Do Now: Try #1 a, b and c

Name: ______________________________ Date: __________

PCH – Evaluating Functions Ms. Loughran

1. Given $f(x) = x^2 + x - 4$

(a) Find $f(3)$. $= 3^2 + 3 - 4 = 8$

(b) Find $f(3+h)$ $= (3+h)^2 + (3+h) - 4$

$= 9 + 6h + h^2 + 3 + h - 4$

$= h^2 + 7h + 8$

(c) Find $\dfrac{f(3+h) - f(3)}{h}$

$\dfrac{h^2 + 7h \cancel{+8} \cancel{-8}}{h} = \dfrac{\cancel{h}(h+7)}{\cancel{h}} = h+7$

(d) Find $f(x+h)$ $= (x+h)^2 + x + h - 4$

$x^2 + 2hx + h^2 + x + h - 4$

(e) Find $f(x+h) - f(x)$

$x^2 + 2hx + h^2 + x + h - 4 - (x^2 + x - 4)$

$\cancel{x^2} + 2hx + h^2 \cancel{+x} + h \cancel{-4} \cancel{-x^2} \cancel{-x} \cancel{+4}$

$2hx + h^2 + h$

(f) Find $\dfrac{f(x+h) - f(x)}{h}$ ← Difference Quotient

$\dfrac{2hx + h^2 + h}{h}$

$\dfrac{\cancel{h}(2x + h + 1)}{\cancel{h}} = 2x + h + 1$

2. Given $f(x)=\begin{cases} 4x & if \quad -2\le x<1 \\ x+2 & if \quad 1\le x\le 4 \\ 10 & if \quad 4<x<5 \end{cases}$

Find :

(a) $f(2)$ $= 2+2=4$

(b) $f(1)$ $= 1+2=3$

(c) $f(0)$ $= 4(0)=0$

(d) $f(4.3)$ $= 10$

(e) $f(5)$ undefined or not defined

3. Let $f(x)=\dfrac{1+x}{x}$.

Find:

(a) $f(-x)$ $= \dfrac{1-x}{-x} \Rightarrow -\left(\dfrac{1-x}{x}\right) = -\left(\dfrac{-(x-1)}{x}\right) = \dfrac{x-1}{x}$

(b) $f\left(-\dfrac{1}{x}\right)$ $= \dfrac{1-\frac{1}{x}}{-\frac{1}{x}} \cdot \dfrac{x}{x} \Rightarrow \dfrac{x-1}{-1} = -x+1$

(c) $\dfrac{1}{x}\cdot f\left(\dfrac{1}{x}\right)$ $= \dfrac{1}{x}\cdot\left(\dfrac{1+\frac{1}{x}}{\frac{1}{x}}\right) \Rightarrow \dfrac{1}{x}\left[\dfrac{x+1}{1}\right] = \dfrac{x+1}{x}$

(d) Determine which, if any, from parts a – c are equivalent to $f(x)$

(c)

Multiple Choice:

4. If $g(x) = ax + b$, then $\dfrac{g(b) - g(a)}{b - a} =$ $\qquad \dfrac{ab+\cancel{b}-(a^2+\cancel{b})}{b-a} = \dfrac{-a^2+ab}{b-a}$

(A) a (B) b (C) x (D) ax (E) $ax + b$

$$= \frac{-a(\cancel{a-b})^{-1}}{\cancel{b-a}}$$

5. If $f(x) = 5^x$, then $\dfrac{f(a)}{f(b)} =$ $\qquad \dfrac{5^a}{5^b} = 5^{a-b}$

(A) $f(a+b)$ (B) $f\left(\dfrac{a}{b}\right)$ (C) $f(ab)$ (D) $f(a-b)$

6. If $g(x) = 3^x$, then $g(x+1) - g(x) =$ $\qquad 3^{x+1} - 3^x = 3^x \cdot 3 - 3^x$

(A) $g(x)$ (B) $2g(x)$ (C) $3g(x)$ (D) 0

$3^x(3-1)$

$3^x(2)$

$g(x)\cdot 2$

More Practice:

7. Given $f(x) = -2x^2 - 3x + 1$

(a) Find $f(2)$.

