Do Now: From the Solving Equations Using Logarithms sheet #s 26 and 28

* change of base

26. Find the value of the product: $(\log_2 3)(\log_3 4)(\log_4 5)\ldots(\log_{31} 32)$

$$\frac{\cancel{\log 3}}{\log 2} \cdot \frac{\cancel{\log 4}}{\cancel{\log 3}} \cdot \frac{\cancel{\log 5}}{\cancel{\log 4}} \cdots \frac{\log 32}{\cancel{\log 31}}$$

$$\frac{\log 32}{\log 2} = \log_2 32 = 5$$

28. Find the numerical value of $(\log_a b^2)(\log_b c^2)(\log_c a^2)$

$$(2\log_a b)(2\log_b c)(2\log_c a)$$

$$\frac{2\cancel{\log b}}{\cancel{\log a}} \cdot \frac{2\cancel{\log c}}{\cancel{\log b}} \cdot \frac{2\cancel{\log a}}{\cancel{\log c}} = 8$$

Also: Evaluate

1. $4^{\log_4 16} = 16$

2. $3^{\log_3 27} = 27$

3. $4^{\log_2 8} = 2^{2\log_2 8}$

   or $4^3 = 64$ $\quad 2^{\log_2 8^2} = 64$

4. $81^{\log_3 \frac{1}{3}} = 3^{4\log_3 \frac{1}{3}}$

   or $81^{-1} = \frac{1}{81}$ $\quad 3^{\log_3 \left(\frac{1}{3}\right)^4} = \frac{1}{81}$

Classwork:

15. $\log x^2 = (\log x)^2$

$2\log x = (\log x)^2$ Substitution

let $y = \log x$

$2y = y^2$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0$ | $y = 2$

$\log x = 0$ | $\log x = 2$

$x = 10^0 = 1$ | $x = 10^2 = 100$

put in exponential form

20. $x^{\log x} = 1000x^2$

$\log(x^{\log x}) = \log(1000x^2)$

$(\log x)(\log x) = \log 1000 + \log x^2$

$(\log x)^2 = 3 + 2\log x$

$\log x = y$

$y^2 = 3 + 2y$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y = 3$ | $y = -1$

