Name: ____ Date: __________

PCH: Even More General Solutions to Trig Equations Ms. Loughran

Do Now:

Find : (a) all solutions of the equation.

(b) all solutions of the equation n the interval $[-\pi, \pi]$.

$\left[-\frac{4\pi}{4}, \frac{4\pi}{4}\right]$ ↑

↓ $\left[-\frac{6\pi}{6}, \frac{6\pi}{6}\right]$

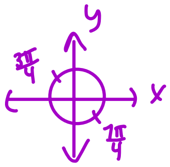
1. $2\sin x\tan x - \tan x = 1 - 2\sin x$

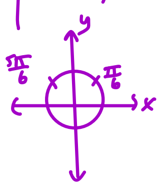
$$2\sin x\tan x - \tan x + 2\sin x - 1 = 0$$

$$\tan x(2\sin x - 1) + 1(2\sin x - 1) = 0$$

$$(\tan x + 1)(2\sin x - 1) = 0$$

$\tan x = -1$ | $\sin x = \frac{1}{2}$





(a) $x = \frac{3\pi}{4} + \pi k$, $\frac{\pi}{6} + 2\pi k$, $\frac{5\pi}{6} + 2\pi k$, $k \in \mathbb{Z}$

(b) $K=-1$: $\frac{3\pi}{4} - \pi = -\frac{\pi}{4}$

$K=1$: $\cancel{\frac{3\pi}{4} + \pi}$

$K=-1$: $\cancel{\frac{\pi}{6} - 2\pi}$

$K=-1$: $\frac{5\pi}{6} - 2\pi$

$\left\{\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$

Classwork:

Find: (a) all solutions of each equation.

(b) all solutions of the equation in the indicated interval.

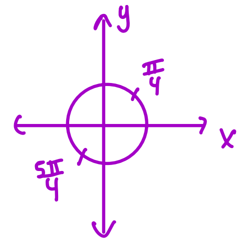
1. $\sqrt{3}\csc x - 2 = 0$ $[-3\pi, \pi]$

2. $\cos^2 x - \cos x = 0$ $[0, 4\pi]$

3. $\cos 2x = \dfrac{1}{2}$ $\quad \left[-\dfrac{\pi}{2}, \pi\right]$

4. $\sin x = \cos x$ $\quad \left[0, 3\pi\right]$

$\left[0, \frac{12\pi}{4}\right]$



$x = \frac{\pi}{4} + \pi k,\ k \in \mathbb{Z}$

$\left\{ \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \right\}$

5. $\dfrac{\sec x}{\cos x} - \dfrac{1}{2}\sec x = 0$ $\quad \left[-2\pi, 2\pi\right]$

$\sec x \left(\frac{1}{\cos x} - \frac{1}{2} \right) = 0$

$\sec x = 0$

$\emptyset$

b/c cos x is never undefined

$\frac{1}{\cos x} - \frac{1}{2} = 0$

$\cos x - 2 = 0$

$\cos x = 2$

$\emptyset$

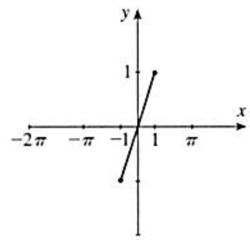
$\emptyset$

6. $\cos^2 x + \dfrac{1}{2}\sin x - \dfrac{1}{2} = 0$ $\quad \left[0, 2\pi\right]$

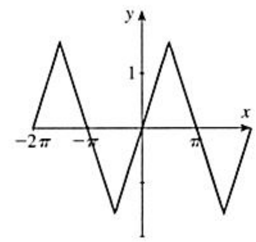
57. (a) $\theta = \sin^{-1}\left(\dfrac{1}{(2\cdot 3+1)\tan 10^\circ}\right) = \sin^{-1}\left(\dfrac{1}{7\tan 10^\circ}\right) \approx \sin^{-1} 0.8102 \approx 54.1^\circ$

(b) For $n = 2$, $\theta = \sin^{-1}\left(\dfrac{1}{5\tan 15^\circ}\right) \approx 48.3^\circ$. For $n = 3$, $\theta = \sin^{-1}\left(\dfrac{1}{7\tan 15^\circ}\right) \approx 32.2^\circ$. For $n = 4$, $\theta = \sin^{-1}\left(\dfrac{1}{9\tan 15^\circ}\right) \approx 24.5^\circ$. $n = 0$ and $n = 1$ are outside of the domain for $\beta = 15^\circ$, because $\dfrac{1}{\tan 15^\circ} \approx 3.732$ and $\dfrac{1}{3\tan 15^\circ} \approx 1.244$, neither of which is in the domain of $\sin^{-1}$.

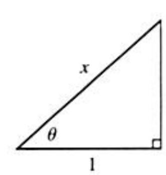
58. $f(x) = \sin(\sin^{-1} x) = x$ has domain $[-1, 1]$ and range $[-1, 1]$.



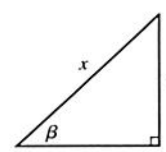
$g(x) = \sin^{-1}(\sin x)$ has domain $(-\infty, \infty)$ and range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Note that $g(x) = x$ only for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



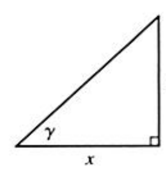
59. (a) Let $\theta = \sec^{-1} x$. Then $\sec\theta = x$, as shown in the figure. Then $\cos\theta = \dfrac{1}{x}$, so $\theta = \cos^{-1}\left(\dfrac{1}{x}\right)$. Thus $\sec^{-1} x = \cos^{-1}\left(\dfrac{1}{x}\right)$, $x \geq 1$.



(b) Let $\gamma = \csc^{-1} x$. Then $\csc\gamma = x$, as shown in the figure. Then $\sin\gamma = \dfrac{1}{x}$, so $\gamma = \sin^{-1}\left(\dfrac{1}{x}\right)$. Thus $\csc^{-1} x = \sin^{-1}\left(\dfrac{1}{x}\right)$, $x \geq 1$.



(c) Let $\beta = \cot^{-1} x$. Then $\cot\beta = x$, as shown in the figure. Then $\tan\beta = \dfrac{1}{x}$, so $\beta = \tan^{-1}\left(\dfrac{1}{x}\right)$. Thus $\cot^{-1} x = \tan^{-1}\left(\dfrac{1}{x}\right)$, $x \geq 1$.



Key to all of section 7.5

7.5 Trigonometric Equations

1. $\cos x + 1 = 0 \quad\Leftrightarrow\quad \cos x = -1$. In the interval $[0, 2\pi)$ the only solution is $x = \pi$.Thus the solutions are $x = (2k+1)\pi$ for any integer k.

2. $\sin x + 1 = 0 \quad\Leftrightarrow\quad \sin x = -1$. In the interval $[0, 2\pi)$ the only solution is $x = \frac{3\pi}{2}$. Therefore, the solutions are $x = \frac{3\pi}{2} + 2k\pi$ for any integer k.

