

Name: \_\_\_\_\_  
PCH – “e” and “ln” in Preparation for Calculus

Date: \_\_\_\_\_

Many questions in Calculus require you to work with “e” and “ln”. The following questions will help you brush up on the skills you will need in this area next year.

1. Given  $h(x) = 5x + 4e^x$  find  $h(0)$ . Simplify your answer.

$$h(0) = 5(0) + 4e^0 = 4$$

2. Solve for  $x$ :  $\ln 3 - \frac{1}{3} \ln x = 0$

$$\begin{aligned} \ln 3 &= \ln x^{\frac{1}{3}} \\ 3 &= x^{\frac{1}{3}} \\ 27 &= x \end{aligned}$$

3. Solve for  $x$ :  $\ln x = 3$

$$x = e^3$$

4. Given  $r(x) = \frac{1}{8}e^{\frac{x}{8}}$ , find  $r(8 \ln 5)$ , find in simplest form.

$$\begin{aligned} r(8 \ln 5) &= \frac{1}{8} e^{\frac{8 \ln 5}{8}} = \frac{1}{8} e^{\ln 5} \\ \frac{1}{8}(5) &= \frac{5}{8} \end{aligned}$$

5. Solve for  $x$ :  $\ln e^2 + \ln e^3 = x$ . Express your answer in simplest form.

$$\begin{aligned} \ln e^5 &= x \\ 5 &= x \end{aligned}$$

6. Given  $g(x) = \frac{1}{(x+6+e^{-5x})} \cdot (1-5e^{-5x})$ , find  $g(0)$  in simplest form.

$$g(0) = \frac{1}{0+6+e^{-5(0)}} \cdot (1-5e^{-5(0)}) = \frac{-4}{7}$$

7. Solve for  $x$ :  $e^{4x} + 4e^{2x} - 21 = 0$ .

$$\begin{array}{l} (e^{2x} + 7)(e^{2x} - 3) = 0 \\ \hline e^{2x} + 7 = 0 \quad | \quad e^{2x} - 3 = 0 \\ e^{2x} \neq -7 \quad | \quad e^{2x} = 3 \\ \text{impossible} \quad | \quad \ln 3 = 2x \\ \quad \quad \quad \quad | \quad x = \frac{\ln 3}{2} \end{array}$$

8. Solve for  $x$ :  $3xe^x + x^2e^x = 0$ .

$$\begin{array}{l} xe^x(3+x) = 0 \\ \hline xe^x = 0 \quad | \quad 3+x = 0 \\ x = 0 \quad | \quad x = -3 \end{array}$$

9. Find the value of  $k$  for which  $f(x)=0$  when  $x=\frac{3}{5}$  given  $f(x)=x^2ke^{kx}+3xe^{kx}$ .

$$f(x) = x e^{kx} (xk + 3)$$

$$0 = \frac{3}{5} e^{k(\frac{3}{5})} \left( \frac{3}{5}k + 3 \right)$$

$$\frac{3}{5} e^{\frac{3k}{5}} = 0 \quad \left| \quad 5 \left( \frac{3}{5}k + 3 = 0 \right) \right.$$

impossible

$$3k + 15 = 0$$

$$k = -5$$

10. Given  $\ln(y) = \ln(3x+1) + c$  and  $y(0) = e$ .

(a) Find  $C$ .

$$\ln y = \ln(3x+1) + c$$

$$\ln e = \ln(3(0)+1) + c$$

$$1 = \ln 1 + c$$

$$c = 1$$

$$1 = 0 + c$$

$y(0) = e$   
 ← plugin  $(0, e)$

(b) Using this value of  $C$ , show the work that would lead to how this equation leads to  $y = 3xe + e$ .

$$\ln y = \ln(3x+1) + 1$$

$$e^{\ln y} = e^{\ln(3x+1) + 1}$$

$$y = e^{\ln(3x+1)} e^1$$

$$y = e(3x+1) = 3xe + e$$

11. Given  $-e^{-y} = e^x + C$

(a) Find  $C$  if  $y(0) = -\ln 2$

$$(0, -\ln 2)$$

$$-e^{\ln 2} = e^0 + c$$

$$-e^{\ln 2} = 1 + c$$

$$e^{\ln 2} = -1 - c$$

$$2 = -1 - c$$

$$-c = 3$$

$$c = -3$$

(b) Using this value of  $C$ , rewrite the equation in the form " $y =$ ".

$$-e^{-y} = e^x - 3$$

$$e^{-y} = -e^x + 3$$

$$\ln e^{-y} = \ln(-e^x + 3)$$

$$-y = \ln(-e^x + 3) \rightarrow \begin{matrix} -e^x + 3 > 0 \\ -e^x > -3 \end{matrix}$$

$$y = -\ln(-e^x + 3)$$