Name:____ Date:_____________

PCH: Hyperbolas Ms. Loughran

Do Now:

1. Find the axis of symmetry, and vertex of the following parabola. Then, make a sketch of its graph including the vertex and 2 other points.

$$y = 2x^2 - 5x - 12$$

axis of symmetry: $x = \frac{-b}{2a}$

vertex: $\left(\frac{-b}{2a}, y\left(\frac{-b}{2a}\right)\right)$

$x = \frac{5}{4}$

vertex: $\left(\frac{5}{4}, \frac{-121}{8}\right)$

$y = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) - 12$

$y = \frac{50}{16} - \frac{25}{4} - 12$

$y = \frac{50}{16} - \frac{100}{16} - 12$

$y = -\frac{50}{16} - 12$

$y = \frac{-50}{16} - \frac{192}{16}$

$y = -\frac{242}{16} = \frac{-121}{8}$

$y = a(x-h)^2 + k$

$\begin{array}{r} 25 \\ 96 \\ \hline 121 \end{array}$

$-\frac{25}{16} - \frac{96}{16}$

$y = 2\left(\underbrace{x^2 - \frac{5}{2}x + \frac{25}{16}} - \frac{25}{16} - 6\right)$

$y = 2\left(x - \frac{5}{4}\right)^2 + \cancel{2}\left(-\frac{121}{\cancel{16}_{8}}\right)$

$y = 2\left(x - \frac{5}{4}\right)^2 - \frac{121}{8}$

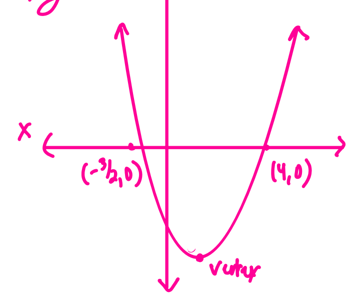
vertex: $\left(\frac{5}{4}, -\frac{121}{8}\right)$

axis of sym: $x = \frac{5}{4}$

x-int: $0 = 2x^2 - 5x - 12$

$0 = (2x+3)(x-4)$

$x = 4, -3/2$



A **hyperbola** is the locus of all points in the plane such that the absolute value of the differences of the distance from two given points in the plane, called foci, is constant. The center of the hyperbola is the midpoint of the segment whose endpoints are the foci. The asymptotes of a hyperbola are lines that the curve approaches as it recedes from the center. A hyperbola has two axes of symmetry. The **transverse** axis joins the vertices and has a length of $2a$. The segment perpendicular to the transverse axis through the center is called the **conjugate** axis and has a length of $2b$. The distance from the center to a vertex is a units, and the distance from the center to a foci is c units.

* a is no longer always the bigger # *

The standard form of the equation of a hyperbola with center (h,k) and transverse axis of length $2a$ units, where $c^2 = a^2 + b^2$ is as follows:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, when the transverse axis is parallel to the x-axis

(opens left and right)

Horizontal transverse axis
(HTA)

with asymptotes $y - k = \pm\frac{b}{a}(x-h)$

vertices $(h \pm a, k)$ and foci $(h \pm c, k)$

OR

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, when the transverse axis is parallel to the y-axis

(opens up and down)

Vertical transverse axis
(VTA)

with asymptotes $y - k = \pm\frac{a}{b}(x-h)$

"y ab"

vertices $(h, k \pm a)$ and foci $(h, k \pm c)$

1. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\frac{x^2}{25} - \frac{y^2}{16} = 1$. Then graph the hyperbola.