3. $2\sin x - 1 = 0 \quad\Leftrightarrow\quad 2\sin x = 1 \quad\Leftrightarrow\quad \sin x = \frac{1}{2}$. In the interval $[0, 2\pi)$ the solutions are $x = \frac{\pi}{6}, \frac{5\pi}{6}$. Therefore, the solutions are $x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

4. $\sqrt{2}\cos x - 1 = 0 \quad\Leftrightarrow\quad \sqrt{2}\cos x = 1 \Leftrightarrow \cos x = \frac{1}{\sqrt{2}}$. The solutions in the interval $[0, 2\pi)$ are $x = \frac{\pi}{4}, \frac{7\pi}{4}$. Thus the solutions are $x = \frac{\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$ for any integer k.

5. $\sqrt{3}\tan x + 1 = 0 \quad \Leftrightarrow \quad \sqrt{3}\tan x = -1 \Leftrightarrow \tan x = -\frac{\sqrt{3}}{3}$. In the interval $[0, \pi)$ the only solution is $x = \frac{5\pi}{6}$. Therefore, the solutions are $x = \frac{5\pi}{6} + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

6. $\cot x + 1 = 0 \quad \Leftrightarrow \quad \cot x = -1$. The solution in the interval $(0, \pi)$ is $x = \frac{3\pi}{4}$. Thus, the solutions are $x = -\frac{\pi}{4} + k\pi$ for any integer k.

7. $4\cos^2 x - 1 = 0 \quad \Leftrightarrow \quad 4\cos^2 x = 1 \Leftrightarrow 4\cos^2 x = 1 \quad \Leftrightarrow \quad \cos^2 x = \frac{1}{4} \Leftrightarrow \cos x = \pm\frac{1}{2}$. In the interval $[0, 2\pi)$ the solutions are $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$. So the solutions are $x = \frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$, which can be expressed more simply as $x = \frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$ for any integer k.

8. $2\cos^2 x - 1 = 0 \quad \Leftrightarrow \quad \cos^2 x = \frac{1}{2} \Leftrightarrow \cos x = \pm\frac{1}{\sqrt{2}} \quad \Leftrightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi)$. Thus, the solutions are $x = \frac{\pi}{4} + k\frac{\pi}{2}$ for any integer k.

9. $\sec^2 x - 2 = 0 \quad \Leftrightarrow \quad \sec^2 x = 2 \Leftrightarrow \sec x = \pm\sqrt{2}$. In the interval $[0, 2\pi)$ the solutions are $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$. Thus, the solutions are $x = \frac{\pi}{4} + k\frac{\pi}{2}$ for any integer k.

10. $\csc^2 x - 4 = 0 \quad \Leftrightarrow \quad \csc^2 x = 4 \Leftrightarrow \csc x = \pm 2$. In the interval $[0, 2\pi)$ the solutions are $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$. So the solutions are $x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$ for any integer k.

11. $3\csc^2 x - 4 = 0 \quad \Leftrightarrow \quad \csc^2 x = \frac{4}{3} \Leftrightarrow \csc x = \pm\frac{2}{\sqrt{3}} \quad \Leftrightarrow \quad \sin x = \pm\frac{\sqrt{3}}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ for x in $[0, 2\pi)$. Thus, the solutions are $x = \frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$ for any integer k.

12. $1 - \tan^2 x = 0 \quad \Leftrightarrow \quad \tan^2 x = 1 \Leftrightarrow \tan x = \pm 1 \quad \Leftrightarrow \quad x = -\frac{\pi}{4}, \frac{\pi}{4}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Thus, the solutions are $\frac{\pi}{4} + k\frac{\pi}{2}$ for any integer k.

13. $\cos x\,(2\sin x + 1) = 0 \quad \Leftrightarrow \quad \cos x = 0$ or $2\sin x + 1 = 0 \quad \Leftrightarrow \quad \sin x = -\frac{1}{2}$. On $[0, 2\pi)$, $\cos x = 0 \quad \Leftrightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$ and $\sin x = -\frac{1}{2} \quad \Leftrightarrow \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$. Thus the solutions are $x = \frac{\pi}{2} + k\pi$, $x = \frac{7\pi}{6} + 2k\pi$, $x = \frac{11\pi}{6} + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

14. $\sec x\left(2\cos x - \sqrt{2}\right) = 0 \quad \Leftrightarrow \quad \sec x = 0$ or $2\cos x - \sqrt{2} = 0$. Since $|\sec x| \geq 1$, $\sec x = 0$ has no solution. Thus $2\cos x - \sqrt{2} = 0 \Leftrightarrow 2\cos x = \sqrt{2} \quad \Leftrightarrow \quad \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow x = \frac{\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi)$. Thus $x = \frac{\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$ for any integer k.

15. $\left(\tan x + \sqrt{3}\right)(\cos x + 2) = 0 \Leftrightarrow \tan x + \sqrt{3} = 0$ or $\cos x + 2 = 0$. Since $|\cos x| \leq 1$ for all x, there is no solution for $\cos x + 2 = 0$. Hence, $\tan x + \sqrt{3} = 0 \quad \Leftrightarrow \quad \tan x = -\sqrt{3} \Leftrightarrow x = -\frac{\pi}{3}$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Thus the solutions are $x = -\frac{\pi}{3} + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

16. $\left(2\cos x + \sqrt{3}\right)(2\sin x - 1) = 0 \Leftrightarrow 2\cos x + \sqrt{3} = 0$ or $2\sin x - 1 = 0 \quad \Leftrightarrow \quad \cos x = -\frac{\sqrt{3}}{2}$ or $\sin x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ in $[0, 2\pi)$. Thus, $x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + 2k\pi$ for any integer k.

17. $\cos x \sin x - 2\cos x = 0 \quad \Leftrightarrow \quad \cos x\,(\sin x - 2) = 0 \quad \Leftrightarrow \quad \cos x = 0$ or $\sin x - 2 = 0$. Since $|\sin x| \leq 1$ for all x, there is no solution for $\sin x - 2 = 0$. Hence, $\cos x = 0 \quad \Leftrightarrow \quad x = \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \quad \Leftrightarrow \quad x = \frac{\pi}{2} + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

18. $\tan x \sin x + \sin x = 0 \quad \Leftrightarrow \quad \sin x\,(\tan x + 1) = 0 \quad \Leftrightarrow \quad \sin x = 0$ or $\tan x + 1 = 0$. Now $\sin x = 0$ when $x = k\pi$ and $\tan x + 1 = 0 \quad \Leftrightarrow \quad \tan x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$. Thus, the solutions are $x = k\pi, \frac{3\pi}{4} + k\pi$ for any integer k.

19. $4\cos^2 x - 4\cos x + 1 = 0 \quad \Leftrightarrow \quad (2\cos x - 1)^2 = 0 \quad \Leftrightarrow \quad 2\cos x - 1 = 0 \quad \Leftrightarrow \quad \cos x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

20. $2\sin^2 x - \sin x - 1 = 0 \quad \Leftrightarrow \quad (2\sin x + 1)(\sin x - 1) = 0 \quad \Leftrightarrow \quad 2\sin x + 1 = 0$ or $\sin x - 1 = 0$. Since $2\sin x + 1 = 0 \quad \Leftrightarrow \quad 2\sin x = -\frac{1}{2} \Leftrightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$ in $[0, 2\pi)$ and $\sin x = 1 \quad \Leftrightarrow \quad x = \frac{\pi}{2}$ in $[0, 2\pi)$. Thus the solutions are $x = \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi$ for any integer k.

