Name: _____ Date: _____

PCH – Nonlinear Inequalities Ms. Loughran

Do Now:

1. Find the solution set and state it:
   a) Graphically
   b) In set builder notation
   c) In interval notation

$$\frac{4}{x+5} > \frac{1}{2x+3}$$

$\frac{4}{x+5} - \frac{1}{2x+3} > 0$

$\frac{4(2x+3) - 1(x+5)}{(x+5)(2x+3)} > 0$ ($8x+12$, $-x-5$)

$\frac{7x+7}{(x+5)(2x+3)} > 0$ ($7(x+1)$; $+$, $+$, $+$)

b) SB: $\{x \mid -5 < x < -\frac{3}{2} \text{ V } x > -1\}$

c) IN: $(-5, -\frac{3}{2}) \cup (1, \infty)$

Classwork

For each example, find the solution set and state it

a) Graphically
b) In set builder notation
c) In interval notation

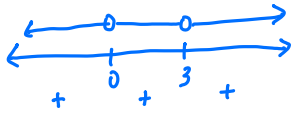
1. $x^2\left(x^2+9\right) > 6x^3$

$x^4 + 9x^2 > 6x^3$

$x^4 - 6x^3 + 9x^2 > 0$

$x^2(x^2 - 6x + 9) > 0$

$x^2(x-3)^2 > 0$



IN: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

SB: $\{x \mid x < 0 \text{ V } 0 < x < 3 \text{ V } x > 3\}$

or

$\{x \mid x \in \mathbb{R}, x \neq 0, 3\}$

real #'s ← $\in$ is an element of

$\{x \mid x \in \mathbb{R} / 0, 3\}$

2. $100 - 29x^2 \leq -x^4$

3. $15-2x^2>7x$

4. $\dfrac{25x^2-30x+9}{2x^2+3x-35}\geq 0$

5. $\dfrac{x+3}{4-x}\leq 3$

6. $\dfrac{9x-45}{x^2+6x}<\dfrac{1}{x}-1$

$\dfrac{9x-45}{x^2+6x} - \dfrac{1}{x} + 1 < 0$

$x(x+6)$

$\dfrac{9x-45-x-6+x^2+6x}{x(x+6)} < 0$

$\dfrac{x^2+14x-51}{x(x+6)} < 0$

$\dfrac{(x+17)(x-3)}{x(x+6)} < 0$

+ −17 − −6 + 0 − 3 +

a)

b) $\{x \mid -17<x<-6 \text{ or } 0<x<3\}$

c) $(-17,-6)\cup(0,3)$

(16) $16x \le x^3$

$-x^3 + 16x \le 0$

$x^3 - 16x \ge 0$

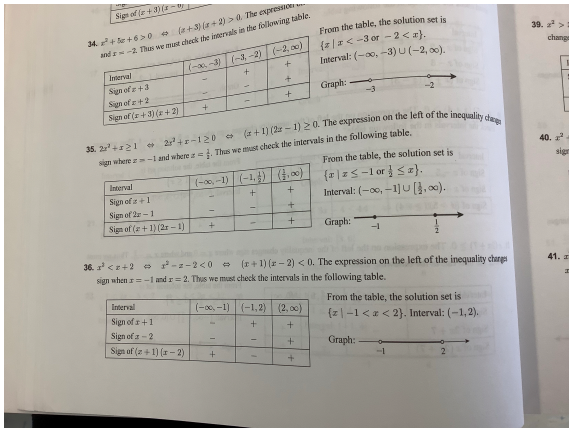
$x(x^2 - 16) \ge 0$

$x(x-4)(x+4) \ge 0$

SB: $\{x \mid -4 \le x \le 0 \text{ or } x \ge 4\}$

IN: $[-4, 0] \cup [4, \infty)$

Homework 10-05



34. $x^2 + 5x + 6 > 0 \Leftrightarrow (x+3)(x+2) > 0$. The expression on the left of the inequality changes sign where $x = -3$ and where $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, \infty)$
Sign of $x+3$	$-$	$+$	$+$
Sign of $x+2$	$-$	$-$	$+$
Sign of $(x+3)(x+2)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid x < -3 \text{ or } -2 < x\}$. Interval: $(-\infty, -3) \cup (-2, \infty)$.

