Name: ______________________________________ Date: ___________

PCH Ms. Loughran

Do Now:

For all values of the angle for which the expressions are defined, choose an equivalent expression.

1. $\frac{-1}{\cos A}$ is equivalent to

(1) $\sec A$ (2) $-\sec A$ (3) $\sin A$ (4) $-\sin A$

2. $\frac{\cot\theta}{\csc\theta}$ is equivalent to

(1) $\sec\theta$ (2) $\sin\theta$ (3) $\cos\theta$ (4) $\csc\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1}$

3. $\frac{\sec\theta}{\csc\theta}$ is equivalent to

(1) $\sin\theta$ (2) $\cos\theta$ (3) $\tan\theta$ (4) $\cot\theta$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \frac{\sin\theta}{\cos\theta}$

4. $\frac{\sin\theta}{\tan\theta}$ is equivalent to

(1) $-\cos\theta$ (3) $1 - \cos\theta$

(2) $\cos\theta$ (4) $1 + \cos\theta$

$\cancel{\sin\theta} \cdot \frac{\cos\theta}{\cancel{\sin\theta}}$

5. $\frac{\sin^2 A}{\tan A}$ is equivalent to

(1) $\frac{\sin A}{\cos A}$ (3) $\frac{1}{\sin A \cos A}$

(2) $\sin A \cos A$ (4) $\frac{\cos A}{\sin A}$

$\sin^{\cancel{2}} A \cdot \frac{\cos A}{\cancel{\sin A}}$

6. $\sin\theta$ is equivalent to

(1) $\frac{\tan\theta}{\sec\theta}$ (2) $\frac{1}{\sec\theta}$ (3) $\sec\theta$ (4) $\frac{\sec\theta}{\tan\theta}$

$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

7. The expression $\frac{\tan x}{\sec^2 x}$ is equivalent to

(1) $\sin x$ (3) $\frac{\sin^3 x}{\cos x}$

(2) $\sin x \cos x$ (4) $\frac{\cos^3 x}{\sin x}$

$\frac{\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x$

Name:___________________________ Date:___________

PCH: Trigonometric Identities and Proofs Ms. Loughran

The Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Double Angle Formulas:

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

You are familiar with the following reciprocal identities:

$$\sec\theta = \frac{1}{\cos\theta}, \cos\theta \neq 0 \qquad \csc\theta = \frac{1}{\sin\theta}, \sin\theta \neq 0 \qquad \cot\theta = \frac{1}{\tan\theta}, \tan\theta \neq 0$$

And the quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}, \sin\theta \neq 0$$

An identity is an equation that is true for all permissible replacements of the variable.

Proving an identity:
To prove that a trigonometric statement is an identity, note:

1. **The object is to show that the two sides of the statement are equivalent.**
   - ⇒ You may work on only one side and show that it is equivalent to the other.
     - ➢ Work on the more complicated side.
   - ⇒ You may work on the two sides independently until you arrive at equivalent expressions.
     - ➢ You may not perform operations involving the two sides simultaneously. You are not solving an equation. That is, never cross the equal sign for any purpose. As a reminder, use a line between sides.

2. **Use the basic identities to transform one or both sides of the proposed identity.**
   - ⇒ A general starting point is to rewrite expressions in terms of sine and cosine, but be alert to situations when a Pythagorean substitution is appropriate.

3. **After replacements have been made, do the algebra suggested by the form of the expression.**
   - ⇒ If there is a complex fraction, simplify it.
   - ⇒ If there are two fractions, combine them.
   - ⇒ Look for possibilities of factoring.

Classwork

1. Simplify the expression: $\cos t + \tan t \sin t$

$$\cos t + \frac{\sin t}{\cos t} \cdot \sin t$$

$$\cos t + \frac{\sin^2 t}{\cos t}$$

$$\frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t} \text{ or } \sec t$$

2. Simplify the expression: $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta}$

$$\frac{\sin\theta(1+\sin\theta) + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cancel{\sin\theta + 1}}{\cos\theta(\cancel{1+\sin\theta})} = \frac{1}{\cos\theta} \text{ or } \sec\theta$$

3. Verify the identity: $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

$$\cos\theta(\sec\theta) - \cos^2\theta$$

$$1 - \cos^2\theta$$

$$\sin^2\theta = \sin^2\theta$$

4. Verify the identity: $2\tan x \sec x = \dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x}$

$$\frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\frac{2\sin x}{1-\sin^2 x}$$