21. $\sin^2 x = 2\sin x + 3 \quad \Leftrightarrow \quad \sin^2 x - 2\sin x - 3 = 0 \Leftrightarrow (\sin x - 3)(\sin x + 1) = 0 \Leftrightarrow \sin x - 3 = 0$ or $\sin x + 1 = 0$. Since $|\sin x| \leq 1$ for all x, there is no solution for $\sin x - 3 = 0$. Hence $\sin x + 1 = 0 \quad \Leftrightarrow \quad \sin x = -1 \quad \Leftrightarrow \quad x = \frac{3\pi}{2} + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

22. $3\tan^3 x = \tan x \quad\Leftrightarrow\quad 3\tan^3 x - \tan x = 0 \Leftrightarrow \tan x\left(3\tan^2 x - 1\right) = 0 \quad\Leftrightarrow\quad \tan x = 0$ or $3\tan^2 x - 1 = 0$. Now $\tan x = 0 \Rightarrow x = k\pi$ and $3\tan^2 x - 1 = 0 \quad\Leftrightarrow\quad \tan^2 x = \frac{1}{3} \Rightarrow \tan x = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$. Thus the solutions are $x = k\pi, \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$ for any integer k.

23. $\sin^2 x = 4 - 2\cos^2 x \quad\Leftrightarrow\quad \sin^2 x + \cos^2 x + \cos^2 x = 4 \quad\Leftrightarrow\quad 1 + \cos^2 x = 4 \quad\Leftrightarrow\quad \cos^2 x = 3$ Since $|\cos x| \le 1$ for all x, it follows that $\cos^2 x \le 1$ and so there is no solution for $\cos^2 x = 3$.

24. $2\cos^2 x + \sin x = 1 \quad\Leftrightarrow\quad 2\left(1 - \sin^2 x\right) + \sin x - 1 = 0 \quad\Leftrightarrow\quad -2\sin^2 x + \sin x + 1 = 0 \Leftrightarrow 2\sin^2 x - \sin x - 1 = 0$ which we solved in Exercise 20. Thus $x = \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi$ for any integer k.

25. $2\sin 3x + 1 = 0 \quad\Leftrightarrow\quad 2\sin 3x = -1 \Leftrightarrow \sin 3x = -\frac{1}{2}$. In the interval $[0, 6\pi)$ the solution are $3x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6} \Leftrightarrow x = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$. So $x = \frac{7\pi}{18} + 2k\frac{\pi}{3}, \frac{11\pi}{18} + 2k\frac{\pi}{3}$ for any integer k.

26. $2\cos 2x + 1 = 0 \quad\Leftrightarrow\quad \cos 2x = -\frac{1}{2} \Leftrightarrow 2x = \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi \Leftrightarrow x = \frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$ for any integer k.

27. $\sec 4x - 2 = 0 \quad\Leftrightarrow\quad \sec 4x = 2 \quad\Leftrightarrow\quad 4x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi \quad\Leftrightarrow\quad x = \frac{\pi}{12} + \frac{1}{2}k\pi, \frac{5\pi}{12} + \frac{1}{2}k\pi$ for any integer k.

28. $\sqrt{3}\tan 3x + 1 = 0 \quad\Leftrightarrow\quad \tan 3x = -\frac{1}{\sqrt{3}} \quad\Leftrightarrow\quad 3x = \frac{5\pi}{6} + k\pi \quad\Leftrightarrow\quad x = \frac{5\pi}{18} + \frac{1}{3}k\pi$ for any integer k.

29. $\sqrt{3}\sin 2x = \cos 2x \quad\Leftrightarrow\quad \tan 2x = \frac{1}{\sqrt{3}}$ (if $\cos 2x \neq 0$) $\quad\Leftrightarrow\quad 2x = \frac{\pi}{6} + k\pi \quad\Leftrightarrow\quad x = \frac{\pi}{12} + \frac{1}{2}k\pi$ for any integer k.

30. $\csc 3x = \sin 3x \quad\Leftrightarrow\quad \dfrac{1}{\sin 3x} = \sin 3x \quad\Leftrightarrow\quad \sin^2 3x = 1 \quad\Leftrightarrow\quad \sin 3x = \pm 1 \Leftrightarrow 3x = \frac{\pi}{2} + k\pi \quad\Leftrightarrow\quad x = \frac{\pi}{6} + k\frac{\pi}{3}$ for any integer k. Notice that multiplying by $\sin 3x$ in the first step did not introduce extraneous roots.

31. $\cos\frac{x}{2} - 1 = 0 \quad\Leftrightarrow\quad \cos\frac{x}{2} = 1 \Leftrightarrow \frac{x}{2} = 2k\pi \quad\Leftrightarrow\quad x = 4k\pi$ for any integer k.

32. $2\sin\frac{x}{3} + \sqrt{3} = 0 \quad\Leftrightarrow\quad 2\sin\frac{x}{3} = -\sqrt{3} \quad\Leftrightarrow\quad \sin\frac{x}{3} = -\frac{\sqrt{3}}{2} \Leftrightarrow \frac{x}{3} = \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi \quad\Leftrightarrow\quad x = 4\pi + 6k\pi, 5\pi + 6k\pi$ for any integer k.

33. $\tan\frac{x}{4} + \sqrt{3} = 0 \quad\Leftrightarrow\quad \tan\frac{x}{4} = -\sqrt{3} \quad\Leftrightarrow\quad \frac{x}{4} = \frac{2\pi}{3} + k\pi \Leftrightarrow x = \frac{8\pi}{3} + 4k\pi$ for any integer k.

34. $\sec\frac{x}{2} = \cos\frac{x}{2} \quad\Leftrightarrow\quad \cos^2\frac{x}{2} = 1 \Rightarrow \cos\frac{x}{2} = \pm 1 \Leftrightarrow \frac{x}{2} = k\pi \quad\Leftrightarrow\quad x = 2k\pi$ for any integer k.

35. $\tan^5 x - 9\tan x = 0 \quad\Leftrightarrow\quad \tan x\left(\tan^4 x - 9\right) = 0 \quad\Leftrightarrow\quad \tan x = 0$ or $\tan^4 x = 9 \Leftrightarrow \tan x = 0$ or $\tan x = \pm\sqrt{3}$ $\quad\Leftrightarrow\quad x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ in $[0, 2\pi)$. Thus, $x = \frac{k\pi}{3}$ for any integer k.

