

Name: \_\_\_\_\_

Date: \_\_\_\_\_

PCH – Using Advanced Completing the Square to Factor Expressions

Ms. Loughran

Do Now:

1. Factor completely:  $x^2 + y^2 + 2xy - 49$

$$(x+y)^2 - 49$$
$$(x+y+7)(x+y-7)$$

Factor each of the following completely.

1.  $x^4 + 2x^2 + 9 + 4x^2 - 4x^2$

$$x^4 + 6x^2 + 9 - 4x^2$$
$$(x^2+3)^2 - 4x^2$$
$$(x^2+3-2x)(x^2+3+2x)$$
$$(x^2-2x+3)(x^2+2x+3)$$

2.  $x^4 + 4x^2 + 16 + 4x^2 - 4x^2$

$$x^4 + 8x^2 + 16 - 4x^2$$
$$(x^2+4)^2 - 4x^2$$
$$(x^2+4-2x)(x^2+4+2x)$$
$$(x^2-2x+4)(x^2+2x+4)$$

3.  $x^4 + 11x^2 + 36 + x^2 - x^2$

$$x^4 + 12x^2 + 36 - x^2$$
$$(x^2+6)^2 - x^2$$
$$(x^2-x+6)(x^2+x+6)$$

4.  $x^4 + 13x^2y^2 + 121y^4 + 9x^2y^2 - 9x^2y^2$

$$x^4 + 22x^2y^2 + 121y^4 - 9x^2y^2$$
$$(x^2+11y^2)^2 - 9x^2y^2$$
$$(x^2+11y^2-3xy)(x^2+11y^2+3xy)$$
$$(x^2-3xy+11y^2)(x^2+3xy+11y^2)$$

$$\begin{aligned}
 5. \quad & a^4 + a^2 + 1 + a^2 - a^2 \\
 & a^2 + 2a^2 + 1 - a^2 \\
 & (a^2 + 1)^2 - a^2 \\
 & (a^2 + 1 - a)(a^2 + 1 + a) \\
 & (a^2 - a + 1)(a^2 + a + 1)
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2 \\
 & x^4 + 2x^2y^2 + y^4 - x^2y^2 \\
 & (x^2 + y^2)^2 - x^2y^2 \\
 & (x^2 + y^2 - xy)(x^2 + y^2 + xy) \\
 & (x^2 - xy + y^2)(x^2 + xy + y^2)
 \end{aligned}$$

Same question now

$$\begin{aligned}
 7. \quad & 16x^4 + 16x^2y^2 + 16y^4 \\
 & 16(x^4 + x^2y^2 + y^4) \\
 & 16(x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2) \\
 & 16[(x^2 + y^2)^2 - x^2y^2] \\
 & 16[(x^2 + y^2 - xy)(x^2 + y^2 + xy)] \\
 & 16(x^2 - xy + y^2)(x^2 + xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & x^4 + 3x^2 + 4 + x^2 - x^2 \\
 & x^4 + 4x^2 + 4 - x^2 \\
 & (x^2 + 2)^2 - x^2 \\
 & (x^2 + x + 2)(x^2 - x + 2)
 \end{aligned}$$

Let's revisit #6 from the More Factoring Fun sheet and #2 from the Factoring Practice sheet

$$\begin{aligned}
 6. \quad & x^9 - x^6 - x^3 + 1 \\
 & x^6(x^3 - 1) - 1(x^3 - 1) \\
 & (x^3 - 1)(x^6 - 1) \quad \text{difference of cubes} \\
 & (x - 1)(x^2 + x + 1)(x^2 - 1)(x^4 + x^2 + 1) \\
 & (x - 1)(x^2 + x + 1)(x - 1)(x + 1)(x^4 + x^2 + 1) \\
 & \uparrow \text{ we can factor further, to be continued...}
 \end{aligned}$$

$$\left\{ \begin{aligned}
 & (x^6 - 1)(x^3 - 1) \\
 & (x^3 - 1)(x^3 + 1)(x^3 - 1) \\
 & (x^3 - 1)^2(x^3 + 1) \\
 & (x - 1)^2(x^2 + x + 1)^2(x + 1)(x^2 - x + 1)
 \end{aligned} \right.$$

$$12. u^{18} - m^{18}$$

$$(u^9 - m^9)(u^9 + m^9)$$

$$(u^3 - m^3)(u^6 + m^3u^3 + m^6)(u^3 + m^3)(u^6 - m^3u^3 + m^6)$$

$$(u-m)(u^2+um+m^2)(u^6+m^3u^3+m^6)(u+m)(u^2-um+m^2)(u^6-m^3u^3+m^6)$$

OR

$$12. u^{18} - m^{18}$$

$$(u^6 - m^6)(u^{12} + m^6u^6 + m^{12})$$

$$(u^2 - m^2)(u^4 + m^2u^2 + m^4)(u^{12} + m^6u^6 + m^{12})$$

$$(u-m)(u+m)$$

both of these can be factored further using a method we will call Advanced Completing the Square

$$(u-m)(u+m)(u^2 - mu + m^2)(u^2 + mu + m^2)(u^6 + m^3u^3 + m^6)(u^6 - m^3u^3 + m^6)$$