$$\frac{2\sin x}{\cos^2 x}$$

$$2\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$$

$$2\tan x\sec x = 2\tan x\sec x$$

5. Verify the identity: $\dfrac{\cos u}{1-\sin u} = \sec u + \tan u$

$$\frac{1}{\cos u}+\frac{\sin u}{\cos u}$$

$$\frac{(1+\sin u)(1-\sin u)}{\cos u\,(1-\sin u)}$$

$$\frac{1-\sin^2 u}{\cos u\,(1-\sin u)}$$

$$\frac{\cos^{\cancel{2}} u}{\cancel{\cos u}\,(1-\sin u)}$$

$$\frac{\cos u}{1-\sin u}$$

$$\frac{\cos u}{1-\sin u}$$

or

$\dfrac{\cos u}{1-\sin u} = \sec u + \tan u$

$$\frac{\cos u\,(1+\sin u)}{(1-\sin u)(1+\sin u)}$$

$$\frac{\cos u\,(1+\sin u)}{1-\sin^2 u}$$

$$\frac{\cancel{\cos u}\,(1+\sin u)}{\cos^{\cancel{2}} u}$$

$$\frac{1+\sin u}{\cos u}$$

$$\frac{1}{\cos u}+\frac{\sin u}{\cos u}$$

$$\sec u+\tan u \qquad \sec u+\tan u$$

$\tan^2\theta + 1 = \sec^2\theta$

6. Verify the identity: $\dfrac{1+\cos\theta}{\cos\theta} = \dfrac{\tan^2\theta}{\sec\theta - 1}$

$$\frac{\sec^2\theta - 1}{\sec\theta - 1}$$

$$\frac{(\sec\theta + 1)\cancel{(\sec\theta - 1)}}{\cancel{\sec\theta - 1}}$$

$$\sec\theta + 1$$

$$\frac{1}{\cos\theta} + 1$$

$$\frac{1+\cos\theta}{\cos\theta} = \frac{1+\cos\theta}{\cos\theta}$$

7. Verify the identity: $(\sin x + \cos x)^2 = 1 + \sin 2x$

Homework 04-19
Last 4 from DM

11 $\quad \sin^2 x + 2 - \cos^2 x = 3\sin x$

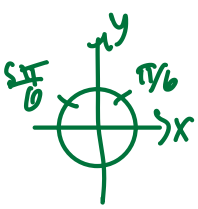
$$\sin^2 x + 2 - (1 - \sin^2 x) = 3\sin x$$

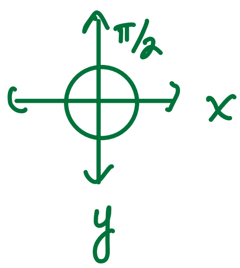
$$\sin^2 x + 2 - 1 + \sin^2 x = 3\sin x$$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$\sin x = \frac{1}{2} \qquad \sin x = 1$





$$x = \begin{cases} \frac{\pi}{6} + 2\pi k \\ \frac{5\pi}{6} + 2\pi k \\ \frac{\pi}{2} + 2\pi k \end{cases}, \quad k \in \mathbb{Z}$$

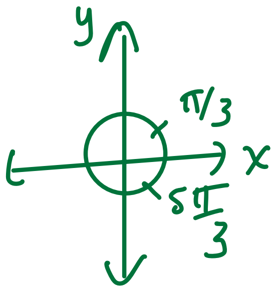
(12) $2\cos^2 x + 7\cos x = 4$

$2\cos^2 x + 7\cos x - 4 = 0$

$(2\cos x - 1)(\cos x + 4) = 0$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = -4$

$\quad\quad\quad\quad\quad\quad\quad\quad \cancel{0}$



$x = \frac{\pi}{3} + 2\pi k$

$\quad\ \ \frac{5\pi}{3} + 2\pi k$, $k \in \mathbb{Z}$

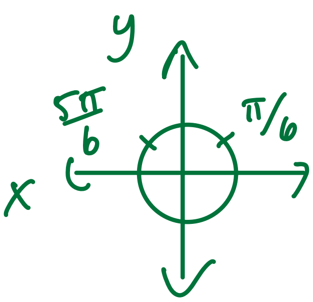
(13) $\csc^2 x - \csc x + 3 = 5$

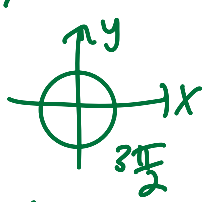
$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$ $\quad\quad$ $\csc x = -1$

$\sin x = \frac{1}{2}$ $\quad\quad$ $\sin x = -1$





$x = \frac{\pi}{6} + 2\pi k$

$\quad\ \ 5\pi/6 + 2\pi k$, $k \in \mathbb{Z}$

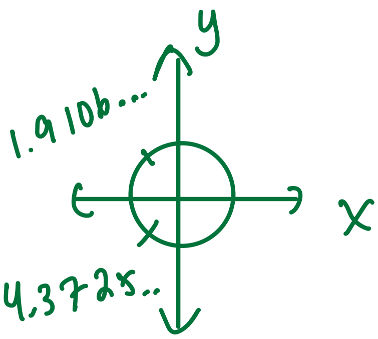
$\quad\ \ 3\pi/2 + 2\pi k$

(14) $6\cos^2 x + 6\cos x + 2 = 1 + \cos x$

$6\cos^2 x + 5\cos x + 1 = 0$

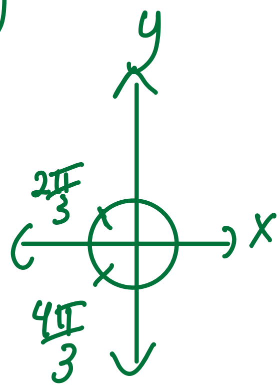
$(3\cos x + 1)(2\cos x + 1) = 0$

$\cos x = -\frac{1}{3}$ $\qquad$ $\cos x = -\frac{1}{2}$



↑ need calc.

$\cos^{-1}\left(\frac{1}{3}\right) = 1.2309...$



$$x = \begin{cases} \frac{2\pi}{3} + 2\pi k \\ \frac{4\pi}{3} + 2\pi k \\ 1.9106... + 2\pi k \\ 4.3725... + 2\pi k \end{cases}, \quad k \in \mathbb{Z}$$