36. $3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0 \quad\Leftrightarrow\quad (\tan x - 1)\left(3\tan^2 x - 1\right) = 0 \quad\Leftrightarrow\quad \tan x = 1$ or $3\tan^2 x = 1 \quad\Leftrightarrow\quad \tan x = 1$ or $\tan x = \pm\frac{1}{\sqrt{3}} \quad\Leftrightarrow\quad x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}$, for $x \in [0, \pi)$. Thus $x = \frac{\pi}{6} + k\pi, \frac{\pi}{4} + k\pi, \frac{5\pi}{6} + k\pi$ for any integer k.

37. $4\sin x\cos x + 2\sin x - 2\cos x - 1 = 0 \quad\Leftrightarrow\quad (2\sin x - 1)(2\cos x + 1) = 0 \quad\Leftrightarrow\quad 2\sin x - 1 = 0$ or $2\cos x + 1 = 0$ $\quad\Leftrightarrow\quad \sin x = \frac{1}{2}$ or $\cos x = -\frac{1}{2} \quad\Leftrightarrow\quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{2\pi}{3}, \frac{4\pi}{3}$ in $[0, 2\pi)$. Thus $x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$ for any integer k.

38. $\sin 2x = 2\tan 2x \quad\Leftrightarrow\quad \sin 2x = \dfrac{2\sin 2x}{\cos 2x} \quad\Leftrightarrow\quad \sin 2x\cos 2x - 2\sin 2x = 0 \quad\Leftrightarrow\quad \sin 2x(\cos 2x - 2) = 0$ $\quad\Leftrightarrow\quad \cos 2x = 2$ (which is impossible since $|\cos u| \le 1$) or $\sin 2x = 0 \Leftrightarrow 2x = k\pi \quad\Leftrightarrow\quad x = \frac{1}{2}k\pi$ for any integer k.

39. $\cos^2 2x - \sin^2 2x = 0 \quad\Leftrightarrow\quad (\cos 2x - \sin 2x)(\cos 2x + \sin 2x) = 0 \quad\Leftrightarrow\quad \cos 2x = \pm\sin 2x \quad\Leftrightarrow\quad \tan 2x = \pm 1$ $\quad\Leftrightarrow\quad 2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$ in $[0, 4\pi) \quad\Leftrightarrow\quad x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ in $[0, 2\pi)$. So the solution can be expressed as the odd multiples of $\frac{\pi}{8}$ which are $x = \frac{\pi}{8} + \frac{k\pi}{4}$ for any integer k.

40. $\sec x - \tan x = \cos x \quad\Leftrightarrow\quad \cos x(\sec x - \tan x) = \cos x(\cos x) \quad\Leftrightarrow\quad 1 - \sin x = \cos^2 x \quad\Leftrightarrow\quad 1 - \sin x = 1 - \sin^2 x \quad\Leftrightarrow\quad \sin x = \sin^2 x \quad\Leftrightarrow\quad \sin^2 x - \sin x = 0 \Leftrightarrow \sin x(\sin x - 1) = 0 \quad\Leftrightarrow\quad \sin x = 0$ or $\sin x = 1 \quad\Leftrightarrow\quad x = 0, \pi$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ in $[0, 2\pi)$. However, since the equation is undefined when $x = \frac{\pi}{2}, \frac{3\pi}{2}$, the solutions are $x = k\pi$ for any integer k.

41. $2\cos 3x = 1 \quad\Leftrightarrow\quad \cos 3x = \frac{1}{2} \Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$ on $[0, 6\pi) \quad\Leftrightarrow\quad x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$ on $[0, 2\pi)$.

42. $3\csc^2 x = 4 \quad\Leftrightarrow\quad \csc^2 x = \frac{4}{3} \Rightarrow \csc x = \pm\frac{2}{\sqrt{3}} \quad\Leftrightarrow\quad \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$ in $[0, 2\pi)$.

general solution:

$$3x = \begin{cases} \frac{\pi}{3} + 2\pi k \\ \frac{5\pi}{3} + 2\pi k \end{cases}, k \in \mathbb{Z} \qquad x = \begin{cases} \frac{\pi}{9} + \frac{2\pi}{3}k \\ \frac{5\pi}{9} + \frac{2\pi}{3}k \end{cases}, k \in \mathbb{Z}$$

43. $2\sin x\tan x - \tan x = 1 - 2\sin x \Leftrightarrow 2\sin x\tan x - \tan x + 2\sin x - 1 = 0 \Leftrightarrow (2\sin x - 1)(\tan x + 1) = 0$ $\Leftrightarrow 2\sin x - 1 = 0$ or $\tan x + 1 = 0 \Leftrightarrow \sin x = \frac{1}{2}$ or $\tan x = -1 \Leftrightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{3\pi}{4}, \frac{7\pi}{4}$. Thus, the solutions in $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}$.

44. $\sec x\tan x - \cos x\cot x = \sin x \Leftrightarrow \frac{1}{\cos x}\frac{\sin x}{\cos x} - \cos x \cdot \frac{\cos x}{\sin x} = \sin x \Leftrightarrow \frac{\sin x}{\cos^2 x} - \frac{\cos^2 x}{\sin x} = \sin x$. Multiplying both sides by the common denominator $\cos^2 x\sin x$ gives $\sin^2 x - \cos^4 x = \sin^2 x\cos^2 x \Leftrightarrow$ $\sin^2 x - \cos^4 x = (1 - \cos^2 x)\cos^2 x \Leftrightarrow \sin^2 x - \cos^4 x = \cos^2 x - \cos^4 x \Leftrightarrow \sin^2 x = \cos^2 x \Leftrightarrow$ $\sin^2 x = 1 - \sin^2 x \Leftrightarrow 2\sin^2 x = 1 \Leftrightarrow \sin x = \pm\frac{1}{\sqrt{2}} \Leftrightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$. Since we multiplied the above equation by $\cos^2 x\sin x$ (which could be zero) we must check to see if we have introduced extraneous solutions. However, each of the values of x satisfies the original equation and so the solutions on $[0, 2\pi)$ are $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

45. $\tan x - 3\cot x = 0 \Leftrightarrow \frac{\sin x}{\cos x} - \frac{3\cos x}{\sin x} = 0 \Leftrightarrow \frac{\sin^2 x - 3\cos^2 x}{\cos x\sin x} = 0 \Leftrightarrow \frac{\sin^2 x + \cos^2 x - 4\cos^2 x}{\cos x\sin x} = 0$ $\Leftrightarrow \frac{1 - 4\cos^2 x}{\cos x\sin x} = 0 \Leftrightarrow 1 - 4\cos^2 x = 0 \Leftrightarrow 4\cos^2 x = 1 \Leftrightarrow \cos x = \pm\frac{1}{2} \Leftrightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ in $[0, 2\pi)$.