(b) Find $f(2+h)$

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13. $f(1) = 2(1) + 1 = 3$; $f(-2) = 2(-2) + 1 = -3$; $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$; $f(a) = 2(a) + 1 = 2a + 1$; $f(-a) = 2(-a) + 1 = -2a + 1$; $f(a+b) = 2(a+b) + 1 = 2a + 2b + 1$.

14. $f(0) = 0^2 + 2(0) = 0$; $f(3) = 3^2 + 2(3) = 9 + 6 = 15$; $f(-3) = (-3)^2 + 2(-3) = 9 - 6 = 3$; $f(a) = a^2 + 2(a) = a^2 + 2a$; $f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$; $f\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)^2 + 2\left(\frac{1}{a}\right) = \frac{1}{a^2} + \frac{2}{a}$.

15. $g(2) = \frac{1-(2)}{1+(2)} = \frac{-1}{3} = -\frac{1}{3}$; $g(-2) = \frac{1-(-2)}{1+(-2)} = \frac{3}{-1} = -3$; $g\left(\frac{1}{2}\right) = \frac{1-\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$; $g(a) = \frac{1-(a)}{1+(a)} = \frac{1-a}{1+a}$; $g(a-1) = \frac{1-(a-1)}{1+(a-1)} = \frac{1-a+1}{1+a-1} = \frac{2-a}{a}$; $g(-1) = \frac{1-(-1)}{1+(-1)} = \frac{2}{0}$, so $g(-1)$ is not defined.

17. $f(0) = 2(0)^2 + 3(0) - 4 = -4$; $f(2) = 2(2)^2 + 3(2) - 4 = 8 + 6 - 4 = 10$; $f(-2) = 2(-2)^2 + 3(-2) - 4 = 8 - 6 - 4 = -2$; $f(\sqrt{2}) = 2(\sqrt{2})^2 + 3(\sqrt{2}) - 4 = 4 + 3\sqrt{2} - 4 = 3\sqrt{2}$; $f(x+1) = 2(x+1)^2 + 3(x+1) - 4 = 2x^2 + 4x + 2 + 3x + 3 - 4 = 2x^2 + 7x + 1$; $f(-x) = 2(-x)^2 + 3(-x) - 4 = 2x^2 - 3x - 4$.

23. Since $-4 \le -1$, we have $f(-4) = (-4)^2 + 2(-4) = 16 - 8 = 8$. Since $-\frac{3}{2} \le -1$, we have $f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 2\left(-\frac{3}{2}\right) = \frac{9}{4} - 3 = -\frac{3}{4}$. Since $-1 \le -1$, we have $f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$. Since $-1 < 0 \le 1$, we have $f(0) = 0$. Since $25 > 1$, we have $f(25) = -1$.

24. Since $-5 < 0$, we have $f(-5) = 3(-5) = -15$. Since $0 \le 0 \le 2$, we have $f(0) = 0 + 1 = 1$. Since $0 \le 1 \le 2$, we have $f(1) = 1 + 1 = 2$. Since $0 \le 2 \le 2$, we have $f(2) = 2 + 1 = 3$. Since $5 > 2$, we have $f(5) = (5-2)^2 = 9$.

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23. **(a)** $h(-2) = 1$; $h(0) = -1$; $h(2) = 3$; $h(3) = 4$.

(b) Domain: $[-3, 4]$. Range: $[-1, 4]$.

24. **(a)** $g(-4) = 3$; $g(-2) = 2$; $g(0) = -2$; $g(2) = 1$; $g(4) = 0$.

(b) Domain: $[-4, 4]$. Range: $[-2, 3]$.

25. **(a)** $f(0) = 3 > \frac{1}{2} = g(0)$. So $f(0)$ is larger.

(b) $f(-3) \approx -\frac{3}{2} < 2 = g(-3)$. So $g(-3)$ is larger.