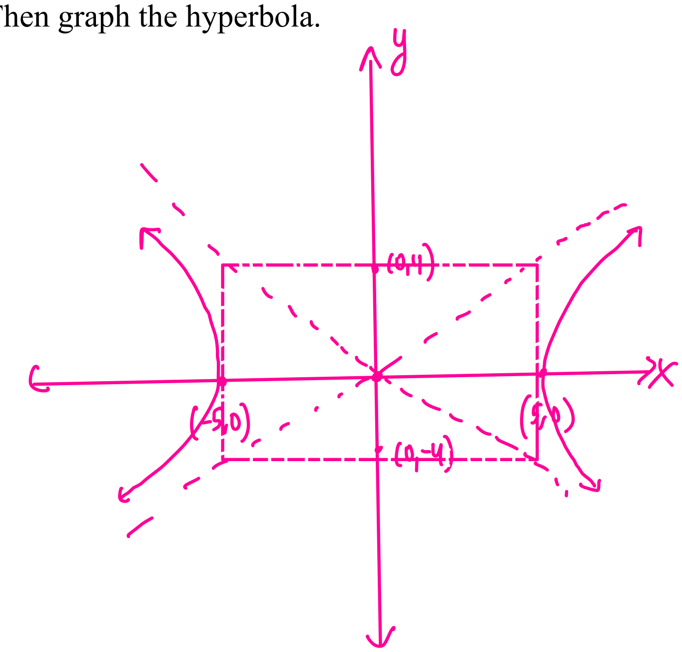
center: $(0,0)$

HTA (b/c x term is $\oplus$)

$a = 5$ ⇄

$b = 4$

$c = \sqrt{41}$ ⇄



Vertices: $(\pm 5, 0)$

foci: $(\pm\sqrt{41}, 0)$

asymptotes: $y - 0 = \pm \frac{4}{5}(x - 0)$

$y = \pm \frac{4}{5}x$

2. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\frac{y^2}{25} - \frac{x^2}{16} = 1$. Then graph the hyperbola.

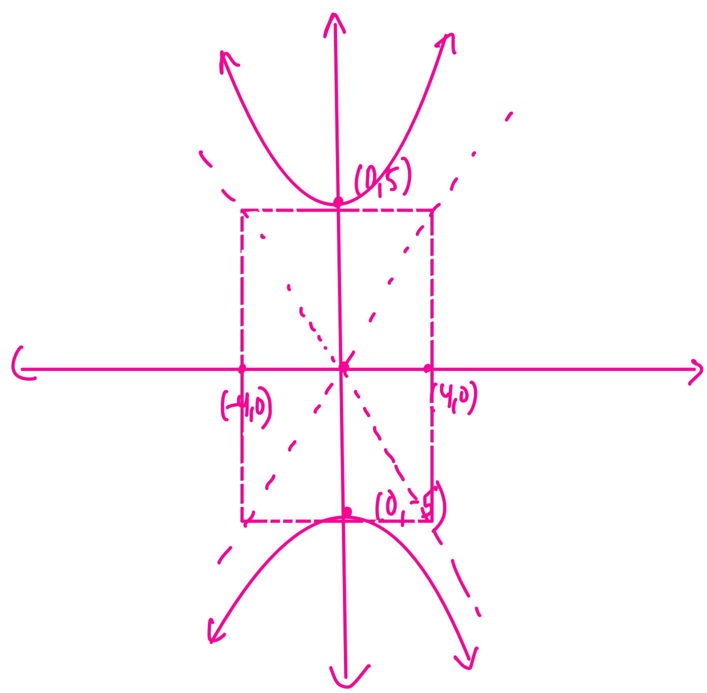
center: $(0,0)$

VTA

$a = 5$ ↑↓

$b = 4$

$c = \sqrt{41}$ ↑↓



Vertices: $(0, \pm 5)$

foci: $(0, \pm\sqrt{41})$

asym: $y - 0 = \pm \frac{5}{4}(x - 0)$

$y = \pm \frac{5}{4}x$

3. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$. Then graph the hyperbola.

