

Name: _____
 PCH: More General Solutions to Trig Equations

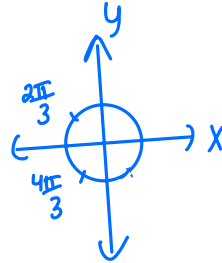
Date: _____
 Ms. Loughran

Do Now:

Find all solutions of the equation.

1. $2 \cos 2x + 1 = 0$

$$\cos 2x = -\frac{1}{2}$$



$$2x = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

$$\frac{4\pi}{3} + 2\pi k$$

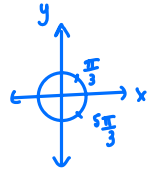
$$x = \frac{\pi}{3} + \pi k, k \in \mathbb{Z}$$

$$\frac{2\pi}{3} + \pi k$$

2. $\sec 4x - 2 = 0$

$$\sec 4x = 2$$

$$\cos 4x = \frac{1}{2}$$



$$4x = \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

$$\frac{5\pi}{3} + 2\pi k$$

$$x = \frac{\pi}{12} + \frac{\pi}{2} k, k \in \mathbb{Z}$$

$$\frac{5\pi}{12} + \frac{\pi}{2} k$$

Classwork:

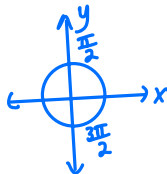
Find all solutions of each equation.

1. $\csc 3x = \sin 3x$

$$\frac{1}{\sin 3x} = \sin 3x$$

$$\sin^2 3x = 1$$

$$\sin 3x = \pm 1$$

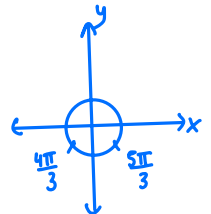


$$3x = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$$

$$x = \frac{\pi}{6} + \frac{\pi}{3} k, k \in \mathbb{Z}$$

2. $2 \sin \frac{x}{3} + \sqrt{3} = 0$

$$\sin \frac{x}{3} = -\frac{\sqrt{3}}{2}$$



$$\frac{x}{3} = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

$$\frac{5\pi}{3} + 2\pi k$$

$$x = 4\pi + 6\pi k, k \in \mathbb{Z}$$

$$5\pi + 6\pi k$$

For this section, find:

- (a) all solutions of the equation.
 (b) all solutions of the equation in the interval $(0, 2\pi]$.

3. $3\sec^2 x + 4\cos^2 x = 7$

$$\frac{3}{\cos^2 x} + 4\cos^2 x = 7$$

$$\left(0, \frac{12\pi}{6}\right]$$

$$3 + 4\cos^4 x = 7\cos^2 x$$

$$(b) \left\{ \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$4\cos^4 x - 7\cos^2 x + 3 = 0$$

$$4\cos^4 x - 4\cos^2 x - 3\cos^2 x + 3 = 0$$

$$4\cos^2 x (\cos^2 x - 1) - 3(\cos^2 x - 1) = 0$$

$(4\cos^2 x - 3)(\cos^2 x - 1) = 0$
 $\cos^2 x = \frac{3}{4} \quad \cos x = \pm \frac{\sqrt{3}}{2}$
 $\cos x = \pm 1$
 $x = \frac{\pi}{6} + \pi k, \frac{5\pi}{6} + \pi k, \pi k, k \in \mathbb{Z}$

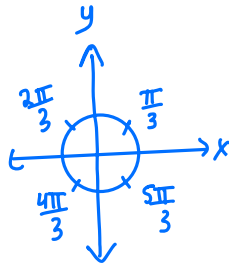
4. $\tan x - 3\cot x = 0$

$$\tan x - \frac{3}{\tan x} = 0$$

$$\tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$



$$(b) \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

$$x = \frac{\pi}{3} + \pi k, \frac{2\pi}{3} + \pi k, k \in \mathbb{Z}$$

$$5. \sec x \tan x - \cos x \cot x = \sin x$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \cos x \cdot \frac{\cos x}{\sin x} = \sin x$$