(c) For $x = -2$ and $x = 2$.

53. $f(x) = \begin{cases} -2 & \text{if } x < -2 \\ x & \text{if } -2 \le x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$

54. $f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ 1 - x & \text{if } -1 < x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$

55. The curves in parts (a) and (c) are graphs of a function of x, by the Vertical Line Test.

56. The curves in parts (b) and (c) are graphs of functions of x.

57. The given curve is the graph of a function of x. Domain: $[-3, 2]$. Range: $[-2, 2]$.

58. No, the given curve is not the graph of a function of x.

59. No, the given curve is not the graph of a function of x, by the Vertical Line Test.

60. The given curve is the graph of a function of x. Domain: $[-3, 2]$. Range: $\{-2\} \cup (0, 3]$.

2.1 Exercises

1–4 ■ Express the rule in function notation. (For example, the rule "square, then subtract 5" is expressed as the function $f(x) = x^2 - 5$.)

1. Add 3, then multiply by 2

2. Divide by 7, then subtract 4

3. Subtract 5, then square

4. Take the square root, add 8, then multiply by $\frac{1}{3}$

5–8 ■ Express the function (or rule) in words.

5. $f(x) = \dfrac{x - 4}{3}$

6. $g(x) = \dfrac{x}{3} - 4$

7. $h(x) = x^2 + 2$

8. $k(x) = \sqrt{x + 2}$

9–10 ■ Draw a machine diagram for the function.

9. $f(x) = \sqrt{x - 1}$

10. $f(x) = \dfrac{3}{x - 2}$

11–12 ■ Complete the table.

11. $f(x) = 2(x - 1)^2$

x	$f(x)$
-1	
0	
1	
2	
3	

12. $g(x) = \lvert 2x + 3 \rvert$

x	$g(x)$
-3	
-2	
0	
1	
3	

13–20 ■ Evaluate the function at the indicated values.

13. $f(x) = 2x + 1$;

$f(1), f(-2), f(\frac{1}{2}), f(a), f(-a), f(a + b)$

14. $f(x) = x^2 + 2x$;

$f(0), f(3), f(-3), f(a), f(-x), f\left(\dfrac{1}{a}\right)$

15. $g(x) = \dfrac{1 - x}{1 + x}$;

$g(2), g(-2), g(\frac{1}{2}), g(a), g(a - 1), g(-1)$

16. $h(t) = t + \dfrac{1}{t}$;

$h(1), h(-1), h(2), h(\frac{1}{2}), h(x), h\left(\dfrac{1}{x}\right)$

17. $f(x) = 2x^2 + 3x - 4$;

$f(\sqrt{2}) = 2(\sqrt{2})^2 + 3(\sqrt{2}) - 4$

$\cancel{4} + 3\sqrt{2} \cancel{-4}$

$f(0), f(2), f(-2), f(\sqrt{2}), f(x + 1), f(-x)$

18. $f(x) = x^3 - 4x^2$;

$f(0), f(1), f(-1), f(\frac{3}{2}), f\left(\dfrac{x}{2}\right), f(x^2)$

19. $f(x) = 2\lvert x - 1 \rvert$;

$f(-2), f(0), f(\frac{1}{2}), f(2), f(x + 1), f(x^2 + 2)$

20. $f(x) = \dfrac{\lvert x \rvert}{x}$;

$f(-2), f(-1), f(0), f(5), f(x^2), f\left(\dfrac{1}{x}\right)$

21–24 ■ Evaluate the piecewise defined function at the indicated values.

21. $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$

$f(-2), f(-1), f(0), f(1), f(2)$

22. $f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$

$f(-3), f(0), f(2), f(3), f(5)$

23. $f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq -1 \\ x & \text{if } -1 < x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$

$f(-4), f(-\frac{3}{2}), f(-1), f(0), f(25)$

24. $f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x + 1 & \text{if } 0 \leq x \leq 2 \\ (x - 2)^2 & \text{if } x > 2 \end{cases}$

$f(-5), f(0), f(1), f(2), f(5)$

25–28 ■ Use the function to evaluate the indicated expressions and simplify.

