Do Now: #2 from the More Work with Complex Fractions sheet

2. $$\frac{\dfrac{1}{2(x+h)+5}+\dfrac{-1}{2x+5}}{h}$$

$(2x+2h+5)(2x+5)$ $(2x+2h+5)(2x+5)$

$(2x+2h+5)(2x+5)$

$h \neq 0$

$x \neq -\frac{5}{2}, \frac{5-2h}{2}$

$$\frac{\cancel{2x}+\cancel{5}-\cancel{2x}-2h-\cancel{5}}{h(2x+2h+5)(2x+5)}$$

$$\frac{-2\cancel{h}}{\cancel{h}(2x+2h+5)(2x+5)}$$

$$\frac{-2}{(2x+2h+5)(2x+5)}$$

$(5-x-h)$ $(3+x)(3+\cancel{(x+h)})$ $(3+x+h)$ $(\cancel{3+x})(3+(x+h))$

4. $$\frac{\dfrac{5-(x+h)}{3+(\cancel{x}+h)}-\dfrac{5-x}{3\cancel{+}x}}{h} \quad (3+x)(3+(x+h))$$

$h \neq 0$

$x \neq -3, -h-3$

$$\frac{\cancel{15}+\cancel{5x}-\cancel{3x}-\cancel{x^2}-3h-\cancel{xh}-(\cancel{15}+\cancel{5x}+5h-\cancel{3x}-\cancel{x^2}-\cancel{xh})}{h(3+x)(3+x+h)} = \frac{-8\cancel{h}}{\cancel{h}(3+x)(3+x+h)}$$

$$\frac{-8}{(3+x)(3+x+h)}$$

5. $\dfrac{(x+h)^{-2}-x^{-2}}{h}$

$x \neq 0, -h$

$h \neq 0$

$$\frac{\frac{1}{\cancel{(x+h)^2}}\cancel{(x+h)^2}x^2 - \frac{1}{\cancel{x^2}}(x+h)^2\cancel{x^2}}{h(x+h)^2x^2}$$

$$\frac{x^2-(x+h)^2}{h(x+h)^2x^2}$$

$$\frac{\cancel{x^2}-(\cancel{x^2}+2xh+h^2)}{h(x+h)^2x^2}$$

$$\frac{-2xh-h^2}{h(x+h)^2x^2}$$

$$\frac{-\cancel{h}(2x+h)}{\cancel{h}(x+h)^2x^2} = \frac{-2x-h}{(x+h)^2x^2}$$

6. $\dfrac{(x+h)^{-3}-x^{-3}}{h}$

$x \neq 0, -h$
$h \neq 0$

$$\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}$$

$$\frac{x^3-(x+h)^3}{x^3h(x+h)^3} = \frac{\cancel{x^3}-(\cancel{x^3}+3x^2h+3xh^2+h^3)}{x^3h(x+h)^3}$$

$$\frac{-3x^2h-3xh^2-h^3}{x^3h(x+h)^3} = \frac{\cancel{h}(-3x^2-3xh-h^2)}{x^3\cancel{h}(x+h)^3} = \frac{-1(3x^2+3xh+h^2)}{x^3(x+h)^3}$$

Homework 09-27

51. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}} = \dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{y^2-x^2}{x^2y^2}} = \dfrac{x^2-y^2}{xy}\cdot\dfrac{x^2y^2}{y^2-x^2} = \dfrac{xy}{-1} = -xy$. An alternative method is to multiply the numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2:

$x, y \neq 0$

$x \neq \pm y$

$$\frac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}} = \frac{\left(\dfrac{x}{y}-\dfrac{y}{x}\right)}{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}\cdot\frac{x^2y^2}{x^2y^2} = \frac{x^3y-xy^3}{y^2-x^2} = \frac{xy\left(x^2-y^2\right)}{y^2-x^2} = -xy.$$

52. $x - \dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}} = x - \dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}}\cdot\dfrac{xy}{xy} = x - \dfrac{xy^2}{x^2+y^2} = \dfrac{x\left(x^2+y^2\right)}{x^2+y^2} - \dfrac{xy^2}{x^2+y^2} = \dfrac{x^3+xy^2-xy^2}{x^2+y^2} = \dfrac{x^3}{x^2+y^2}$

$x, y \neq 0$

53. $\dfrac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}} = \dfrac{\dfrac{c-1}{c-1}+\dfrac{1}{c-1}}{\dfrac{c-1}{c-1}+\dfrac{-1}{c-1}} = \dfrac{\dfrac{c}{c-1}}{\dfrac{c-2}{c-1}} = \dfrac{c}{c-1}\cdot\dfrac{c-1}{c-2} = \dfrac{c}{c-2}$. Using the alternative method, we obtain

$c \neq 1, 2$

$$\frac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}} = \frac{\left(1+\dfrac{1}{c-1}\right)}{\left(1-\dfrac{1}{c-1}\right)}\cdot\frac{c-1}{c-1} = \frac{c-1+1}{c-1-1} = \frac{c}{c-2}.$$

54. $1+\dfrac{1}{1+\dfrac{1}{1+x}} = 1+\dfrac{1}{1+\dfrac{1}{1+x}}\cdot\dfrac{1+x}{1+x} = 1+\dfrac{1+x}{1+x+1} = 1+\dfrac{1+x}{2+x} = \dfrac{2+x}{2+x}+\dfrac{1+x}{2+x} = \dfrac{3+2x}{2+x}$