$$\cos^2 x \sin x \left(\frac{\sin x}{\cos^2 x} - \frac{\cos^2 x}{\sin x} = \sin x \right)$$

$$\sin^2 x - \cos^4 x = \sin^2 x \cos^2 x$$

$$1 - \cos^2 x - \cos^4 x = (1 - \cos^2 x) \cos^2 x$$

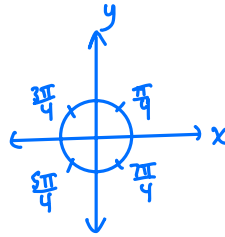
$$1 - \cos^2 x - \cancel{\cos^4 x} = \cos^2 x - \cancel{\cos^4 x}$$

$$1 - 2\cos^2 x = 0$$

$$1 = 2\cos^2 x$$

$$\cos^2 x = \frac{1}{2}$$

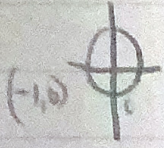
$$\cos x = \pm \frac{1}{\sqrt{2}}$$



$$x = \frac{\pi}{4} + \frac{\pi}{2}k, k \in \mathbb{Z}$$

$$(b) \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

Homework 04-15



$$\textcircled{1} \quad \begin{aligned} \cos x + 1 &= 0 \\ \cos x &= -1 \end{aligned}$$

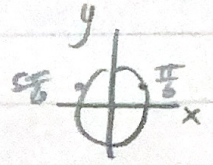
$$x = \pi + 2\pi k, \quad k \in \mathbb{Z}$$

$$\textcircled{2} \quad 2\sin x - 1 = 0$$

$$\begin{aligned} 2\sin x &= 1 \\ \sin x &= \frac{1}{2} \end{aligned}$$

$$x = \frac{\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}$$

$$x = \frac{5\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}$$



$$\textcircled{5} \quad \sqrt{3} \tan x + 1 = 0$$

$$\begin{aligned} \sqrt{3} \tan x &= -1 \\ \tan x &= \frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3} \end{aligned}$$

ref $\frac{\pi}{6}$



$$x = \frac{5\pi}{6} + \pi k, \quad k \in \mathbb{Z}$$

$$\textcircled{8} \quad 2\cos^2 x - 1 = 0$$

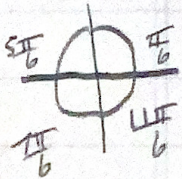
$$\begin{aligned} \cos^2 x &= \frac{1}{2} \\ \cos x &= \pm \frac{\sqrt{2}}{2} \end{aligned}$$

$$x = \frac{\pi}{4} + \frac{\pi}{2} k, \quad k \in \mathbb{Z}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\textcircled{10} \quad \csc^2 x - 4 = 0$$

$$\begin{aligned} \csc^2 x &= 4 \\ \csc x &= \pm 2 \\ \sin x &= \pm \frac{1}{2} \end{aligned}$$



$$x = \frac{\pi}{6} + \pi k, \quad k \in \mathbb{Z}$$

$$x = \frac{5\pi}{6} + \pi k, \quad k \in \mathbb{Z}$$

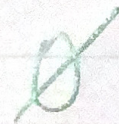
$$\textcircled{23} \quad \sin^2 x = 4 - 2\cos^2 x$$

$$\sin^2 x = 4 - 2(1 - \sin^2 x)$$

$$\sin^2 x = 4 - 2 + 2\sin^2 x$$

$$-\sin^2 x = 2$$

$$\sin^2 x = -2$$



$$4) \quad 2\cos^2 x + \sin x = 1$$

$$2(1 - \sin^2 x) + \sin x = 1$$

$$2 - 2\sin^2 x + \sin x = 1$$

$$0 = 2\sin^2 x - \sin x - 1$$

$$0 = (2\sin x + 1)(\sin x - 1)$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{\pi}{6} + 2\pi k$$