25. $f(x) = x^2 + 1$; $f(x + 2), f(x) + f(2)$

26. $f(x) = 3x - 1$; $f(2x), 2f(x)$

27. $f(x) = x + 4$; $f(x^2), (f(x))^2$

28. $f(x) = 6x - 18$; $f\left(\dfrac{x}{3}\right), \dfrac{f(x)}{3}$

29–36 ■ Find $f(a)$, $f(a + h)$, and the difference quotient $\dfrac{f(a + h) - f(a)}{h}$, where $h \neq 0$.

29. $f(x) = 3x + 2$

30. $f(x) = x^2 + 1$

38. $f(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x + 1 & \text{if } x > 1 \end{cases}$

39. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \ge 2 \end{cases}$

40. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \ge -2 \end{cases}$

41. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

42. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \ge -1 \end{cases}$

43. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$

44. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$

45. $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

46. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$

47. $f(x) = \begin{cases} 0 & \text{if } |x| \le 2 \\ 3 & \text{if } |x| > 2 \end{cases}$

48. $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

49. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x + 6 & \text{if } x > 2 \end{cases}$

50. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ 9 - x^2 & \text{if } 0 < x \le 3 \\ x - 3 & \text{if } x > 3 \end{cases}$

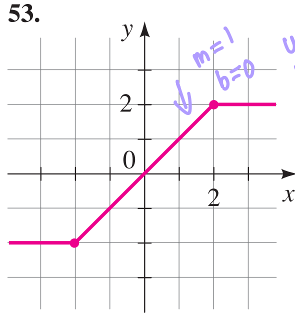
51–52 ■ Use a graphing device to draw the graph of the piecewise defined function. (See the margin note on page 162.)

51. $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

52. $f(x) = \begin{cases} 2x - x^2 & \text{if } x > 1 \\ (x - 1)^3 & \text{if } x \le 1 \end{cases}$

53–54 ■ The graph of a piecewise defined function is given. Find a formula for the function in the indicated form.

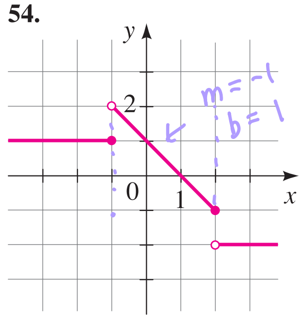
53.



y=mx+b
x+0

$f(x) = \begin{cases} -2 & \text{if } x < -2 \\ x & \text{if } -2 \le x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$

54.

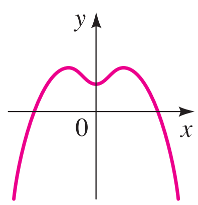


y=mx+b
-x+1

$f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ -x+1 & \text{if } -1 < x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$

55–56 ■ Determine whether the curve is the graph of a function of x.

55. (a)



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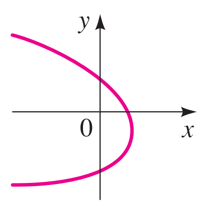
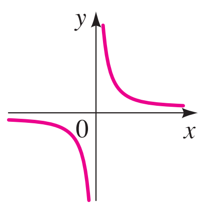
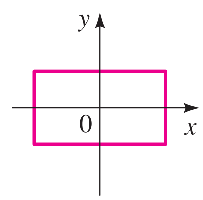


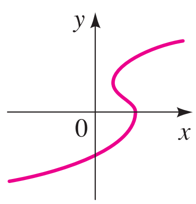
(c)



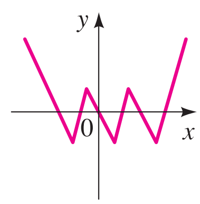
(d)



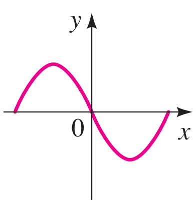
56. (a)



(b)



(c)



(d)