

Name: _____

Date: _____

PCH Factoring Expressions with Negative/Fractional Exponents

Ms. Loughran

Do Now: #s 1 and 2

Factor completely.

1. $x^{\frac{1}{2}} - x^{\frac{5}{2}}$

$$x^{\frac{1}{2}}(1 - x^2)$$

$$x^{\frac{1}{2}}(1 - x)(1 + x)$$

2. $x^{-\frac{1}{2}}(x+3)^{\frac{1}{2}} + x^{\frac{1}{2}}(x+3)^{-\frac{1}{2}}$

$$x^{-\frac{1}{2}}(x+3)^{-\frac{1}{2}} \left((x+3) + x \right)$$

$$x^{-\frac{1}{2}}(x+3)^{-\frac{1}{2}} (2x+3)$$

3. $x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + 15x^{-\frac{1}{2}}$

$$x^{-\frac{1}{2}}(x^2 + 8x + 15)$$

$$x^{-\frac{1}{2}}(x+3)(x+5)$$

4. $2x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

$$x^{-\frac{3}{2}}(2x^2 + 5x + 2)$$

$$x^{-\frac{3}{2}}(2x+1)(x+2)$$

$$\frac{x^{-\frac{1}{2}}}{x^{-\frac{3}{2}}}$$

5. $(x+2)^{\frac{7}{2}} - (x+2)^{\frac{3}{2}}$

$$(x+2)^{\frac{3}{2}} \left((x+2)^2 - 1 \right)$$

$$(x+2)^{\frac{3}{2}} (x+2-1)(x+2+1)$$

$$(x+2)^{\frac{3}{2}} (x+1)(x+3)$$

6. $4(x+1)^{\frac{1}{2}} - 5(x+1)^{\frac{3}{2}} + (x+1)^{\frac{5}{2}}$

$$(x+1)^{\frac{1}{2}} \left(4 - 5(x+1) + (x+1)^2 \right)$$

let $c = x+1$

$$(x+1)^{\frac{1}{2}} (4 - 5c + c^2)$$

$$(x+1)^{\frac{1}{2}} (c-1)(c-4)$$

$$(x+1)^{\frac{1}{2}} (x+1-1)(x+1-4) = x(x+1)^{\frac{1}{2}}(x-3)$$

$$7. x^{\frac{1}{2}} - 3x^{\frac{1}{3}} - 3x^{\frac{1}{6}} + 9$$

$$x^{\frac{2}{6}} - 3x^{\frac{2}{6}} - 3x^{\frac{1}{6}} + 9$$

$$x^{\frac{2}{6}}(x^{\frac{1}{6}} - 3) - (x^{\frac{1}{6}} - 3)$$

$$(x^{\frac{1}{6}} - 3)(x^{\frac{1}{6}} - 3)$$

$$8. 12x^{\frac{4}{3}} - 5x^{\frac{2}{3}} - 2$$

$$12(x^{\frac{2}{3}})^2 - 5x^{\frac{2}{3}} - 2$$

$$\text{let } y = x^{\frac{2}{3}}$$

$$12y^2 - 5y - 2$$

$$(3y - 2)(4y + 1)$$

$$(3x^{\frac{2}{3}} - 2)(4x^{\frac{2}{3}} + 1)$$

$$9. x^{\frac{3}{5}} + 5x^{\frac{2}{5}} - 3x^{\frac{1}{5}} - 15$$

$$x^{\frac{2}{5}}(x^{\frac{1}{5}} + 5) - 3(x^{\frac{1}{5}} + 5)$$

$$(x^{\frac{2}{5}} - 3)(x^{\frac{1}{5}} + 5)$$

$$10. 3x(x+2)^{-\frac{1}{3}} + 4(x+2)^{\frac{2}{3}}$$

$$(x+2)^{-\frac{1}{3}}(3x + 4(x+2))$$

$$(x+2)^{-\frac{1}{3}}(7x + 8)$$

$$11. 2(5x-1)^{\frac{3}{4}} + 7x(5x-1)^{-\frac{1}{4}}$$

$$(5x-1)^{-\frac{1}{4}}[2(5x-1) + 7x]$$

$$(5x-1)^{-\frac{1}{4}}(17x - 2)$$

$$12. (x^2+2)^{\frac{5}{2}} + 2x(x^2+2)^{\frac{3}{2}} + x^2(x^2+2)^{\frac{1}{2}}$$