center: $(2,3)$

VTA

$a = 5$ ↑↓

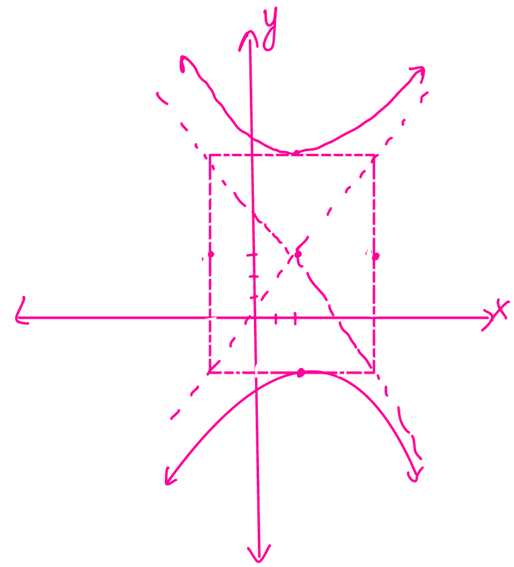
$b = 4$

$c = \sqrt{41}$ ↑↓

vertices: $(2, 3 \pm 5)$ — $(2,-2)$, $(2,8)$

foci: $(2, 3 \pm \sqrt{41})$

asymp: $y - 3 = \pm \frac{5}{4}(x-2)$



5. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $25y^2 - 9x^2 - 100y - 72x - 269 = 0$.. Then graph the hyperbola.

$25y^2 - 100y \quad -9x^2 - 72x \quad = 269$

$25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) \quad = 269 + 100 - 144$

$25(y-2)^2 - 9(x+4)^2 = 225$

$\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$

center: $(-4,2)$

VTA

$a = 3$ ↑↓

$b = 5$

$c = \sqrt{34}$ ↑↓

vertices: $(-4, 2 \pm 3)$ — $(-4,-1)$, $(-4,5)$

foci: $(-4, 2 \pm \sqrt{34})$

asymptotes: $y - 2 = \pm \frac{3}{5}(x+4)$

* you get the idea with the graphs

Homework 03-01

Name: ____ Date: __________
PCH Ellipses Ms. Loughran

Do Now:
Sketch the graph of each ellipse. State the coordinates of the center, vertices, covertices, and foci. State the length of the major axis and the length of the minor axis.

1. $25(x+2)^2 + 9y^2 = 225$

$$\frac{25(x+2)^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$$

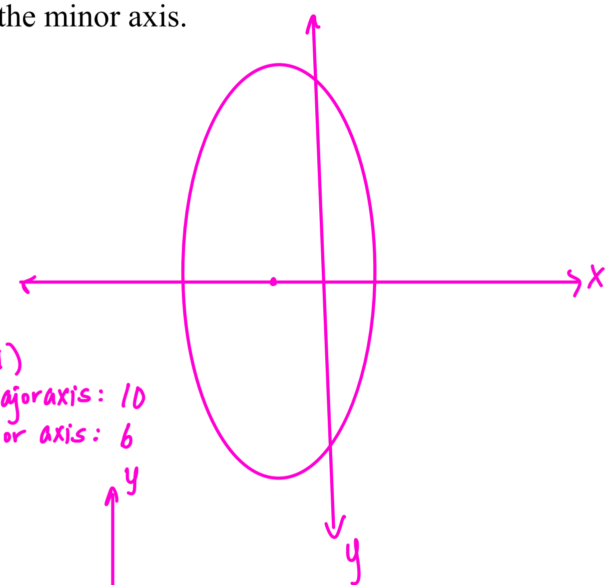
$$\frac{(x+2)^2}{9} + \frac{y^2}{25} = 1$$