46. $2\sin^2 x - \cos x = 1 \Leftrightarrow 2(1 - \cos^2 x) - \cos x - 1 = 0 \Leftrightarrow -2\cos^2 x - \cos x + 1 = 0 \Leftrightarrow$ $(2\cos x - 1)(\cos x + 1) = 0 \Leftrightarrow 2\cos x - 1 = 0$ or $\cos x + 1 = 0 \Leftrightarrow \cos x = \frac{1}{2}$ or $\cos x = -1 \Leftrightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$, π in $[0, 2\pi)$.

general solution
$\frac{\pi}{3} + 2\pi k$
$X = \frac{5\pi}{3} + 2\pi k, \; k \in \mathbb{Z}$
$\pi + \pi k$

47. $\tan 3x + 1 = \sec 3x \Rightarrow (\tan 3x + 1)^2 = \sec^2 3x \Leftrightarrow \tan^2 3x + 2\tan 3x + 1 = \sec^2 3x \Leftrightarrow$ $\sec^2 3x + 2\tan 3x = \sec^2 3x \Leftrightarrow 2\tan 3x = 0 \Leftrightarrow 3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$ in $[0, 6\pi) \Leftrightarrow x = 0, \frac{\pi}{3}, \frac{2\pi}{3}$, $\pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ in $[0, 2\pi)$ .Since squaring both sides is an operation that can introduce extraneous solutions, we must check each of the possible solution in the original equation. We see that $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ are not solutions. Hence the only solutions are $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

48. $3\sec^2 x + 8\cos^2 x = 7 \Leftrightarrow \frac{3}{\cos^2 x} + 4\cos^2 x - 7 = 0$ (for $\cos x \neq 0$) $\Leftrightarrow 3 + 4\cos^4 x - 7\cos^2 x = 0$ $\Leftrightarrow 4\cos^4 x - 7\cos^2 x + 3 = 0 \Leftrightarrow (4\cos^2 x - 3)(\cos^2 x - 1) = 0 \Leftrightarrow 4\cos^2 x - 3 = 0$ or $\cos^2 x - 1 = 0 \Leftrightarrow$ $\cos x = \pm\frac{\sqrt{3}}{2}$ or $\cos x = \pm 1 \Leftrightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ or $x = 0, \pi$ in $[0, 2\pi)$. So the solutions are $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$.

49. (a) Since the period of cosine is 2π the solutions are $x \approx 1.15928 + 2k\pi$ and $x \approx 5.12391 + 2k\pi$ for any integer k.

(b) $\cos x = 0.4 \Rightarrow x = \cos^{-1} 0.4 \approx 1.15928$. The other solution is $2\pi - \cos^{-1} 0.4 \approx 5.12391$.

50. (a) Since the period is π, the solutions are of the form $x \approx 1.41815 + k\pi$ for any integer k.

(b) $2\tan x = 13 \Leftrightarrow \tan x = \frac{13}{2} \Rightarrow x \approx 1.41815$. Since the period for tangent is π, the other solution in $[0, 2\pi]$ is $x \approx 1.41815 + \pi \approx 4.55974$.

51. (a) Since the period of secant is 2π the solutions are $x \approx 1.36943 \pm 2k\pi$ and $\approx 4.91375 \pm 2k\pi$ for any integer k.

(b) $\sec x = 5 \Leftrightarrow \cos x = \frac{1}{5} \Rightarrow x = \cos^{-1}\frac{1}{5} \approx 1.36944$. The other solution is $2\pi - \cos^{-1}\frac{1}{5} \approx 4.91375$.

52. (a) Since the period is π, the solutions are of the form $x \approx 1.16590 + k\pi$ for any integer k.

(b) Since $\cos x = 0$ is not a solution, we divide both sides by $\cos x$. $3\sin x = 7\cos x \Leftrightarrow \tan x = \frac{7}{3} \Rightarrow x \approx 1.16590$. Since the period for tangent is π, the other solution in $[0, 2\pi]$ is $x \approx 1.16590 + \pi \approx 4.30750$.

53. (a) Since $\sin(\pi + x) = -\sin x$, we can express the general solutions as $x \approx 0.46365 + k\pi$ and $x \approx 2.67795 + k\pi$ for any integer k.

(b) $5\sin^2 x - 1 = 0 \Rightarrow \sin^2 x = \frac{1}{5} \Rightarrow \sin x = \pm\frac{1}{\sqrt{5}}$. Now $\sin x = \frac{1}{\sqrt{5}} \Rightarrow x = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 0.46365$,and $x \approx \pi - 0.46365 \approx 2.67795$. Also, $\sin x = -\frac{1}{\sqrt{5}} \Rightarrow x = \sin^{-1}\left(-\frac{1}{\sqrt{5}}\right) \approx -0.46365$, so in the interval $[0, 2\pi)$, $x \approx \pi - (-0.46365) \approx 3.60524$ and $x \approx 2\pi - 0.46365 \approx 5.81954$.

47 $(\tan 3x + 1)^2 = (\sec 3x)^2$

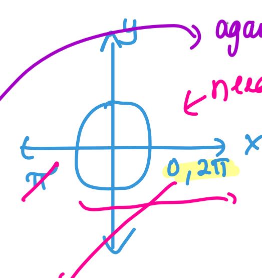
$\tan^2 x + 1 = \sec^2 x$

$\tan^2(3x) + 2\tan 3x + 1 = \sec^2 3x$

$\cancel{\tan^2(3x)} + 2\tan 3x + 1 = \cancel{\tan^2(3x)} + 1$

$2\tan 3x = 0$

$\tan 3x = 0$



again still need to check for extraneous root

in $[0, 2\pi)$

$\tan 3\pi + 1 = \sec 3\pi$

$0 + 1 \neq -1 \quad \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

$\frac{\sin 3x}{\cos 3x} + 1 = \frac{1}{\cos 3x}$

$(\sin 3x + \cos 3x)^2 \stackrel{?}{=} 1^2$

$\sin^2 3x + 2\sin 3x \cos 3x + \cos^2 3x = 1$

or

$1 + 2\sin 3x \cos 3x = 1$

$2\sin 3x \cos 3x = 0$

$\sin 3x = 0$ | $\cos 3x = 0$

$\cancel{\pi}$ $\oplus$ 0 | $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\tan 0 + 1 = \sec 0$

$0 + 1 = 1$ ✓

$3x = 0 + 2\pi k, k \in \mathbb{Z}$

$x = \frac{2\pi}{3}k, k \in \mathbb{Z}$

54. (a) Since the period is 2π, the solutions are of the form $x \approx 1.57080 + k\pi, x \approx 0.25268 + 2k\pi, x \approx 2.88891 + 2k\pi$ for any integer k.

tan is undefined at both these angles

(b) $2\sin 2x - \cos x = 0 \Leftrightarrow 2(2\sin x \cos x) - \cos x = 0 \Leftrightarrow 4\sin x \cos x - \cos x = 0 \Leftrightarrow \cos x (4\sin x - 1) = 0 \Leftrightarrow \cos x = 0$ or $4\sin x - 1 = 0$. Now $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ and $4\sin x - 1 = 0 \Leftrightarrow \sin x = \frac{1}{4} \Rightarrow x \approx 0.25268$ or $x \approx \pi - 0.25268 \approx 2.88891$. Thus the solutions in $[0, 2\pi)$ are $x \approx 0.25268$, 1.57080, 2.88891, 4.71239.