$$(x^2+2)^{\frac{1}{2}}[(x^2+2)^2 + 2x(x^2+2) + x^2]$$

$$\text{let } y = x^2 + 2$$

$$(x^2+2)^{\frac{1}{2}}(y^2 + 2xy + x^2) = (x^2+2)^{\frac{1}{2}}(y+x)^2$$

$$= (x^2+2)^{\frac{1}{2}}(x^2+2+x)^2 \text{ or } (x^2+2)^{\frac{1}{2}}(x^2+x+2)^2$$

Homework 09-07

Name: _____
PreCalculus Honors – Factoring Practice Sheet

Date: _____

Factor each expression completely.

1. $3x^2 - 5xy - 2y^2$

2. $x^6 - 2x^3y^3 + y^6$
 $(x^3 - y^3)(x^3 - y^3)$
 $(x - y)^2 (x^2 + xy + y^2)^2$

3. $125x^3 - 64y^9$

4. $(x - 1)^2 - 25$
 $(x - 1 - 5)(x - 1 + 5)$
 $(x - 6)(x + 4)$

5. $(2x + y)x^2 - (2x + y)y^2$

6. $(2x+1)^2 - 6(2x+1) + 8$

$$\begin{aligned} & \text{let } y = 2x+1 \\ & y^2 - 6y + 8 \\ & (y-4)(y-2) \\ & (2x+1-4)(2x+1-2) \\ & (2x-3)(2x-1) \end{aligned}$$

7. $16u^4 - 200u^2 + 625$

8. $6g^2 - 23gh + 15h^2$

$$(6g - 5h)(g - 3h)$$

9. $12st + 56r - 96rt - 7s$

$$\begin{aligned} & 12st - 96rt - 7s + 56r \\ & 12t(s - 8r) - 7(s - 8r) \\ & (12t - 7)(s - 8r) \end{aligned}$$

10. $6c^{2x} + 8c^x - 64$

11. $(s^2 - 2st)^2 - 2t^2(s^2 - 2st) - 3t^4$

$$\begin{aligned} & \text{let } x = s^2 - 2st \\ & x^2 - 2t^2x - 3t^4 \\ & (x + t^2)(x - 3t^2) \\ & (s^2 - 2st + t^2)(s^2 - 2st - 3t^2) \\ & (s-t)(s+t)(s-3t)(s+t) \end{aligned}$$

$$12. u^{18} - m^{18}$$

$$(u^9 - m^9)(u^9 + m^9)$$

$$(u^3 - m^3)(u^6 + m^3u^3 + m^6)(u^3 + m^3)(u^6 - m^3u^3 + m^6)$$

$$(u-m)(u^2+um+m^2)(u^6+m^3u^3+m^6)(u+m)(u^2-um+m^2)(u^6-m^3u^3+m^6)$$

OR

$$12. u^{18} - m^{18}$$

$$(u^6 - m^6)(u^{12} + m^6u^6 + m^{12})$$

$$(u^2 - m^2)(u^4 + m^2u^2 + m^4)(u^{12} + m^6u^6 + m^{12})$$

$$(u-m)(u+m)$$

both of these can be factored further using a method we will call Advanced Completing the Square

$$(u-m)(u+m)(u^2 - mu + m^2)(u^2 + mu + m^2)(u^6 + m^3u^3 + m^6)(u^6 - m^3u^3 + m^6)$$