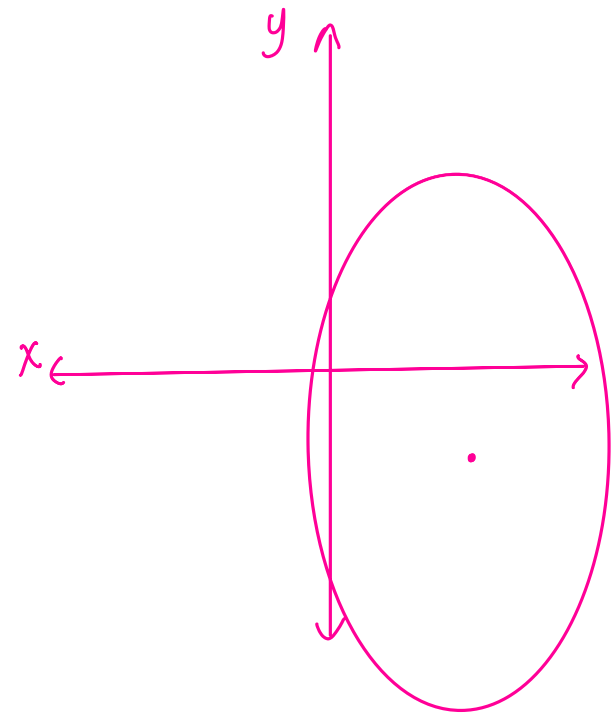
$b=4$ ⇄

$c=3$ ↑↓

vertices: $(3,-2\pm5)$ → $(3,-7)$, $(3,3)$

covertices: $(3\pm4,-2)$ → $(-1,-2)$, $(7,-2)$

foci: $(3,-2\pm3)$ → $(3,-5)$, $(3,1)$




Find the coordinates of the center, the foci, the vertices and covertices of the ellipse with the equation:

5. $4x^2+y^2-8x+6y+9=0$

$$4x^2-8x+y^2+6y=-9$$

$$4(x^2-2x+1)+y^2+6y+9=-9+4+9$$

$$\frac{4(x-1)^2}{4}+\frac{(y+3)^2}{4}=\frac{4}{4}$$

$$(x-1)^2+\frac{(y+3)^2}{4}=1$$

center: $(1,-3)$

VMA

$a=2$ ↑↓

$b=1$ ⇄

$c=\sqrt{3}$ ↑↓

vertices: $(1,-3\pm2)$ → $(1,-5)$, $(1,-1)$

covertices: $(1\pm1,-3)$ → $(0,-3)$, $(2,-3)$

foci: $(1,-3\pm\sqrt{3})$

6. $4x^2+9y^2-8x-54y+49=0$

$4x^2-8x+9y^2-54y=-49$

$4(1)$ $9(9)$

$4(x^2-2x+1)+9(y^2-6y+9)=-49+4+81$

$4(x-1)^2+9(y-3)^2=36$

$$\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4}=1$$

center: $(1,3)$

HMA

$a=3$ $\rightleftarrows$

$b=2$ $\uparrow\downarrow$

$c=\sqrt{5}$ $\rightleftarrows$

vertices: $(1\pm3,3)$ $(-2,3)$ $(4,3)$

covertices: $(1,3\pm2)$ $(1,1)$ $(1,5)$

foci: $(1\pm\sqrt{5},3)$

7. $9x^2+4y^2-18x+16y=11$

$9x^2-18x+4y^2+16y=11$

$9(x^2-2x+1)+4(y^2+4y+4)=11+9+16$

$9(x-1)^2+4(y+2)^2=36$

$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$$

c: $(1,-2)$

VMA

$a=3$ $\uparrow\downarrow$

$b=2$ $\rightleftarrows$

$c=\sqrt{5}$ $\uparrow\downarrow$

v: $(1,-2\pm3)$ $(1,1)$ $(1,-5)$

co: $(1\pm2,-2)$ $(-1,-2)$ $(3,-2)$

f: $(1,-2\pm\sqrt{5})$

# Homework 02-28

① $\underbrace{(-3-h)^2+(-2-k)^2 = (-2-h)^2 + \cancel{(-3-k)^2} = (-4-h)^2 + \cancel{(-3-k)^2}}$