center: $(-2,0)$
VMA
$a = 5$ ↑↓
$b = 3$ ⇄

vertices: $(-2, \pm 5)$
covertices: $(1,0), (-5,0)$

foci: $(-2, \pm 4)$
length of major axis: 10
length of minor axis: 6

$c^2 = 25 - 9$
$c^2 = 16$
$c = 4$ ↑↓



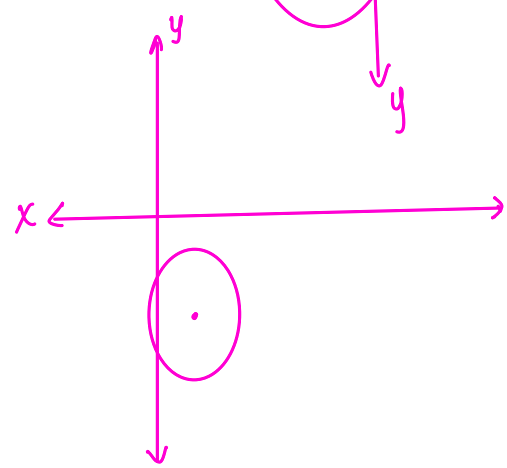
2. $4(x-1)^2 + 2(y+5)^2 = 8$

$$\frac{(x-1)^2}{2} + \frac{(y+5)^2}{4} = 1$$

center: $(1,-5)$
VMA
$a = 2$ ↑↓
$b = \sqrt{2}$ ⇄
$c = \sqrt{2}$ ↑↓

vertices: $(1,-7), (1,-3)$
covertices: $(1 \pm \sqrt{2}, -5)$
foci: $(1, -5 \pm \sqrt{2})$

length of major axis: 4
length of minor axis: $2\sqrt{2}$



3. $9x^2 + 16y^2 - 18x + 96y + 9 = 0$

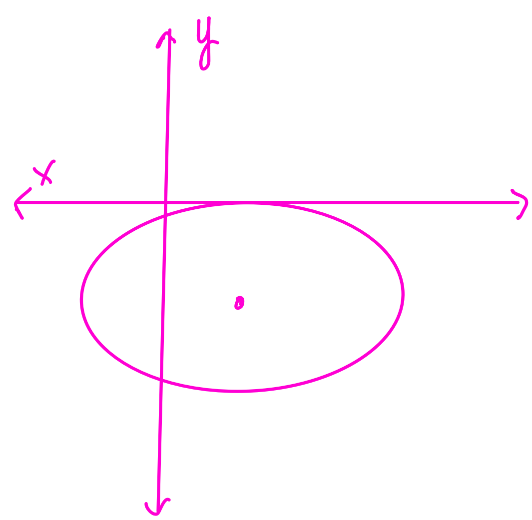
$$9(x^2 - 2x + 1) + 16(y^2 + 6y + 9) = -9 + 9 + 169$$

$$\frac{(x-1)^2}{16} + \frac{(y+3)^2}{9} = 1$$

center: $(1,-3)$
HMA
$a = 4$ ⇄
$b = 3$ ↑↓

vertices: $(-3,-3), (5,-3)$
covertices: $(1,-6), (1,0)$
foci: $(1 \pm \sqrt{7}, -3)$
length of major axis: 8
length of minor axis: 6

$c^2 = 16 - 9$
$c^2 = 7$
$c = \sqrt{7}$ ⇄



For #s 1– 6, write the standard form equation of an ellipse having the given properties.

