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PCH – Intro to Complex Rational Expressions

Do Now:

1. Factor completely: $(x^2+2)^{\frac{5}{2}} + 2x(x^2+2)^{\frac{3}{2}} + x^2\sqrt{x^2+2}$

$y^2+2xy+x^2$

$(y+x)^2$

$(x^2+2)^{\frac{1}{2}}\left((x^2+2)^2 + 2x(x^2+2) + x^2\right)$

$(x^2+2)^{\frac{1}{2}}\left(x^2+2+x\right)^2$

$(x^2+2)^{\frac{1}{2}}(x^2+x+2)^2$

2. Simplify: $\dfrac{p}{2p^2+3pq-2q^2} + \dfrac{-3q}{4p^2-4pq+q^2}$

$(2p-q)$ $(p+2q)$

$(2p-q)(2p-q)(p+2q)$ $\quad (2p-q)^2(p+2q)$

$\dfrac{2p^2-pq-3pq-6q^2}{(2p-q)^2(p+2q)}$

$\dfrac{2p^2-4pq-6q^2}{(2p-q)^2(p+2q)}$

$\dfrac{2(p^2-2pq-3q^2)}{(2p-q)^2(p+2q)}$ $\quad p \neq \frac{q}{2}, -2q$

Examples:

1. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}}$ $\quad xy(x+y)$ $\quad xy(x+y)$

$x, y \neq 0$

$x \neq -y$

$\dfrac{y(x+y)+x(x+y)}{xy}$

$\dfrac{(y+x)(x+y)}{xy}$

2. $\dfrac{x^{-2}-y^{-2}}{x^{-2}+2(xy)^{-1}+y^{-2}}$ $= \dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}}$ $\cdot \dfrac{x^2y^2}{x^2y^2}$

$x, y \neq 0$

$x \neq -y$

$\dfrac{y^2-x^2}{y^2+2xy+x^2} = \dfrac{(y-x)(y+x)}{(y+x)^2} = \dfrac{y-x}{y+x}$

Complex rational expressions are rational expressions that contain other rational expressions within them.

To simplify these types of problems:

If there are negative exponents used, rewrite as an equivalent expression with positive exponents.

Write restrictions for the denominators of the "mini" rational expressions. (I use the term "mini" to mean the rational expressions that are in the numerator and denominator of the "main" rational expression. I use the term "main" rational expression to mean the problem as a whole.)

Multiply each term of the main rational expression by the LCD of the mini rational expressions and simplify.

Now that the mini rational expressions are gone, factor the numerator and denominator of the main rational expression completely.

Write any additional restrictions you need for the denominator of the main rational expression.

Simplify once more if possible.

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PCH – More Work with Complex Fractions

Simplify each expression.

1. $\dfrac{\dfrac{1}{x+h-8}-\dfrac{1}{x-8}}{h}$

$(x+h-8)(x-8)$ $(x+h-8)(x-8)$ $(x+h-8)(x-8)$

$h \neq 0$

$x \neq -h+8, 8$

$$\frac{x-8-(x+h-8)}{h(x+h-8)(x-8)} = \frac{x-8-x-h+8}{h(x+h-8)(x-8)} = \frac{-h}{h(x+h-8)(x-8)}$$

$$= \frac{-1}{(x+h-8)(x-8)}$$

2. $\dfrac{\dfrac{1}{2(x+h)+5}-\dfrac{1}{2x+5}}{h}$

3. $\dfrac{\dfrac{x+h-1}{x+h-2}-\dfrac{x-1}{x-2}}{h}$

$(x+h-2)(x-2)$ $(x+h-2)(x-2)$ $(x+h-2)(x-2)$

$x \neq -h+2, 2$

$h \neq 0$

$$\frac{(x+h-1)(x-2)-(x-1)(x+h-2)}{h(x+h-2)(x-2)}$$

$$\frac{x^2+xh-x-2x-2h+2-(x^2+xh-2x-x-h+2)}{h(x+h-2)(x-2)} = \frac{-2h-(-h)}{h(x+h-2)(x-2)} = \frac{-h}{h(x+h-2)(x-2)} = \frac{-1}{(x+h-2)(x-2)}$$

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3. $\dfrac{x-3y}{x^2-y^2}+\dfrac{x-2y}{x^2+xy-2y^2}+\dfrac{-3y}{x^2+3xy+2y^2}$

$x \neq \pm y, -2y$

$\dfrac{x^2-xy-6y^2+x^2-xy-2y^2-3xy+3y^2}{(x-y)(x+y)(x+2y)}$

$\dfrac{2x^2-5xy-5y^2}{(x-y)(x+y)(x+2y)}$

4. $\dfrac{2h-t}{h^2-2ht-15t^2}+\dfrac{h+3t}{h^2-3ht-10t^2}+\dfrac{4h-t}{h^2+5ht+6t^2}$

$h \neq -2t, -3t, 5t$

$\dfrac{2h^2+3ht-2t^2+h^2+6ht+9t^2+4h^2-21ht+5t^2}{(h+3t)(h-5t)(h+2t)}$

$\dfrac{7h^2-12ht+12t^2}{(h+3t)(h-5t)(h+2t)}$

5. $\dfrac{3(3x+2)}{6x^2-x-1}+\dfrac{28(x+3)}{3x^2+7x+2}+\dfrac{-7(x-2)}{2x^2+3x-2}$

$(3x+1)(2x-1)$ $(3x+1)(x+2)$ $(2x-1)(x+2)$

$$\frac{9x^2+24x+12+56x^2+140x-84-21x^2+35x+14}{(3x+1)(2x-1)(x+2)}$$

$$\frac{44x^2+199x-58}{(3x+1)(2x-1)(x+2)}$$

$x \neq -\frac{1}{3}, \frac{1}{2}, -2$

6. $\dfrac{15(a+4)}{a^2+a-2}-\dfrac{36(a-4)}{a^2-4a+3}+\dfrac{50(a+1)}{a^2-a-6}$

$(a+2)(a-1)$ $(a-1)(a-3)$ $(a-3)(a+2)$

a^2+a-12 a^2-2a-8 a^2-1

$$\frac{15(a+4)(a-3)-36(a-4)(a+2)+50(a+1)(a-1)}{(a+2)(a-1)(a-3)}$$

$a \neq -2, 1, 3$

✓ ✓

$$\frac{15a^2+15a-180-36a^2+72a+288+50a^2-50}{(a+2)(a-1)(a-3)}$$

$$\frac{29a^2+87a+58}{(a+2)(a-1)(a-3)}$$

$(29a+29)(a+2)$

$$\frac{29a+29}{(a-1)(a-3)}$$

7. $\dfrac{7x^3y-7x^2y+7xy}{x^3+1}-\dfrac{(x-1)^2-49x^2y^2}{x^2-1-7x^2y-7xy}$

$(x-1)(x+1)-7xy(x+1)$

$(x+1)(x-1-7xy)$

$7x^3y-7x^2y+7xy$

$$\frac{7xy(x^2-x+1)}{(x+1)(x^2-x+1)}-\frac{(x-1-7xy)(x-1+7xy)}{(x+1)(x-1-7xy)}$$

$x \neq -1, 1+7xy$

$x^2-x+1 \neq 0$

$$\frac{7xy-x+1-7xy}{x+1}=\frac{-(x-1)}{x+1}$$

$$\frac{2xy}{x+1} - \frac{(x-1+2xy)}{x+1} = \frac{2xy - x + 1 - 2xy}{x+1}$$