Name: _____ Date: _____

PCH: General Solutions to Trig Equations Ms. Loughran

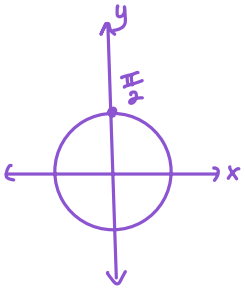
Do Now:

Find all solutions of the equation in the interval $(0, 2\pi]$.

1. $\sin x = 1$

particular solution

$x = \frac{\pi}{2}$



general solutions:

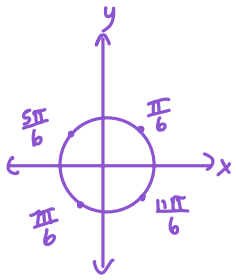
$x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$

↖ "k is an element of the Integers"

2. $\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



general solutions:

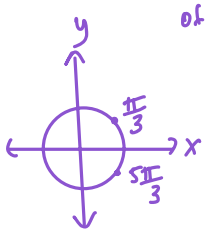
$x = \frac{\pi}{6} + \pi k$ or $\frac{5\pi}{6} + \pi k$, $k \in \mathbb{Z}$

3. $\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



general solution:

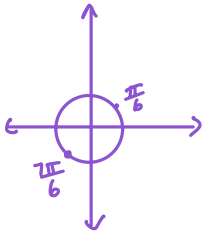
$x = \frac{\pi}{3} + 2\pi k$ or $\frac{5\pi}{3} + 2\pi k$, $k \in \mathbb{Z}$

Now let's write what is called the general solutions for questions 1-3.

Classwork:

Find all solutions of each equation.

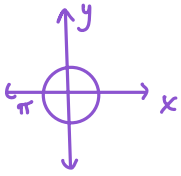
1. $\tan 2x = \frac{1}{\sqrt{3}}$



$2x = \frac{\pi}{6} + \pi k, k \in \mathbb{Z}$

$x = \frac{\pi}{12} + \frac{\pi}{2} k, k \in \mathbb{Z}$

2. $\cos \frac{x}{2} + 1 = 0$



$\cos \frac{x}{2} = -1$

$\frac{x}{2} = \pi + 2\pi k, k \in \mathbb{Z}$

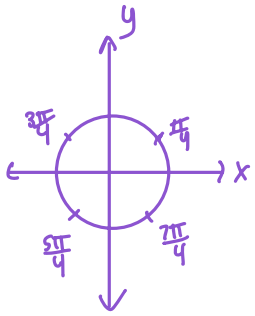
$x = 2\pi + 4\pi k, k \in \mathbb{Z}$

3. $2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

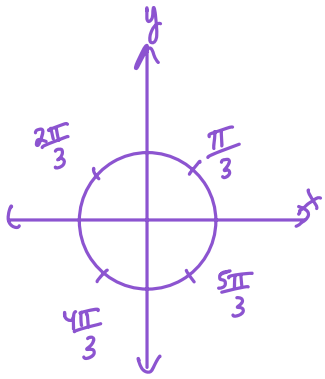


$x = \frac{\pi}{4} + \frac{\pi}{2}k, \; k \in \mathbb{Z}$

4. $4\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$



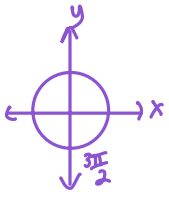
$x = \begin{matrix} \frac{\pi}{3} + \pi k \\ \frac{2\pi}{3} + \pi k \end{matrix}, \; k \in \mathbb{Z}$

5. $\sin^2 x = 2\sin x + 3$

$\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 3)(\sin x + 1) = 0$

$\sin x = 3$ $\quad \sin x = -1$

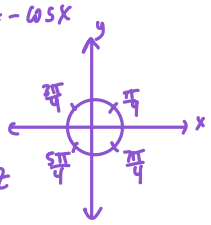


$x = \frac{3\pi}{2} + 2\pi k, \; k \in \mathbb{Z}$

6. $\sin^2 x - \cos^2 x = 0$

$(\sin x - \cos x)(\sin x + \cos x) = 0$

$\sin x = \cos x$ $\quad \sin x = -\cos x$



$x = \frac{\pi}{4} + \frac{\pi}{2}k, \; k \in \mathbb{Z}$

From your textbook:

7.5 Exercises

1–40 ■ Find all solutions of the equation.

1. $\cos x + 1 = 0$
2. $\sin x + 1 = 0$
3. $2 \sin x - 1 = 0$
4. $\sqrt{2} \cos x - 1 = 0$
5. $\sqrt{3} \tan x + 1 = 0$
6. $\cot x + 1 = 0$
7. $4 \cos^2 x - 1 = 0$
8. $2 \cos^2 x - 1 = 0$
9. $\sec^2 x - 2 = 0$
10. $\csc^2 x - 4 = 0$
11. $3 \csc^2 x - 4 = 0$
12. $1 - \tan^2 x = 0$
13. $\cos x (2 \sin x + 1) = 0$
14. $\sec x (2 \cos x - \sqrt{2}) = 0$
15. $(\tan x + \sqrt{3})(\cos x + 2) = 0$
16. $(2 \cos x + \sqrt{3})(2 \sin x - 1) = 0$
17. $\cos x \sin x - 2 \cos x = 0$
18. $\tan x \sin x + \sin x = 0$
19. $4 \cos^2 x - 4 \cos x + 1 = 0$
20. $2 \sin^2 x - \sin x - 1 = 0$