Graph: −3, −2

35. $2x^2 + x \ge 1 \Leftrightarrow 2x^2 + x - 1 \ge 0 \Leftrightarrow (x+1)(2x-1) \ge 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = \frac{1}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
Sign of $x+1$	$-$	$+$	$+$
Sign of $2x-1$	$-$	$-$	$+$
Sign of $(x+1)(2x-1)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid x \le -1 \text{ or } \frac{1}{2} \le x\}$. Interval: $(-\infty, -1] \cup [\frac{1}{2}, \infty)$.

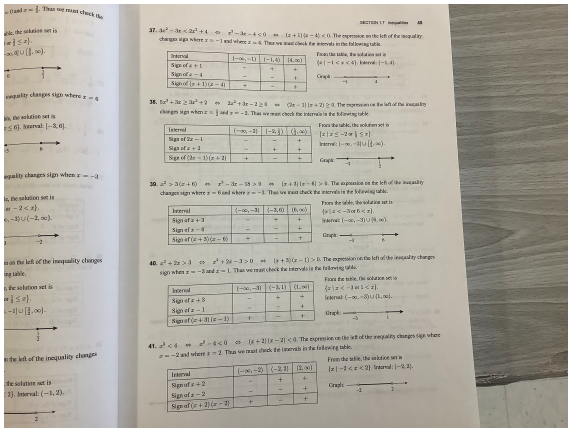
Graph: −1, $\frac{1}{2}$

36. $x^2 < x + 2 \Leftrightarrow x^2 - x - 2 < 0 \Leftrightarrow (x+1)(x-2) < 0$. The expression on the left of the inequality changes sign when $x = -1$ and $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 2)$	$(2, \infty)$
Sign of $x+1$	$-$	$+$	$+$
Sign of $x-2$	$-$	$-$	$+$
Sign of $(x+1)(x-2)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid -1 < x < 2\}$. Interval: $(-1, 2)$.

Graph: −1, 2



37. $3x^2 - 3x < 2x^2 + 4 \Leftrightarrow x^2 - 3x - 4 < 0 \Leftrightarrow (x+1)(x-4) < 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign of $x+1$	$-$	$+$	$+$
Sign of $x-4$	$-$	$-$	$+$
Sign of $(x+1)(x-4)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid -1 < x < 4\}$. Interval: $(-1, 4)$.

Graph: −1, 4

38. $5x^2 + 3x \ge 2x^2 + 2 \Leftrightarrow 2x^2 + 3x - 2 \ge 0 \Leftrightarrow (2x-1)(x+2) \ge 0$. The expression on the left of the inequality changes sign where $x = \frac{1}{2}$ and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
Sign of $2x-1$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$
Sign of $(2x-1)(x+2)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid x \le -2 \text{ or } \frac{1}{2} \le x\}$. Interval: $(-\infty, -2] \cup [\frac{1}{2}, \infty)$.

Graph: −2, $\frac{1}{2}$

39. $x^2 > 3(x+6) \Leftrightarrow x^2 - 3x - 18 > 0 \Leftrightarrow (x+3)(x-6) > 0$. The expression on the left of the inequality changes sign where $x = 6$ and where $x = -3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x+3$	$-$	$+$	$+$
Sign of $x-6$	$-$	$-$	$+$
Sign of $(x+3)(x-6)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid x < -3 \text{ or } 6 < x\}$. Interval: $(-\infty, -3) \cup (6, \infty)$.

Graph: −3, 6

40. $x^2 + 2x > 3 \Leftrightarrow x^2 + 2x - 3 > 0 \Leftrightarrow (x+3)(x-1) > 0$. The expression on the left of the inequality changes sign where $x = -3$ and $x = 1$. Thus we must check the intervals in the following table.