# Answers to #s 2-8 even from the Classwork section of yesterday's sheet

2.  $5(x+2)^4(x^2-1)^4 + 4(x^2-1)^3(x+2)^5$

$$(x+2)^4(x^2-1)^3 [5(x^2-1) + 4(x+2)]$$

$$(x+2)^4(x^2-1)^3 (5x^2 + 4x + 3)$$

$$(x+2)^4(x-1)^3(x+1)^3(5x^2 + 4x + 3)$$

6.  $-2x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}}(5-2x)$

$$x^{-\frac{1}{3}}(-2x + \frac{2}{3}(5-2x))$$

$$x^{-\frac{1}{3}}(-2x + \frac{10}{3} - \frac{4}{3}x)$$

$$x^{-\frac{1}{3}}(-\frac{10}{3}x + \frac{10}{3})$$

$$x^{-\frac{1}{3}} \cdot \frac{-10}{3}(x-1)$$

$$-\frac{10}{3}x^{-\frac{1}{3}}(x-1) = \frac{-10(x-1)}{3x^{\frac{1}{3}}} \quad x \neq 0$$

4.  $2(x-2)(x-7)^3 + 3(x-7)^2(x-2)$

$$(x-2)(x-7)^2 [2x-14 + 3]$$

$$(x-2)(x-7)^2(2x-11)$$

8.  $\frac{(3x^4-2) \cdot 5(x^3+4)^4 \cdot 3x^2 - (x^3+4)^5 \cdot 12x^3}{(3x^4-2)^2}$

$$\frac{3x^2(x^3+4)^4 [5(3x^4-2) - 4x(x^3+4)]}{(3x^4-2)^2}$$

$$\frac{3x^2(x^3+4)^4 [11x^4 - 16x - 10]}{(3x^4-2)^2}$$

$$\begin{aligned} 3x^4 - 2 &= 0 \\ 3x^4 &= 2 \\ x^4 &= \frac{2}{3} \\ x &= \pm \sqrt[4]{\frac{2}{3}} \end{aligned}$$

$$x \neq \pm \sqrt[4]{\frac{2}{3}}$$

$$x \neq \pm \sqrt[4]{\frac{2}{3}}$$

10.  $\frac{(x+5)^5 \cdot \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2 - (2x+1)^{\frac{1}{2}} \cdot 5(x+5)^4}{(x+5)^{10}}$

$$\frac{-9x}{(2x+1)^{\frac{1}{2}}(x+5)^6}$$

$$x \neq -\frac{1}{2}, -5$$

# Homework 09-13

$$2. x^3(x+4)^5 - 2x^2(x+4)^6$$

$$x^2(x+4)^5 \left[ x - 2(x+4) \right]$$

$$x^2(x+4)^5 (-x-8)$$

$$-x^2(x+4)^5(x+8)$$

$$6. 2x^{\frac{1}{2}}(x+2)^2 + 2x^{\frac{3}{2}}(x+2)^3$$

$$2x^{\frac{1}{2}}(x+2)^2 \left[ 1 + x(x+2) \right]$$

$$2x^{\frac{1}{2}}(x+2)^2 (x^2+2x+1)$$

$$2x^{\frac{1}{2}}(x+2)^2 (x+1)^2$$

$$10. (3x+11)^5(x+5)^2(2x-1)^3 + (3x+11)^4(x+5)^4(2x-1)^3$$

$$(3x+11)^4(x+5)^2(2x-1)^3 \left[ 3x+11 + (x+5)^2 \right]$$

$$(3x+11)^4(x+5)^2(2x-1)^3 (x^2+13x+36)$$

$$(3x+11)^4(x+5)^2(2x-1)^3(x+4)(x+9)$$

$$4. \frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)}$$

$$(x+1)^2 \frac{4x^2-5x-9-16x-9}{(x+1)(4x-9) - (16x+9)}$$

$$\frac{4x^2-21x-18}{(x-6)(x+1)}$$

$$\frac{(x+1)[4x^2-21x-18]}{x-6}$$

$$\frac{(x+1)(4x+3)(x-6)}{(x-6)}$$

$$(x+1)(4x+3) \quad x \neq 6, -1$$

$$8. \frac{3x(x+2)^3 + 9x^2(x+2)^2}{12x^2 + 30x + 12}$$

$$6(2x^2+5x+2)$$

$$3x(x+2)^2 \frac{4x+2}{x+2+3x}$$

$$6(2x+1)(x+2)$$

$$\frac{3x(x+2)^2 \cdot 2(2x+1)}{6(2x+1)(x+2)}$$

$$\frac{x(x+2)^2}{(x+2)} \quad x \neq -\frac{1}{2}, -2$$

$$x(x+2)$$