$x \neq -1, -2$

55. $$\frac{\dfrac{5}{x-1}-\dfrac{2}{x+1}}{\dfrac{x}{x-1}+\dfrac{1}{x+1}} = \frac{\dfrac{5(x+1)}{(x-1)(x+1)}+\dfrac{-2(x-1)}{(x-1)(x+1)}}{\dfrac{x(x+1)}{(x-1)(x+1)}+\dfrac{x-1}{(x-1)(x+1)}} = \frac{\dfrac{5x+5-2x+2}{(x-1)(x+1)}}{\dfrac{x^2+x+x-1}{(x-1)(x+1)}}$$

$$= \frac{3x+7}{(x-1)(x+1)}\cdot\frac{(x-1)(x+1)}{x^2+2x-1} = \frac{3x+7}{x^2+2x-1}$$

$x \neq \pm 1$

$x^2+2x-1 \neq 0$

Alternatively,

$$\frac{\dfrac{5}{x-1}-\dfrac{2}{x+1}}{\dfrac{x}{x-1}+\dfrac{1}{x+1}} = \frac{\left(\dfrac{5}{x-1}-\dfrac{2}{x+1}\right)}{\left(\dfrac{x}{x-1}+\dfrac{1}{x+1}\right)}\cdot\frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{5(x+1)-2(x-1)}{x(x+1)+(x-1)}$$

$$= \frac{5x+5-2x+2}{x^2+x+x-1} = \frac{3x+7}{x^2+2x-1}$$

56. $$\frac{\dfrac{a-b}{a}-\dfrac{a+b}{b}}{\dfrac{a-b}{b}+\dfrac{a+b}{a}} = \frac{\dfrac{a-b}{a}-\dfrac{a+b}{b}}{\dfrac{a-b}{b}+\dfrac{a+b}{a}}\cdot\frac{ab}{ab} = \frac{(a-b)b-(a+b)a}{(a-b)a+(a+b)b} = \frac{ab-b^2-a^2-ab}{a^2-ab+ab+b^2}$$

$a, b \neq 0$

$$= \frac{-a^2-b^2}{a^2+b^2} = \frac{-\left(a^2+b^2\right)}{a^2+b^2} = -1$$

57. $$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \frac{\dfrac{1}{\cancel{x^2}}-\dfrac{1}{\cancel{y^2}}}{\dfrac{1}{\cancel{x}}+\dfrac{1}{\cancel{y}}} = \frac{\dfrac{y^2}{x^2y^2}-\dfrac{x^2}{x^2y^2}}{\dfrac{y}{xy}+\dfrac{x}{xy}} = \frac{y^2-x^2}{x^2y^2}\cdot\frac{xy}{y+x} = \frac{(y-x)(y+x)xy}{x^2y^2(y+x)} = \frac{y-x}{xy}$$

x^2y^2 x^2y^2 x^2y^2 x^2y^2

$x, y \neq 0$

$y \neq -x$

Alternatively, $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}{\left(\dfrac{1}{x}+\dfrac{1}{y}\right)}\cdot\dfrac{x^2y^2}{x^2y^2} = \dfrac{y^2-x^2}{xy^2+x^2y} = \dfrac{(y-x)(y+x)}{xy(y+x)} = \dfrac{y-x}{xy}.$

$$\frac{y^2-x^2}{xy^2+x^2y} = \frac{(y-x)\cancel{(y+x)}}{xy\cancel{(y+x)}}$$

58. $\frac{x^{-1}+y^{-1}}{(x+y)^{-1}} = \frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}} = \frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}} \cdot \frac{xy(x+y)}{xy(x+y)} = \frac{y(x+y)+x(x+y)}{xy}$

$= \frac{xy+y^2+x^2+xy}{xy} = \frac{x^2+2xy+y^2}{xy} = \frac{(x+y)^2}{xy}$

$xy(x+y)$ $xy(x+y)$ $xy(x+y)$

$x, y \neq 0$

$x \neq -y$

$$\frac{y(x+y)+x(x+y)}{xy} = \frac{(x+y)(y+x)}{xy}$$

$x \neq -1, -2$

$(1+x)\,1+\frac{1}{\cancel{1+x}}(1+x)$, $1\,(1+x)$

54. $1+\frac{1}{1+\frac{1}{1+x}}$

$$\frac{1+x}{1+x+1} = \frac{x+1}{x+2}$$

$$1+\frac{x+1}{x+2}$$

$$\frac{x+2+x+1}{x+2} = \frac{2x+3}{x+2}, \quad x \neq -1, -2$$

$\frac{y\ xy}{xy\frac{x}{\cancel{y}}+\frac{y}{\cancel{x}}xy}$

52. $x - \frac{y}{\frac{x}{y}+\frac{y}{x}}$

$x, y \neq 0$

$$\frac{x}{1} - \frac{xy^2}{x^2+y^2}$$

$$\frac{x(x^2+y^2)-xy^2}{x^2+y^2}$$

$$\frac{x^3+\cancel{xy^2}-\cancel{xy^2}}{x^2+y^2} = \frac{x^3}{x^2+y^2}$$