Name:_____ Date:_____

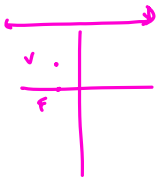
PCH: Radical Equations with Restriction Sets Ms. Loughran

Do Now:

$x = ay^2 + by + c$

$y = ax^2 + bx + c$

Find the equation of the parabola, in standard form, with the following characteristics:



1. vertex: $(-1,2)$
   focus: $(-1,0)$

$4p(y-k) = (x-h)^2$

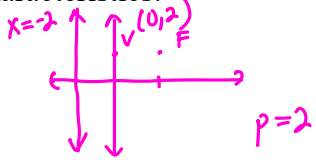
$-8(y-2) = (x+1)^2$

$y-2 = \frac{1}{-8}(x+1)^2$

$y = \frac{1}{-8}(x+1)^2 + 2$

$y = -\frac{1}{8}(x^2+2x+1) + 2$

$y = -\frac{1}{8}x^2 - \frac{1}{4}x + \frac{15}{8}$



2. focus: $(2,2)$
   directrix: $x = -2$

$4p(x-h) = (y-k)^2$

$8x = (y-2)^2$

$x = \frac{1}{8}(y-2)^2$

$x = \frac{1}{8}(y^2 - 4y + 4)$

$x = \frac{1}{8}y^2 - \frac{1}{2}y + \frac{1}{2}$

Model:

1. Solve for x: $x + \sqrt{x} = 1$

old way

$(\sqrt{x})^2 = (1-x)^2$

$x = x^2 - 2x + 1$

$0 = x^2 - 3x + 1$

$x = \frac{3 \pm \sqrt{9-4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{5}}{2}$

nearly impossible to check without a calculator

new way

$\sqrt{x} = 1 - x$

$x = x^2 - 2x + 1$

$x \geq 0$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

final restrictions $[0,1]$

$x = \frac{3+\sqrt{5}}{2}$ reject

$x = \frac{3-\sqrt{5}}{2}$

Classwork:

Solve each equation. Show restriction sets.

1. $\sqrt{x+11}+x=1$

$x+11 \geq 0$
$x \geq -11$

$1-x \geq 0$
$-x \geq -1$
$x \leq 1$

final restrictions
$[-11, 1]$

$\left(\sqrt{x+11}\right)^2 = (1-x)^2$

$x+11 = x^2-2x+1$

$0 = x^2-3x-10$

$0 = (x-5)(x+2)$

$x = \cancel{5}, -2$

reject

b/c it does not fall within our restriction set

2. $\sqrt{1+x}+x=5$

$1+x \geq 0$
$x \geq -1$

$5-x \geq 0$
$-x \geq -5$
$x \leq 5$

Final restrictions:
$[-1, 5]$

$\left(\sqrt{1+x}\right)^2 = (5-x)^2$

$1+x = x^2-10x+25$

$0 = x^2-11x+24$

$0 = (x-8)(x-3)$

$x = \cancel{8}, 3$

reject

b/c it falls outside of our restriction set

3. $\sqrt{x}+\sqrt{x-5}=5$

$x \geq 0$
$x-5 \geq 0$
$x \geq 5$

$5-\sqrt{x} \geq 0$
$-\sqrt{x} \geq -5$
$\sqrt{x} \leq 5$
$x \leq 25$

final restrictions:
$[5, 25]$

$\left(\sqrt{x-5}\right)^2 = (5-\sqrt{x})^2$

$x-5 = x-10\sqrt{x}+25$

$-30 = -10\sqrt{x}$

$3 = \sqrt{x}$

$9 = x$

4. $\sqrt{x-1}+\sqrt{3x+2}=3$

$x-1 \geq 0$ $\quad 3x+2 \geq 0$
$x \geq 1$ $\quad x \geq -2/3$

$3-\sqrt{x-1} \geq 0$
$-\sqrt{x-1} \geq -3$
$\sqrt{x-1} \leq 3$
$x-1 \leq 9$
$x \leq 10$

$x-3 \leq 0$
$x \leq 3$

final restrictions
$[1, 3]$

$\left(\sqrt{3x+2}\right)^2 = (3-\sqrt{x-1})^2$

$3x+2 = 9-6\sqrt{x-1}+x-1$

$3x+2 = 8-6\sqrt{x-1}+x$

$2x-6 = -6\sqrt{x-1}$

$(x-3)^2 = (-3\sqrt{x-1})^2$

$x^2-6x+9 = 9(x-1)$

$x^2-6x+9 = 9x-9$

$x^2-15x+18 = 0$

225
-72

$x = \dfrac{15 \pm \sqrt{225-4(1)(18)}}{2(1)}$

$x = \dfrac{15 \pm \sqrt{153}}{2}$

$\dfrac{15+\sqrt{153}}{2}$ reject

$\dfrac{15-\sqrt{153}}{2}$

Homework 03-13

6. $x^2+8x+4y+8=0$

opens down D F

$x^2+8x+16 = -4y-8+16$

$(x+4)^2 = -4y+8$

$(x+4)^2 = -4(y-2)$

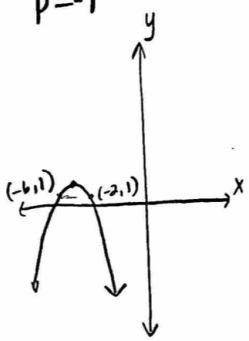
$-4 = 4p$

$p=-1$

Vertex: $(-4,+2)$

Focus: $(-4,1)$

Directrix: $y=3$



Axis of symmetry: $x=-4$

8. $2y^2+x+20y+51=0$

$2y^2+20y = -x-51$

$2(y^2+10y+25) = -x-51+50$

opens left

$2(y+5)^2 = -x-1$ $p=-\frac{1}{8}$

$2(y+5)^2 = -(x+1)$

$(y+5)^2 = -\frac{1}{2}(x+1)$

F: $(-\frac{9}{8},-5)$

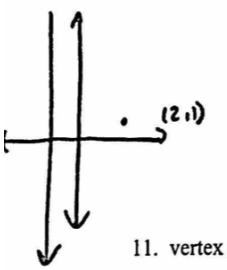
V: $(-1,-5)$

D: $x=-1+\frac{1}{8}=-\frac{7}{8}$

Aut sym: $y=-5$

10. focus: $(2,1)$ and directrix: $x=-2$

opens right



V: $(0,1)$

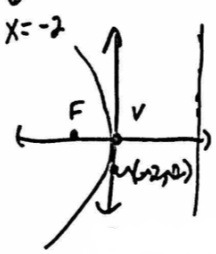
$p=2$

$(y-1)^2 = 4p(x)$

$(y-1)^2 = 8x$

11. vertex at the origin and focus: $(-2,0)$

opens left

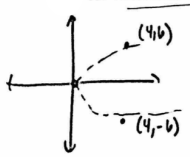


$p=2$

$-8x = y^2$

13. vertex at the origin, horizontal axis of symmetry, and passes through $(4,6)$

opens right



$y^2 = 4px$

$6^2 = 4p(4)$

$36 = 16p$ $p=36$

$y^2 = 4(36)x$

$y^2 = 9x$