$$x = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$$

$$x = \frac{\pi}{2} + 2\pi k$$

$$1) \quad \cos \frac{x}{2} - 1 = 0$$

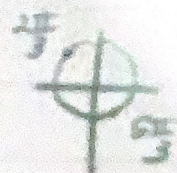
$$\cos \frac{x}{2} = 1$$

$$\frac{x}{2} = 0 + 2\pi k, k \in \mathbb{Z}$$

$$2\pi l, l \in \mathbb{Z}$$

$$x = 4\pi k, k \in \mathbb{Z}$$

$$33) \quad \tan \frac{x}{4} + \sqrt{3} = 0$$



$$\tan \frac{x}{4} = -\sqrt{3}$$

$$\frac{x}{4} = \frac{2\pi}{3} + \pi k, k \in \mathbb{Z}$$

$$x = \frac{8\pi}{3} + 4\pi k, k \in \mathbb{Z}$$

From your textbook:

7.5 Exercises

1–40 ■ Find all solutions of the equation.

- | | |
|------------------------------|------------------------------|
| 1. $\cos x + 1 = 0$ | 2. $\sin x + 1 = 0$ |
| 3. $2 \sin x - 1 = 0$ | 4. $\sqrt{2} \cos x - 1 = 0$ |
| 5. $\sqrt{3} \tan x + 1 = 0$ | 6. $\cot x + 1 = 0$ |
| 7. $4 \cos^2 x - 1 = 0$ | 8. $2 \cos^2 x - 1 = 0$ |
| 9. $\sec^2 x - 2 = 0$ | 10. $\csc^2 x - 4 = 0$ |

- | | |
|---|---|
| 21. $\sin^2 x = 2 \sin x + 3$ | 22. $3 \tan^3 x = \tan x$ |
| 23. $\sin^2 x = 4 - 2 \cos^2 x$ | 24. $2 \cos^2 x + \sin x = 1$ |
| 25. $2 \sin 3x + 1 = 0$ | 26. $2 \cos 2x + 1 = 0$ |
| 27. $\sec 4x - 2 = 0$ | 28. $\sqrt{3} \tan 3x + 1 = 0$ |
| 29. $\sqrt{3} \sin 2x = \cos 2x$ | 30. $\cos 3x = \sin 3x$ |
| 31. $\cos \frac{x}{2} - 1 = 0$ | 32. $2 \sin \frac{x}{3} + \sqrt{3} = 0$ |
| 33. $\tan \frac{x}{4} + \sqrt{3} = 0$ | 34. $\sec \frac{x}{2} = \cos \frac{x}{2}$ |
| 35. $\tan^5 x - 9 \tan x = 0$ | |
| 36. $3 \tan^3 x - 3 \tan^2 x - \tan x + 1 = 0$ | |
| 37. $4 \sin x \cos x + 2 \sin x - 2 \cos x - 1 = 0$ | |
| 38. $\sin 2x = 2 \tan 2x$ | 39. $\cos^2 2x - \sin^2 2x = 0$ |
| 40. $\sec x - \tan x = \cos x$ | |

41–48 ■ Find all solutions of the equation in the interval $[0, 2\pi)$.

- | | |
|---|-----------------------------------|
| 41. $2 \cos 3x = 1$ | 42. $3 \csc^2 x = 4$ |
| 43. $2 \sin x \tan x - \tan x = 1 - 2 \sin x$ | |
| 44. $\sec x \tan x - \cos x \cot x = \sin x$ | |
| 45. $\tan x - 3 \cot x = 0$ | 46. $2 \sin^2 x - \cos x = 1$ |
| 47. $\tan 3x + 1 = \sec 3x$ | 48. $3 \sec^2 x + 4 \cos^2 x = 7$ |

49–56 ■ (a) Find all solutions of the equation. (b) Use a calculator to solve the equation in the interval $[0, 2\pi)$, correct to five decimal places.

- | | |
|---------------------------------------|------------------------------|
| 49. $\cos x = 0.4$ | 50. $2 \tan x = 13$ |
| 51. $\sec x - 5 = 0$ | 52. $3 \sin x = 7 \cos x$ |
| 53. $5 \sin^2 x - 1 = 0$ | 54. $2 \sin 2x - \cos x = 0$ |
| 55. $3 \sin^2 x - 7 \sin x + 2 = 0$ | |
| 56. $\tan^4 x - 13 \tan^2 x + 36 = 0$ | |

57–60 ■ Graph f and g on the same axes, and find their points of intersection.