1. Center (0, 0); horizontal major axis of length 10; minor axis of length 6.

$a=5$ $b=3$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

2. Center (0, 0); foci $(\pm 2, 0)$; vertices $(\pm 5, 0)$

$a=5$ $c=2$

$$\frac{x^2}{25}+\frac{y^2}{21}=1$$

$c^2=a^2-b^2$

$4=25-b^2$

$-21=-b^2$

$21=b^2$

$b=\sqrt{21}$

3. Vertices $(0, \pm 5)$; foci $(0, \pm 3)$

VMA $c:(0,0)$

$a=5$

$c=3$

$c^2=a^2-b^2$

$9=25-b^2$

$-16=-b^2$

$16=b^2$

$b=4$

$$\frac{x^2}{16}+\frac{y^2}{25}=1$$

4. Endpoints of the major and minor axes are $(\pm 8, 0)$ and $(0, \pm 4)$

HMA

$a=8$

$b=4$ $c:(0,0)$

$$\frac{x^2}{64}+\frac{y^2}{16}=1$$

5. Endpoints of the major and minor axes are $(\pm 1, 0)$ and $(0, \pm 3)$

VMA
$C: (0,0)$
$a=3$
$b=1$

$$x^2 + \frac{y^2}{9} = 1$$

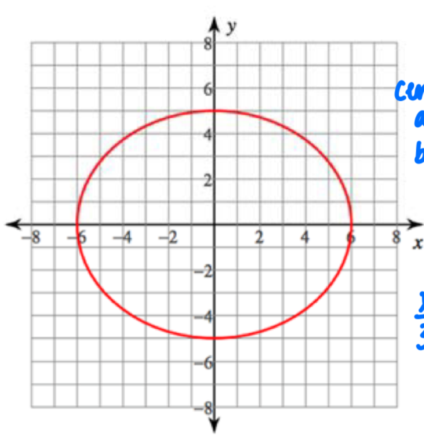
6. Endpoints of the major axis are $(-9,5)$ *and* $(3,5)$; endpoints of the minor axis are $(-3,6)$ *and* $(-3,4)$

$C: (-3, 5)$

HMA
$a=6$
$b=1$

$$\frac{(x+3)^2}{36} + (y-5)^2 = 1$$

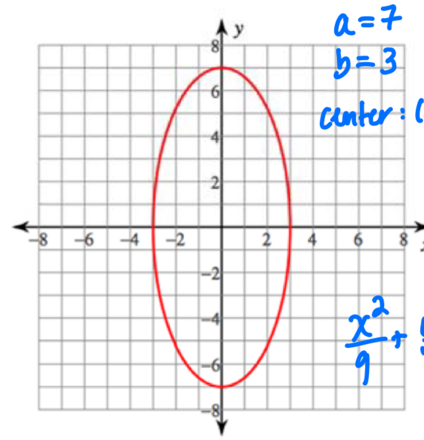
7.



center: $(0,0)$
$a=6$
$b=5$
HMA

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

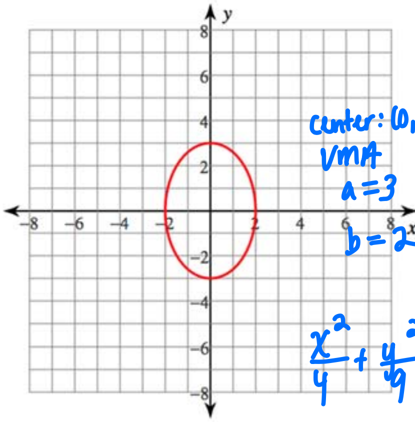
8.



VMA
$a=7$
$b=3$
center: $(0,0)$

$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$

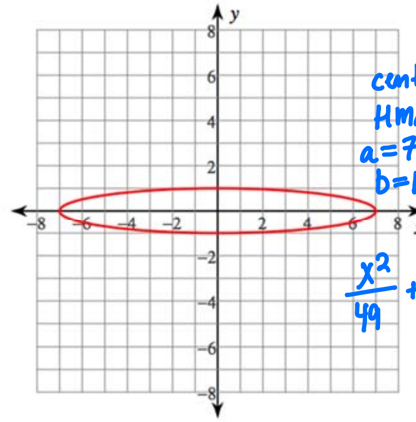
9.



center: (0,0)
VMA
a=3
b=2

$$\frac{x^2}{4}+\frac{y^2}{49}=1$$

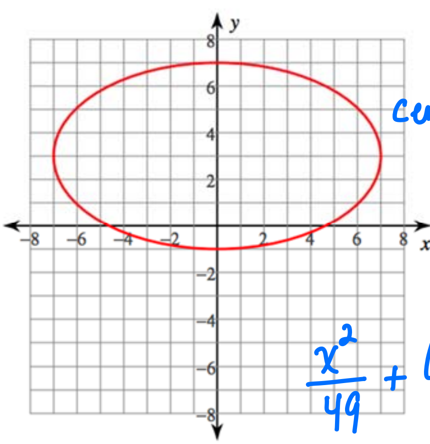
10.