Homework 04-12

② $4(\csc\theta + 2) = \csc\theta + 14$

Let $x = \csc\theta$

$4(x+2) = x+14$

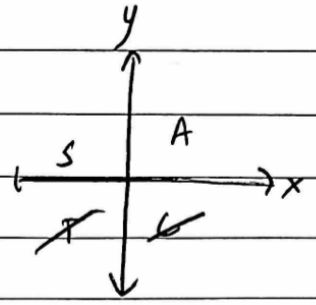
$4x + 8 = x + 14$

$3x = 6$

$x = 2$

$\csc\theta = 2$

$\sin\theta = \frac{1}{2}$



QI $\theta = 30°$

QII $\theta = 180° - 30° = 150°$

④ $2\cos\theta + 5\sqrt{3} = 4\sqrt{3}$

Let $x = \cos\theta$

$2x + 5\sqrt{3} = 4\sqrt{3}$

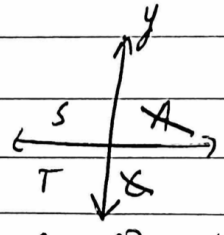
$2x = -\sqrt{3}$

$x = \frac{-\sqrt{3}}{2}$

$\cos\theta = \frac{-\sqrt{3}}{2}$

Remember to find ref $\measuredangle$ ignore the $\ominus$

ref $\measuredangle$: $30°$



$\cos\theta$ is $\ominus$ in Quadrants II, III

QII $\theta = 180° - 30° = 150°$

QIII $\theta = 180° + 30° = 210°$

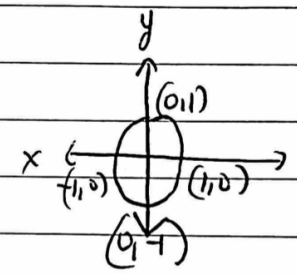
$\{150°, 210°\}$

⑥ $3\sin\theta - 1 = 2$

$3\sin\theta = 3$

$\sin\theta = 1$

Remember when sin or cos $= \pm 1, 0$ θ is a quadrantal ∠ so goto your unit circle.



$\theta = 90°$

⑧ $3\sec\theta = \frac{2}{3}(3\sec\theta - 3)$

let $x = \sec\theta$

$3x = \frac{2}{3}(3x - 3)$

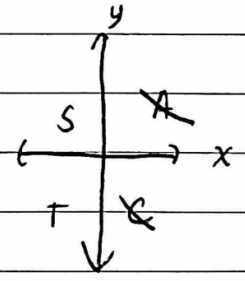
$3x = 2x - 2$

$x = -2$

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

ref ∠: $= 60°$



QII: $\theta = 180° - 60° = 120°$

QIII $\theta = 180° + 60° = 240°$

$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

⑩ $6\cos\theta + \sqrt{3} = -4(\cos\theta + \sqrt{3})$

let $x = \cos\theta$

$6x + \sqrt{3} = -4(x + \sqrt{3})$

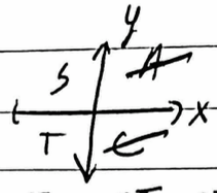
$6x + \sqrt{3} = -4x - 4\sqrt{3}$

$10x = -5\sqrt{3}$

$x = \frac{-5\sqrt{3}}{10} = \frac{-\sqrt{3}}{2}$

$\cos\theta = \frac{-\sqrt{3}}{2}$

ref $\measuredangle$: 30°



$\cos\theta$ is $\ominus$ in QII, QIII

QII $\theta = 180^\circ - 30^\circ = 150^\circ$

QIII $\theta = 180^\circ + 30^\circ = 210^\circ$

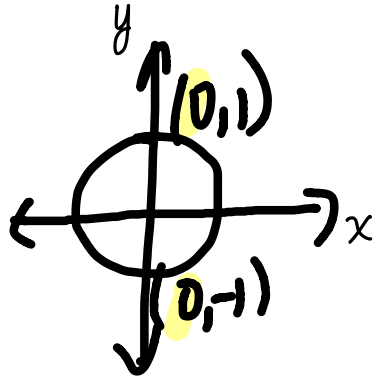
Remember to convert to radians here b/c of the interval given in question.

$\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$

⑫ $4\cos\theta + 3 = 3$

$4\cos\theta = 0$

$\cos\theta = 0$



$\theta = 90^\circ, 270^\circ$ $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

14 $-2(\tan\beta - 4) = 3(4 - \tan\beta)$

$-2\tan\beta + 8 = 12 - 3\tan\beta$

$\tan\beta = 4$

$\tan^{-1}(4) = 75.963...^\circ$

QI $\beta = 75.963...^\circ$

QIII $\beta = 180^\circ + 75.963...^\circ = 255.963...^\circ$

$\{76.0^\circ, 256.0^\circ\}$

16 $\frac{1}{2}\csc\beta + 1 = \frac{1}{4}(\csc\beta + 8)$

$\frac{1}{2}\csc\beta + 1 = \frac{1}{4}\csc\beta + 2$

$4\left(\frac{1}{4}\csc\beta = 1\right)$

$\csc\beta = 4$

$\sin\beta = \frac{1}{4}$

$\sin^{-1}\left(\frac{1}{4}\right) = 14.477...$

QI $\beta = 14.477...^\circ$

QII $\beta = 180 - 14.477...^\circ = 165.522...$

$\{14.5^\circ, 165.5^\circ\}$