Exponential Growth

$$n(t) = n_0 e^{rt}$$

n_0 = initial amount

r: rate of growth

t: time

$n(t)$: amount at time t

Radioactive Decay

$$m(t) = m_0 e^{-rt}$$

m_0: initial mass

r: rate

t: time

$m(t)$: mass at time t

$$\frac{1}{2} = e^{-rh}$$

$$-rh = \ln \frac{1}{2}$$

$$-rh = \ln 2^{-1}$$

$$-rh = -\ln 2$$

$$rh = \ln 2$$

$$r = \frac{\ln 2}{h}$$ ← half life

or

$$h = \frac{\ln 2}{r}$$

Name: _____ Date: _____

PCH: Practice with Natural Growth and Decay

You may use a calculator.

1. Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg. How long will it take for the sample to decay to a mass of 200 mg? Round your answer to 3 decimal places.

$200 = 300e^{-rt}$

* need r: $r = \frac{\ln 2}{140}$

$200 = 300e^{-\frac{\ln 2}{140}t}$

$\frac{2}{3} = e^{-\frac{\ln 2}{140}t}$

$-\frac{140}{\ln 2} \cdot \ln\frac{2}{3} = -\frac{\ln 2}{140}t \cdot \frac{-140}{\ln 2}$

$81.8947... = t$

$t = 81.895$ days

2. The half-life of strontium-90 is 28 years. How long will it take a 50 mg sample to decay to a mass of 32 mg? Round your answer to 3 decimal places.

$32 = 50e^{-\frac{\ln 2}{28}t}$

* need r. $r = \frac{\ln 2}{28}$

$\frac{32}{50} = e^{-\frac{\ln 2}{28}t}$

$-\frac{\ln 2}{28}t = \ln\left(\frac{32}{50}\right)$

$t = \ln\left(\frac{32}{50}\right) \cdot \frac{-28}{\ln 2} = 18.0279...$

$t = 18.028$ years

3. If 250 mg of a radioactive element decays to 200 mg in 48 hours, find the half-life of the element. Round your answer to 3 decimal places.

* $r = \frac{\ln 2}{h}$

$200 = 250e^{-r(48)}$

$200 = 250e^{-\frac{\ln 2}{h}(48)}$

$\frac{200}{250} = e^{-\frac{\ln 2}{h}(48)}$

$\frac{4}{5} = e^{-\frac{\ln 2}{h}(48)}$

$\ln\left(\frac{4}{5}\right) = \frac{-48\ln 2}{h}$

$h\ln\left(\frac{4}{5}\right) = -48\ln 2$

$h = \frac{-48\ln 2}{\ln\left(\frac{4}{5}\right)} = 149.1016...$

149.102 hrs

4. A wooden artifact from an ancient tomb contains 65% of the carbon-14 that is present in living trees. The half-life of carbon-14 is 5730 years. How long ago was the artifact made? Round your answer to 3 decimal places.

$r = \frac{\ln 2}{5730}$

$.65 = e^{-\frac{\ln 2}{5730}t}$

$\ln .65 = -\frac{\ln 2}{5730}t$

3561.128 yrs

$t = \ln .65 \cdot \frac{5730}{-\ln 2} = 3561.12839...$

5. After 3 days a sample of radon-222 has decayed to 58% of its initial amount. Find the half-life of radon-222. Then find how long it will take the sample to decay to 20% of its original amount. Round your answers to 3 decimal places.

$(-\ln 2)\left(\frac{\ln .58}{-3\ln 2}\right)$

$.58 = e^{-\frac{\ln 2}{h}(3)}$

$\ln .58 = -\frac{3\ln 2}{h}$

$h \ln .58 = -3\ln 2$

$h = \frac{-3\ln 2}{\ln .58}$

$h = 3.8174...$ $h = 3.817$ days

$.2 = e^{\frac{-\ln 2}{-3\ln 2/\ln .58}t}$

$\ln .2 = \frac{\ln .58}{3}t$

$t = \ln .2 \cdot \frac{3}{\ln .58} = 8.8637...$

8.864 days

6. The population of the world in 2000 was 6.1 billion and the estimated relative growth rate was 1.4% per year. If the population continues to grow at this rate, during what year will it reach 122 billion?