center: (0,0)
HMA
a=7
b=1

$$\frac{x^2}{49}+y^2=1$$

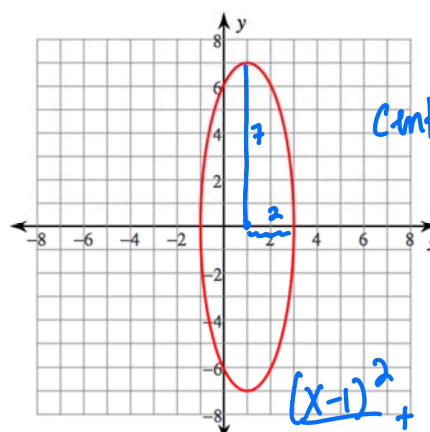
11.



center: (0,3)
a=7
b=4
HMA

$$\frac{x^2}{49}+\frac{(y-3)^2}{16}=1$$

12.



Center (1,0)
a=7
b=2
VMA

$$\frac{(x-1)^2}{4}+\frac{y^2}{49}=1$$

(17) $2x^2+8x+y^2-16y=-52$

$2(x^2+4x+4)+y^2-16y+64=-52+8+64$

20

$2(x+2)^2+(y-8)^2$

$\frac{(x+2)^2}{10}+\frac{(y-8)^2}{20}=1$

$a=\sqrt{20}=2\sqrt{5}$

$b=\sqrt{10}$

$c=\sqrt{10}$

$a^2=20, a=\sqrt{20}$ or $2\sqrt{5}$ ↑↓

$b^2=10, b=\sqrt{10}$ ⇄

$c(-2,8)$

VMA

vertices $(-2,8+2\sqrt{5})$

$(-2,8-2\sqrt{5})$

$(-2+\sqrt{10},8)$

$(-2-\sqrt{10},8)$

foci $(-2,8\pm\sqrt{10})$

(18) $169x^2-338x+4y^2+32y=443$

$169(x^2-2x+1)+4(y^2+8y+16)=443+169+64$

$169(x-1)^2+4(y+4)^2=676$

$\frac{(x-1)^2}{4}+\frac{(y+4)^2}{169}=1$

$c^2=169-4=165$

$c=\sqrt{165}$

VMA

$c:(1,-4)$

$a=13$ ↑↓ $b=2$ ⇄

VMA

vertices: $(1,9),(1,-17)$

$(3,-4),(-1,-4)$

foci: $(1,-4\pm\sqrt{165})$

19 is on next page

(20) $4x^2-8x+9y^2+126y=131$

$4(x^2-2x+1)+9(y^2+14y+49)=131+4+9(49)$

$4(x-1)^2+9(y+7)^2=576$

$\frac{(x-1)^2}{144}+\frac{(y+7)^2}{64}=1$

$a=12$ ⇄

$b=8$ ↑↓

$c:(1,-7)$

HMA

$V:(13,-7),(-11,-7)$

$(1,1)\quad(1,-15)$

$c^2=144-64=80$

$c=\sqrt{80}=\sqrt{16}\sqrt{5}=4\sqrt{5}$

$f:(1\pm4\sqrt{5},-7)$

(19) $\frac{(x+4)^2}{4} + \frac{(y+9)^2}{64} = 1$

VMA $a^2 = 64$, $a = 8 \uparrow\downarrow$
$b^2 = 4$, $b = 2 \rightleftarrows$

Center $(-4, -9)$

Vertices: $(-4, -9 \pm 8)$ $(-4, -17)$ $(-4, -1)$

Covertices: $(-4 \pm 2, -9)$ $(-6, -9)$ $(-2, -9)$

$c^2 = a^2 - b^2$
$c^2 = 64 - 4 = 60$
$c = \sqrt{60}$ or $2\sqrt{15}$ $\uparrow\downarrow$

foci: $(-4, -9 \pm 2\sqrt{15})$ or $(-4, -9 \pm \sqrt{60})$