$\log x = 3$ | $\log x = -1$

$x = 10^3 = 1000$ | $x = 10^{-1} = \frac{1}{10}$

24. $10^{(1+\log x)} = 50$

$10^1 \cdot 10^{\log x}$

$10 \cdot x = 50$

$x = 5$

18. $\log_3 x + \log_9 x + \log_{81} x = 7$

$\frac{\log x}{\log 3} + \frac{\log x}{\log 9} + \frac{\log x}{\log 81} = 7$

$\frac{\log x}{\log 3} + \frac{\log x}{2\log 3} + \frac{\log x}{4\log 3} = 7$

let $y = \frac{\log x}{\log 3}$

$y + \frac{y}{2} + \frac{y}{4} = 7$

$4y + 2y + y = 28$

$7y = 28$

$y = 4$

$\frac{\log x}{\log 3} = 4$

$\log_3 x = 4$ or $4\log 3 = \log x$

$x = 3^4 = 81$ | $\log 3^4 = \log x$

$3^4 = x$

$81 = x$

22. $4^{\log_2 x} + x^2 = 8$

$2^{2\log_2 x} + x^2 = 8$

$2^{\log_2 x^2} + x^2 = 8$

$x^2 + x^2 = 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$ reject -2

Homework 03-19

Do Now

1. $2\log x + \log 5 = \log 125$

$\log x^2 + \log 5 = \log 125$

$\log 5x^2 = \log 125$

$5x^2 = 125$

$x^2 = 25$

$x = \cancel{\pm} 5$

2. $\log(x+3) + \log(x-2) = \log(x-5) + \log(x+2)$

$\log(x+3)(x-2) = \log(x-5)(x+2)$

$\cancel{x^2} + x - 6 = \cancel{x^2} - 3x - 10$

$4x = -4$

$\cancel{x = -1}$ $\varnothing$

3. $\log(x-4) - \log(x+1) = \log 6$

$\log \frac{x-4}{x+1} = \log 6$

$\frac{x-4}{x+1} = 6$

$6x + 6 = x - 4$

$5x = -10$

$\cancel{x = -2}$

$\varnothing$

4. $\ln 3 - \frac{1}{3}\ln x = 0$

$\ln \frac{3}{\sqrt[3]{x}} = 0$

$e^0 = \frac{3}{\sqrt[3]{x}}$

$\sqrt[3]{x} = 3$

$x = 27$

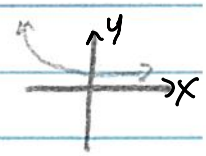
5 $\log(x^2-21x)=2$

$10^2 = x^2-21x$

$0 = x^2-21x-100$

$0 = (x-25)(x+4)$

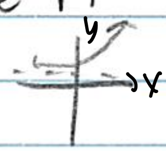
$x=25$ / $x=-4$



CW

1 $y=e^x+1$

$e^x \uparrow 1$



D: $(-\infty,\infty)$

R: $(1,\infty)$

Asym: $y=1$

x-int none

y-int $(0,2)$

2 $y=e^{x+1}$ e^x left one

D $(-\infty,\infty)$

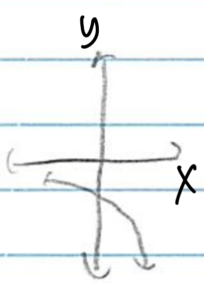
R $(0,\infty)$

Asym: $y=0$

x-int: none

y-int $(0,e)$

3 $y=-e^x$ reflect over x-axis



D: $(-\infty,\infty)$

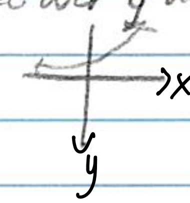
R: $(-\infty,0)$

Asy: $y=0$

x-int none

y-int $(0,-1)$

4 $y=e^{-x}$ reflect over y axis



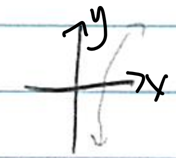
D: $(-\infty,\infty)$

R: $(0,\infty)$

Asy: $y=0$

x-int: none

y-int $(0,1)$



$\ln x \downarrow 1$

⑤ $y = \ln x - 1$

D: $(0, \infty)$
R: $(-\infty, \infty)$
Asym $x = 0$
x-int $(e, 0)$
y-int none

$0 = \ln x - 1$
$1 = \ln x$
$x = e$

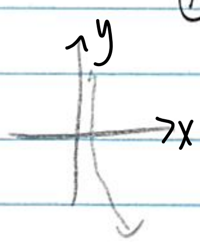
right one

⑥ $y = \ln(x-1)$

D: $x - 1 > 0$

D: $(1, \infty)$
R: $(-\infty, \infty)$
Asy: $x = 1$
x-int $(2, 0)$
y-int none

$0 = \ln(x-1)$
$e^0 = x - 1$



⑦ $y = -\ln x$

reflect over x

D: $(0, \infty)$
R: $(-\infty, \infty)$
Asy: $x = 0$
x-int: $(1, 0)$
y-int: none

$0 = -\ln x$
$0 = \ln x$
$e^0 = x$

⑧ $y = \ln(-x)$

reflect over y

D: $(-\infty, 0)$
R: $(-\infty, \infty)$
Asy: $x = 0$
x-int $(-1, 0)$
y-int none

9. $\log n = 1 + \log 2$

$\log n = \log 10 + \log 2$

$\log n = \log 10(2)$

$\log n = \log 20$

$n = 20$ (D)

10. $x = (\log_8 2)(\log_2 8)$

$x = \frac{\cancel{\log 2}}{\cancel{\log 8}} \cdot \frac{\cancel{\log 8}}{\cancel{\log 2}}$

$x = 1$

$\log_3 1 = 0$

11. a) $e^{\ln 5} = 5$

b) $10^{\log 5} = 5$

$\uparrow$

$10^{\log 5} = x$

$\log 5 = \log x$

$5 = x$

* $b^{\log_b x} = x$

$\log_b x = \log_b x$

12. $3^{\ln x} = 3$

$3^{\ln x} = 3^1$

$\ln x = 1$

$x = e$

13. $5^{\ln x} = 25$

$5^{\ln x} = 5^2$

$\ln x = 2$

$x = e^2$

14. $(\log_3 9)(\log_9 3) = x$

$\frac{\cancel{\log 9}}{\cancel{\log 3}} \cdot \frac{\cancel{\log 3}}{\cancel{\log 9}} = x$