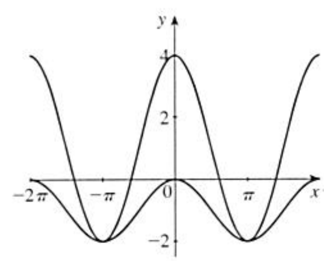
55. (a) Since the period for sine is 2π, the general solution is of the form $x \approx 0.33984 + 2k\pi$ and $x \approx 2.80176 + 2k\pi$ for any integer k.

(b) $3\sin^2 x - 7\sin x + 2 = 0 \Rightarrow (3\sin x - 1)(\sin x - 2) = 0 \Rightarrow 3\sin x - 1 = 0$ or $\sin x - 2 = 0$. Since $|\sin x| \le 1$, $\sin x - 2 = 0$ has no solution. Thus $3\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{3} \Rightarrow x \approx 0.33984$ and $x \approx \pi - 0.33984 \approx 2.80176$.

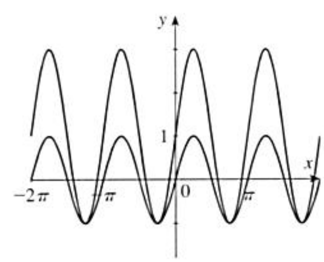
56. (a) Since the period is π, the solutions are of the form $x \approx 1.10715 + k\pi, x \approx 2.03444 + k\pi, x \approx 1.24905 + k\pi, x \approx 1.89255 + k\pi$ for any integer k.

(b) $\tan^4 x - 13\tan^2 x + 36 = 0 \Leftrightarrow (\tan^2 x - 4)(\tan^2 x - 9) = 0 \Leftrightarrow \tan x = \pm 2$ or $\tan x = \pm 3 \Rightarrow x \approx 1.10715, -1.10715, 1.24905, -1.24905$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Since the period for tangent is π, the solutions in $[0, 2\pi)$ are $x \approx 1.10715, 1.24905, 1.89255, 2.03444, 4.24874, 4.39064, 5.03414, 5.17604$.

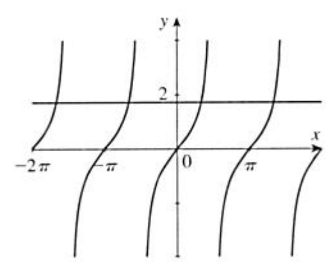
57. $f(x) = 3\cos x + 1$; $g(x) = \cos x - 1$. $f(x) = g(x)$ when $3\cos x + 1 = \cos x - 1 \Leftrightarrow 2\cos x = -2 \Leftrightarrow \cos x = -1 \Leftrightarrow x = \pi + 2k\pi = (2k+1)\pi$. The points of intersection are $((2k+1)\pi, -2)$ for any integer k.



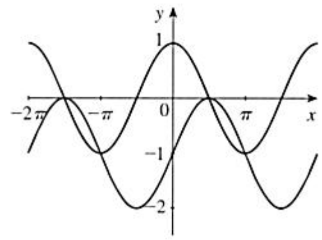
58. $f(x) = \sin 2x$; $g(x) = 2\sin 2x + 1$. $f(x) = g(x)$ when $\sin 2x = 2\sin 2x + 1 \Leftrightarrow \sin 2x = -1 \Leftrightarrow 2x = \frac{3\pi}{2} + 2k\pi \Leftrightarrow x = \frac{3\pi}{4} + k\pi$ for any integer k.



59. $f(x) = \tan x$; $g(x) = \sqrt{3}$. $f(x) = g(x)$ when $\tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} + k\pi$. The intersection points are $\left(\frac{\pi}{3} + k\pi, \sqrt{3}\right)$ for any integer k.



60. $f(x) = \sin x - 1$; $g(x) = \cos x$. $f(x) = g(x)$ when $\sin x - 1 = \cos x \Rightarrow$ $(\sin x - 1)^2 = \cos^2 x \quad \Leftrightarrow \quad \sin^2 x - 2\sin x + 1 = \cos^2 x \quad \Leftrightarrow$ $\sin^2 x - 2\sin x + 1 - \cos^2 x = 0 \Leftrightarrow 2\sin^2 x - 2\sin x = 0 \quad \Leftrightarrow$ $2\sin x(\sin x - 1) = 0 \quad \Leftrightarrow \quad \sin x = 0$ or $\sin x = 1 \Leftrightarrow x = k\pi, \frac{\pi}{2} + 2k\pi$. However, $x = k\pi$ is not a solution when k is even. (The extraneous solutions were introduced by squaring both sides.) So the solutions are $x = (2k+1)\pi, \frac{\pi}{2} + 2k\pi$, and the intersection points are $(\pi + 2k\pi, -1)$, $\left(\frac{\pi}{2} + 2k\pi, 0\right)$ for any integer k.



Another method: $\sin x - 1 = \cos x \quad \Leftrightarrow \quad \sin x - \cos x = 1 \quad \Leftrightarrow \quad \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right) = 1 \quad \Leftrightarrow \quad \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = \frac{\sqrt{2}}{2}$. Since $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, we may write $\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = \cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x = \frac{\sqrt{2}}{2} \quad \Leftrightarrow$ $\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \Leftrightarrow \quad x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4} \quad \Leftrightarrow \quad x = \frac{\pi}{2}, \pi$ in $[0, 2\pi)$. So the solutions are $x = (2k+1)\pi, \frac{\pi}{2} + 2k\pi$, and the intersection points are $(\pi + 2k\pi, -1)$, $\left(\frac{\pi}{2} + 2k\pi, 0\right)$ for any integer k.

61. $\cos x\cos 3x - \sin x\sin 3x = 0 \quad \Leftrightarrow \quad \cos(x + 3x) = 0 \quad \Leftrightarrow \quad \cos 4x = 0 \quad \Leftrightarrow \quad 4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$ in $[0, 8\pi) \quad \Leftrightarrow \quad x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ in $[0, 2\pi)$.

62. $\cos x\cos 2x + \sin x\sin 2x = \frac{1}{2} \quad \Leftrightarrow \quad \cos(x - 2x) = \frac{1}{2} \quad \Leftrightarrow \quad \cos(-x) = \frac{1}{2} \quad \Leftrightarrow \quad \cos x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$ in $[0, 2\pi)$.

63. $\sin 2x\cos x + \cos 2x\sin x = \frac{\sqrt{3}}{2} \Leftrightarrow \sin(2x + x) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin 3x = \frac{\sqrt{3}}{2} \quad \Leftrightarrow \quad 3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$ in $[0, 6\pi) \Leftrightarrow x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$ in $[0, 2\pi)$.

64. $\sin 3x\cos x - \cos 3x\sin x = 0 \quad \Leftrightarrow \quad \sin(3x - x) = 0 \quad \Leftrightarrow \quad \sin 2x = 0 \quad \Leftrightarrow \quad 2x = 0, \pi, 2\pi, 3\pi, 4\pi$ in $[0, 4\pi)$ $\Leftrightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ in $[0, 2\pi)$.

65. $\sin 2x + \cos x = 0 \quad \Leftrightarrow \quad 2\sin x\cos x + \cos x = 0 \Leftrightarrow \cos x(2\sin x + 1) = 0 \quad \Leftrightarrow \quad \cos x = 0$ or $\sin x = -\frac{1}{2} \quad \Leftrightarrow$ $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$.