$$\cancel{h^2}+4h+4 = \cancel{h^2}+8h+16$$
$$-4h=12$$
$$h=-3$$

$$\cancel{(-3+3)^2}+(-2-k)^2 = (-2+3)^2+(-3-k)^2$$
$$\cancel{k^2}+4k+4 = 1+\cancel{k^2}+6k+9$$
$$-2k=6$$
$$k=-3$$

$$(x+3)^2+(y+3)^2=r^2$$

Plug in $(-3,-2)$

$$(-3+3)^2+(-2+3)^2=r^2$$
$$0+1=r^2$$
$$1=r^2$$

$(x+3)^2+(y+3)^2=1$ or $x^2+y^2+6x+6y+17=0$

②

$(-1,0)$ $\quad (-1)^2+0^2+(-1)(B)+0(C)+D=0$

$$-B+D=-1$$

$(2,3)$ $\quad 2^2+3^2+2B+3C+D=0$

$$2B+3C+D=-13$$

$(-1,6)$ $\quad (-1)^2+6^2-B+6C+D=0$

$$-B+6C+D=-37$$

$$-4B-6C-2D=26$$
$$\underline{-B+6C+D=-37}$$
$$-5B-D=-11$$
$$\underline{-B+D=-1}$$
$$-6B=-12$$
$$B=2$$

$$-B+D=-1$$
$$-2+D=-1$$
$$D=1$$

$$2B+3C+D=-13$$
$$4+3C+1=-13$$
$$3C=-18$$
$$C=-6$$

$x^2+y^2+2x-6y+1=0$
or
$(x+1)^2+(y-3)^2=9$

(3) $(-8+11)^2+(14-k)^2=(-16+11)^2+(6-k)^2=(-6+11)^2+(12-k)^2$

$9+k^2-28k+196=\cancel{25}+\cancel{k^2}-12k+36=\cancel{25}+\cancel{k^2}-24k+144$

$12k=108$
$k=9$

$(x+11)^2+(y-9)^2=r^2$

Plug in $(-8,14)$

$(-8+11)^2+(14-9)^2=r^2$
$9+25=r^2$
$34=r^2$

$(x+11)^2+(y-9)^2=34$ or $x^2+y^2+22x-18y+168=0$

(4) $A(-6,5)$
$B(-3,-4)$
$C(2,1)$

$\text{midpt}_{AB}=\left(\frac{-9}{2},\frac{1}{2}\right)$ $\text{midpt}_{BC}\left(\frac{-1}{2},\frac{-3}{2}\right)$

$m_{AB}=\frac{5+4}{-6+3}=\frac{9}{-3}=-3$ $m_{BC}=\frac{-4-1}{-3-2}=\frac{-5}{-5}=1$

$m_{\perp \text{ bisector of } AB}=\frac{1}{3}$ $m_{\perp \text{ bisector of } BC}=-1$

eq. of $\perp$ bisector of AB

$y-\frac{1}{2}=\frac{1}{3}\left(x+\frac{9}{2}\right)$
$y=\frac{1}{3}x+\frac{3}{2}+\frac{1}{2}$
$y=\frac{1}{3}x+2$

eq. of $\perp$ bisector of BC

$y+\frac{3}{2}=-1\left(x+\frac{1}{2}\right)$
$y=-x-\frac{1}{2}-\frac{3}{2}$
$y=-x-2$

$(x+3)^2+(y-1)^2=r^2$

Plug in $(2,1)$
$25+0=r^2$
$r^2=25$

$(x+3)^2+(y-1)^2=25$
or $x^2+y^2+6x-2y-15=0$

$\frac{1}{3}x+2=-x-2$
$x+6=-3x-6$
$4x=-12$
$x=-3$

$y=-x-2$
$y=3-2=1$

$\therefore (-3,1)$ is the center