$1 = x$

15. $(\log_5 7)(\log_7 5) = x$

$\frac{\cancel{\log 7}}{\log 5} \cdot \frac{\cancel{\log 5}}{\log 7} = x$

$1 = x$

16 $(\log_3 9)(\log_9 81)=x$

$$\frac{\cancel{\log 9}}{\log 3}\cdot\frac{\log 81}{\cancel{\log 9}}=x$$

$$\frac{\log 3^4}{\log 3}=x$$

$$\frac{4\log 3}{\log 3}=x$$

$$4=x$$

17 $(\log_2 3)(\log_3 4)=x$

$$\frac{\cancel{\log 3}}{\log 2}\cdot\frac{\log 4}{\cancel{\log 3}}=x$$

$$\frac{2\log 2}{\log 2}=x$$

$$2=x$$

18 $(\log_8 9)(\log_3 64)(\log_{27} 3)=x$

$$\frac{\log 9}{\log 8}\cdot\frac{\log 64}{\log 3}\cdot\frac{\log 3}{\log 27}=x$$

$$\frac{2\cancel{\log 3}}{3\cancel{\log 2}}\cdot\frac{6\cancel{\log 2}}{\cancel{\log 3}}\cdot\frac{\cancel{\log 3}}{3\cancel{\log 3}}=x$$

$$\frac{12}{9}=x$$

$$\frac{4}{3}=x$$

19 $\log_2(\log_3 x)=4$

$$2^4=\log_3 x$$

$$16=\log_3 x$$

$$x=3^{16}$$

20 $\ln(\ln x)=0$

$$e^0=\ln x$$

$$1=\ln x$$

$$x=e^1=e$$

21. $\log_2(\log_4 x) = 1$

$2^1 = \log_4 x$

$x = 4^2 = 16$

22. $\log_5(\log_3 x) = 0$

$5^0 = \log_3 x$

$1 = \log_3 x$

$3^1 = x$

$3 = x$

23. $\log_4(\log_3(\log_2 x)) = 0$

$4^0 = \log_3(\log_2 x)$

$1 = \log_3(\log_2 x)$

$3^1 = \log_2 x$

$2^3 = x$

$8 = x$

24. $\log_2(\log_3(\log_5 x)) = 0$

$2^0 = \log_3(\log_5 x)$

$1 = \log_3(\log_5 x)$

$3^1 = \log_5 x$

$3 = \log_5 x$

$5^3 = x$

$125 = x$

25. $\log(\log_6(\log(\log x))) = 0$

$10^0 = \log_6(\log(\log x))$

$6^1 = \log(\log x)$

$10^6 = \log x$

$10^{10^6} = x$

26) a) $\log 6 = \log(2 \cdot 3)$
$\log 2 + \log 3$
$a + b$

b) $\log \frac{2}{3}$
$\log 2 - \log 3$
$a - b$

$118 \cdot 100$

c) $\log 12$
$\log(2^2 \cdot 3)$
$2\log 2 + \log 3$
$2a + b$

d) $\log 1800$
$\log(3^2 \cdot 2 \cdot 100)$
$\log 3^2 + \log 2 + \log 100$
$2\log 3 + \log 2 + \log 100$
$2b + a + 2$

e) $\log \frac{1}{2}$
$\log 2^{-1}$
$-\log 2$
$-a$

$\log 1 - \log 2$
$0 - a$
$-a$

g) $\log \frac{1}{3}$
$\log 3^{-1}$
$-\log 3$
$-b$

$\log 1 - \log 3$
$0 - b$
$-b$

f) $\log 5$
$\log \frac{10}{2}$
$\log 10 - \log 2$
$1 - a$

h) $\log(3^2 \cdot 2^2)$
$2\log 3 + 2\log 2$
$2b + 2a$