66. $\tan\dfrac{x}{2} - \sin x = 0 \quad \Leftrightarrow \quad \dfrac{\sin x}{1 + \cos x} - \sin x = 0 \quad \Leftrightarrow \quad \sin x - \sin x(1 + \cos x) = 0$ (and $\cos x \neq -1 \quad \Leftrightarrow \quad x \neq \pi$) $\Leftrightarrow \quad \sin x(-\cos x) = 0 \quad \Leftrightarrow \quad \sin x = 0$ or $\cos x = 0 \quad \Leftrightarrow \quad x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ in $[0, 2\pi)$ ($x = \pi$ is inadmissible).

67. $\cos 2x + \cos x = 2 \quad \Leftrightarrow \quad 2\cos^2 x - 1 + \cos x - 2 = 0 \Leftrightarrow 2\cos^2 x + \cos x - 3 = 0 \quad \Leftrightarrow \quad (2\cos x + 3)(\cos x - 1) = 0$ $\Leftrightarrow \quad 2\cos x + 3 = 0$ or $\cos x - 1 = 0 \quad \Leftrightarrow \quad \cos x = -\frac{3}{2}$ (which is impossible) or $\cos x = 1 \quad \Leftrightarrow \quad x = 0$ in $[0, 2\pi)$.

68. $\tan x + \cot x = 4\sin 2x \quad \Leftrightarrow \quad \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = 8\sin x\cos x \quad \Leftrightarrow \quad \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right) \cdot \sin x\cos x = (8\sin x\cos x) \cdot \sin x\cos x \quad \Leftrightarrow \quad \sin^2 x + \cos^2 x = 8\sin^2 x\cos^2 x \quad \Leftrightarrow \quad 1 = 2(2\sin x\cos x)^2 \quad \Leftrightarrow \quad (\sin 2x)^2 = \frac{1}{2} \Leftrightarrow$ $\sin 2x = \pm\frac{1}{\sqrt{2}}$. Therefore, $2x = \frac{\pi}{4} + k\pi$ or $2x = \frac{3\pi}{4} + k\pi \quad \Leftrightarrow \quad x = \frac{\pi}{8} + \frac{k\pi}{2}$ or $x = \frac{3\pi}{8} + \frac{k\pi}{2}$. Thus on the interval $[0, 2\pi)$ the solutions are $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ and $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$. Together we write the solutions as $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$, which are odd multiples of $\frac{\pi}{8}$. So we can express the general solution as $x = \frac{\pi}{8} + k\frac{\pi}{4}$ where k is any integer.

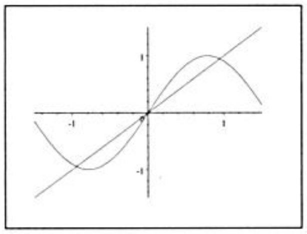
69. $\sin x + \sin 3x = 0 \quad \Leftrightarrow \quad 2\sin 2x\cos(-x) = 0 \quad \Leftrightarrow \quad 2\sin 2x\cos x = 0 \quad \Leftrightarrow \quad \sin 2x = 0$ or $\cos x = 0 \quad \Leftrightarrow$ $2x = k\pi$ or $x = k\frac{\pi}{2} \Leftrightarrow x = k\frac{\pi}{2}$ for any integer k.

70. $\cos 5x - \cos 7x = 0 \quad \Leftrightarrow \quad -2\sin 6x\sin(-x) = 0 \quad \Leftrightarrow \quad \sin 6x\sin x = 0 \quad \Leftrightarrow \quad \sin 6x = 0$ or $\sin x = 0 \quad \Leftrightarrow$ $6x = k\pi$ or $x = k\pi \Leftrightarrow x = k\frac{\pi}{6}$ for any integer k.

71. $\cos 4x + \cos 2x = \cos x \quad \Leftrightarrow \quad 2\cos 3x\cos x = \cos x \quad \Leftrightarrow \quad \cos x(2\cos 3x - 1) = 0 \Leftrightarrow \cos x = 0$ or $\cos 3x = \frac{1}{2}$ $\Leftrightarrow \quad x = \frac{\pi}{2}$ or $3x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi, \frac{7\pi}{3} + 2k\pi, \frac{11\pi}{3} + 2k\pi, \frac{13\pi}{3} + 2k\pi, \frac{17\pi}{3} + 2k\pi \quad \Leftrightarrow \quad x = \frac{\pi}{2}, \frac{\pi}{9} + 2k\frac{\pi}{3}, \frac{5\pi}{9} + 2k\frac{\pi}{3}$.

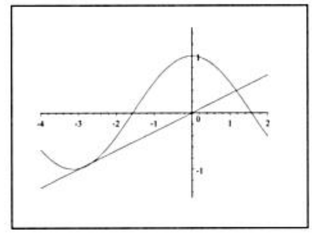
72. $\sin 5x - \sin 3x = \cos 4x \quad\Leftrightarrow\quad 2\cos 4x \sin x = \cos 4x \quad\Leftrightarrow\quad \cos 4x\,(2\sin x - 1) = 0 \Leftrightarrow \cos 4x = 0$ or $\sin x = \frac{1}{2}$ $\quad\Leftrightarrow\quad 4x = \frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{6} + 2k\pi,\ \frac{5\pi}{6} + 2k\pi \Leftrightarrow x = \dfrac{\pi}{8} + k\frac{\pi}{4},\ \frac{\pi}{6} + 2k\pi,\ \frac{5\pi}{6} + 2k\pi.$

73. $\sin 2x = x$



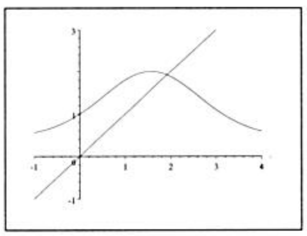
The three solutions are $x = 0$ and $x \approx \pm 0.95$.

74. $\cos x = \dfrac{x}{3}$



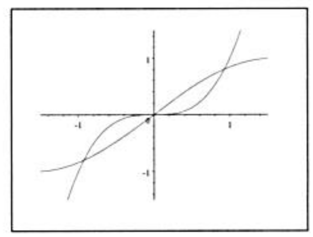
The three solutions are $x \approx 1.17, -2.66,$ and -2.94.

75. $2^{\sin x} = x$



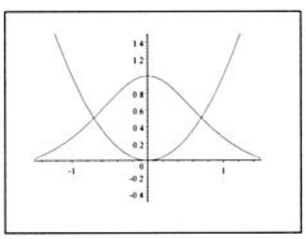
The only solution is $x \approx 1.92$.

76. $\sin x = x^3$



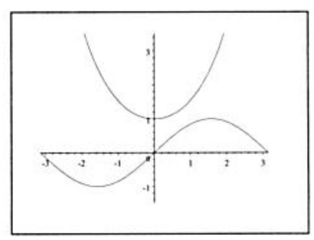
The three solutions are $x = 0$ and $x \approx \pm 0.93$.