57. $f(x) = 3 \cos x + 1$, $g(x) = \cos x - 1$

- | | |
|---|--|
| 11. $3 \csc^2 x - 4 = 0$ | 12. $1 - \tan^2 x = 0$ |
| 13. $\cos x (2 \sin x + 1) = 0$ | 14. $\sec x (2 \cos x - \sqrt{2}) = 0$ |
| 15. $(\tan x + \sqrt{3})(\cos x + 2) = 0$ | |
| 16. $(2 \cos x + \sqrt{3})(2 \sin x - 1) = 0$ | |
| 17. $\cos x \sin x - 2 \cos x = 0$ | 18. $\tan x \sin x + \sin x = 0$ |
| 19. $4 \cos^2 x - 4 \cos x + 1 = 0$ | 20. $2 \sin^2 x - \sin x - 1 = 0$ |

63. $\sin 2x \cos x + \cos 2x \sin x = \sqrt{3}/2$

64. $\sin 3x \cos x - \cos 3x \sin x = 0$

65–68 ■ Use a double- or half-angle formula to solve the equation in the interval $[0, 2\pi)$.

- | | |
|----------------------------|-------------------------------------|
| 65. $\sin 2x + \cos x = 0$ | 66. $\tan \frac{x}{2} - \sin x = 0$ |
| 67. $\cos 2x + \cos x = 2$ | 68. $\tan x + \cot x = 4 \sin 2x$ |

69–72 ■ Solve the equation by first using a sum-to-product formula.

- | | |
|----------------------------------|-----------------------------------|
| 69. $\sin x + \sin 3x = 0$ | 70. $\cos 5x - \cos 7x = 0$ |
| 71. $\cos 4x + \cos 2x = \cos x$ | 72. $\sin 5x - \sin 3x = \cos 4x$ |

73–78 ■ Use a graphing device to find the solutions of the equation, correct to two decimal places.

- | | |
|----------------------|----------------------------|
| 73. $\sin 2x = x$ | 74. $\cos x = \frac{x}{3}$ |
| 75. $2^{\sin x} = x$ | 76. $\sin x = x^3$ |

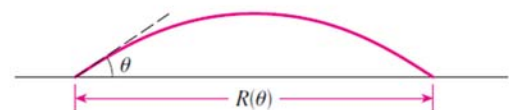
77. $\frac{\cos x}{1 + x^2} = x^2$	78. $\cos x = \frac{1}{2}(e^x + e^{-x})$
------------------------------------	--

Applications

79. **Range of a Projectile** If a projectile is fired with velocity v_0 at an angle θ , then its *range*, the horizontal distance it travels (in feet), is modeled by the function

$$R(\theta) = \frac{v_0^2 \sin 2\theta}{32}$$

(See page 818.) If $v_0 = 2200$ ft/s, what angle (in degrees) should be chosen for the projectile to hit a target on the ground 5000 ft away?



58. $f(x) = \sin 2x$, $g(x) = 2 \sin 2x + 1$

59. $f(x) = \tan x$, $g(x) = \sqrt{3}$

60. $f(x) = \sin x - 1$, $g(x) = \cos x$

61–64 ■ Use an addition or subtraction formula to simplify the equation. Then find all solutions in the interval $[0, 2\pi)$.

61. $\cos x \cos 3x - \sin x \sin 3x = 0$

62. $\cos x \cos 2x + \sin x \sin 2x = \frac{1}{2}$

80. **Damped Vibrations** The displacement of a spring vibrating in damped harmonic motion is given by

$$y = 4e^{-3t} \sin 2\pi t$$

Find the times when the spring is at its equilibrium position ($y = 0$).

81. **Refraction of Light** It has been observed since ancient times that light refracts or “bends” as it travels from one medium to another (from air to water, for example). If v_1 is