$122 = 6.1e^{.014t}$

$\frac{122}{6.1} = e^{.014t}$

$.014t = \ln\left(\frac{122}{6.1}\right)$

$t = \frac{\ln\left(\frac{122}{6.1}\right)}{.014}$

$t = 213.9808...$

$2000 + 213.9808...$

During 2213

7. The half-life of cesium-137 is 30 years. Suppose we have a 10 gram sample. How much of the sample will remain after 80 years? After how long will only 2 grams of the sample remain? Round your answers to 3 decimal places.

a) $y = 10e^{\frac{-\ln 2}{30}(80)}$

$y = 1.5749\ldots$

1.575 grams

b) $2 = 10e^{\frac{-\ln 2}{30}t}$

$.2 = e^{\frac{-\ln 2}{30}t}$

$\ln .2 = \frac{-\ln 2}{30}t$

$t = \ln .2 \left(\frac{30}{-\ln 2}\right)$

$t = 69.6578\ldots$

69.658 yrs

Name: _____

Who Stole the Golden Cone?

WHO?
Mr. Edelman
Mr. Lee
Mrs. Stack
Mr. Carman
Ms. Simon
Ms. Loughran

WHAT?
Compass
Protractor
Ruler
Math Textbook
Smartboard Pen
Graphing Calculator

WHERE?
Math Study Center
Cafeteria
Main Office
Security Desk
Bookstore
Bus Garage
Main Gym
Library
Lecture Room A
Lecture Room B
Band Room

IT WAS (WHO)	WITH A (WHAT)	IN THE (WHERE)
Ms. Simon	Smart board Pen	Main Gym

Name: _____ Date: _____

PCH Conic Section CLUE Questions and Workspace

Answer the following for $x^2 - y - 8x + 19 = 0$

1. Identify the type of conic section.
2. What are the coordinates of the vertex?
3. In what direction does it open?
4. Sketch a graph.

$x^2 - 8x + 16 = y - 19 + 16$

$(x-4)^2 = y - 3$

parabola

vertex: $(4, +3)$

up

Answer the following for $4x^2 - 16y^2 - 16x + 32y - 64 = 0$

5. Identify the type of conic section.
6. What are the coordinates of the center?
7. What are the coordinates of the vertices?
8. Sketch a graph.

$4(x^2 - 4x + 4) - 16(y^2 - 2y + 1) = 64 + 16 - 1$

$4(x-2)^2 - 16(y-1)^2 = 64$

$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{4} = 1$

$a = 4$

hyperbola

center $(2, 1)$

vertices: $(6, 1), (-2, 1)$

Answer the following for $9x^2 + y^2 - 18x - 6y + 9 = 0$

9. Identify the type of conic section.
10. What are the coordinates of the center?
11. What are the coordinates of the vertices?
12. What are the coordinates of the covertices?
13. Sketch the graph.

$9(x^2-2x+1) + y^2 - 6y + 9 = -9+9+9$

$\frac{(x-1)^2}{9} + (y-3)^2 = 1$ VMA

$a=3$

ellipse $b=1$

$(1,3)$

vertices $(1,6), (1,0)$

covertices: $(0,3), (2,3)$

Answer the following for $x^2 + y^2 + 2x + 6y = 26$

14. Identify the type of conic section.
15. What are the coordinates of the center?
16. What is the length of the radius?
17. Sketch the graph.

$x^2+2x+1 + y^2+6y+9 = 26+1+9$

$(x+1)^2 + (y+3)^2 = 36$

center: $(-1,-3)$

$r = 6$