77. $\dfrac{\cos x}{1 + x^2} = x^2$



The two solutions are $x \approx \pm 0.71$.

78. $\sin x = \frac{1}{2}\left(e^x + e^{-x}\right)$



This equation has no solution.

79. We substitute $v_0 = 2200$ and $R(\theta) = 5000$ and solve for θ. So $5000 = \dfrac{(2200)^2 \sin 2\theta}{32} \quad\Leftrightarrow\quad 5000 = 151250 \sin 2\theta$ $\quad\Leftrightarrow\quad \sin 2\theta = 0.03308 \Rightarrow 2\theta = 1.89442^\circ$ or $2\theta = 180^\circ - 1.89442^\circ = 178.10558^\circ$. If $2\theta = 1.89442^\circ$, then $\theta = 0.94721^\circ$, and if $2\theta = 178.10558^\circ$, then $\theta = 89.05279^\circ$.

80. Since $4e^{-3t} > 0$, we have $0 = 4e^{-3t} \sin 2\pi t \Leftrightarrow 0 = \sin 2\pi t \quad\Leftrightarrow\quad 2\pi t = 0, \pi, 2\pi, \ldots \quad\Leftrightarrow\quad t = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$.

81. We substitute $\theta_1 = 70^\circ$ and $\dfrac{v_1}{v_2} = 1.33$ into Snell's Law to get $\dfrac{\sin 70^\circ}{\sin \theta_2} = 1.33 \quad\Leftrightarrow\quad \sin \theta_2 = \dfrac{\sin 70^\circ}{1.33} = 0.7065 \Rightarrow$ $\theta_2 \approx 44.95^\circ$.

82. The index of refraction from glass to air is $\dfrac{1}{1.52} \approx 0.658$, so we substitute $\theta_2 = 90^\circ$ into Snell's Law to get $\dfrac{\sin \theta_1}{\sin 90^\circ} = 0.658 \Rightarrow \sin \theta_1 \approx 0.658 \Rightarrow \theta_1 \approx 41.1^\circ$.

83. **(a)** $10 = 12 + 2.83\sin\left(\frac{2\pi}{3}(t-80)\right) \quad\Leftrightarrow\quad 2.83\sin\left(\frac{2\pi}{3}(t-80)\right) = -2 \quad\Leftrightarrow\quad \sin\left(\frac{2\pi}{3}(t-80)\right) = -0.70671$.
Now $\sin\theta = -0.70671$ and $\theta = -0.78484$. If $\frac{2\pi}{3}(t-80) = -0.78484 \Leftrightarrow t - 80 = 45.6 \quad\Leftrightarrow\quad t = 34.4$.
Now in the interval $[0, 2\pi)$, we have $\theta = \pi + 0.78484 \approx 3.92644$ and $\theta = 2\pi - 0.78484 \approx 5.49834$. If $\frac{2\pi}{3}(t-80) = 3.92644 \quad\Leftrightarrow\quad t - 80 = 228.1 \Leftrightarrow t = 308.1$. And if $\frac{2\pi}{3}(t-80) = 5.49834 \quad\Leftrightarrow\quad t - 80 = 319.4 \quad\Leftrightarrow\quad t = 399.4 \quad (399.4 - 365 = 34.4)$. So according to this model, there should be 10 hours of sunshine on the 34th day (February 3) and on the 308th day (November 4).

(b) Since $L(t) = 12 + 2.83\sin\left(\frac{2\pi}{3}(t-80)\right) \geq 10$ for $t \in [34, 308]$, the number of days with more than 10 hours of daylight is $308 - 34 + 1 = 275$ days.

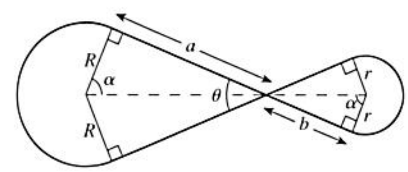
84. **(a)** $F = \frac{1}{2}(1 - \cos\theta) = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

(b) $F = \frac{1}{2}(1 - \cos\theta) = 0.25 \Rightarrow 1 - \cos\theta = 0.5 \Rightarrow \cos\theta = 0.5 \Rightarrow \theta = 60^\circ$ or 120°

(c) $F = \frac{1}{2}(1 - \cos\theta) = 0.5 \Rightarrow 1 - \cos\theta = 1 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$ or 270°

(d) $F = \frac{1}{2}(1 - \cos\theta) = 1 \Rightarrow 1 - \cos\theta = 2 \Rightarrow \cos\theta = -1 \Rightarrow \theta = 180^\circ$

85. **(a)** First note that $\alpha = \dfrac{\pi}{2} - \dfrac{\theta}{2}$. The part of the belt touching the larger pulley has length $2(\pi - \alpha)R = (\theta + \pi)R$ and similarly the part touching the smaller belt has length $(\theta + \pi)r$. To calculate a and b, we write $\cot\dfrac{\theta}{2} = \dfrac{a}{R} = \dfrac{b}{r} \Rightarrow a = R\cot\dfrac{\theta}{2}$ and $b = r\cot\dfrac{\theta}{2}$, so

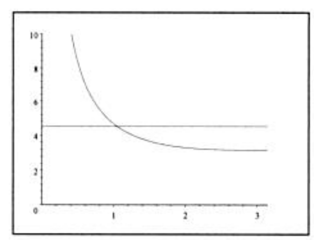


the length of the straight parts of the belt is $2a + 2b = 2(R + r)\cot\dfrac{\theta}{2}$. Thus, the total length of the belt is

$L = (\theta + \pi)R + (\theta + \pi)r + 2(R + r)\cot\dfrac{\theta}{2} = (R + r)\left(\theta + \pi + 2\cot\dfrac{\theta}{2}\right)$ and so $\theta + \pi + 2\cot\dfrac{\theta}{2} = \dfrac{L}{R + r} \Leftrightarrow$

$\theta + 2\cot\dfrac{\theta}{2} = \dfrac{L}{R + r} - \pi$.

(b) We plot $\theta + 2\cot\dfrac{\theta}{2}$ and $\dfrac{L}{R + r} - \pi = \dfrac{27.78}{2.42 + 1.21} - \pi \approx 4.5113$ in the same viewing rectangle. The solution is $\theta \approx 1.047$ rad $\approx 60^\circ$.



86. Statement A is true: every identity is an equation. However, Statement B is false: not every equation is an identity. The difference between an identity and an equation is that an identity is true for all values in the domain, whereas an equation may only be true for certain values in the domain and false for others. For example, $x = 0$ is an equation but not an identity, because it is true for only one value of x.

87. $\sin(\cos x)$ is a function of a function, that is, a composition of trigonometric functions (see Section 2.7). Most of the other equations involve sums, products, differences, or quotients of trigonometric functions.

$\sin(\cos x) = 0 \quad\Leftrightarrow\quad \cos x = 0$ or $\cos x = \pi$. However, since $|\cos x| \leq 1$, the only solution is $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$. The graph of $f(x) = \sin(\cos x)$ is shown.