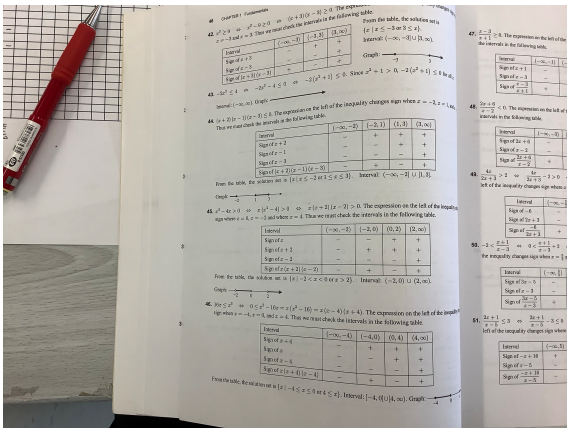
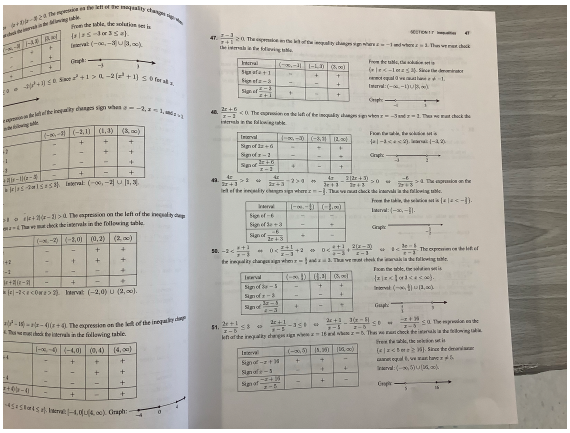
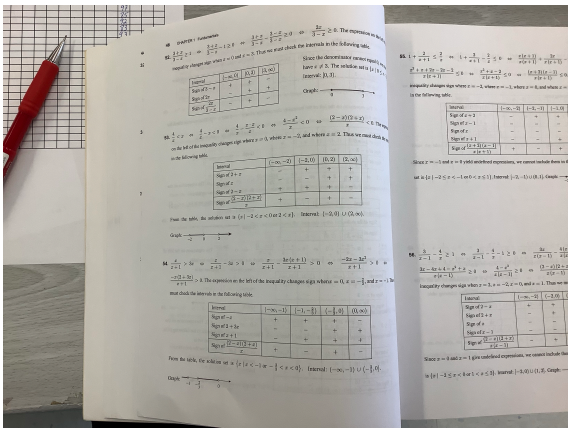
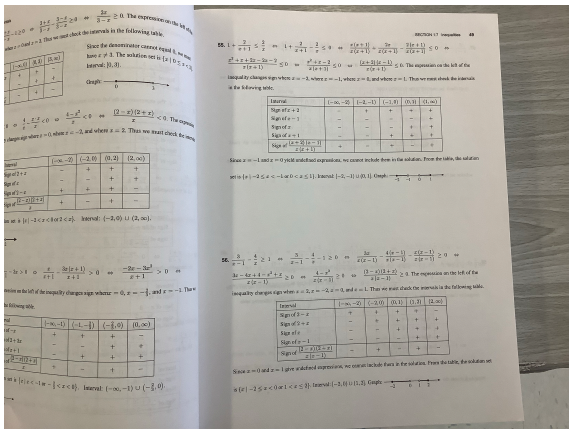
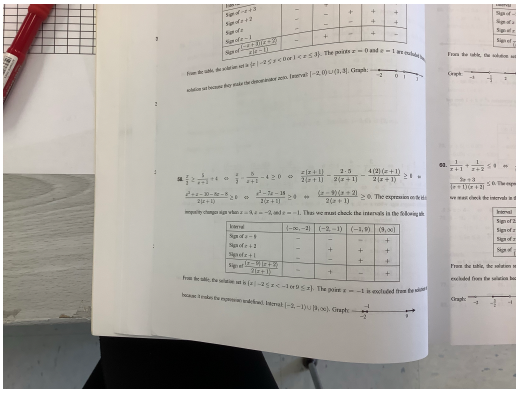
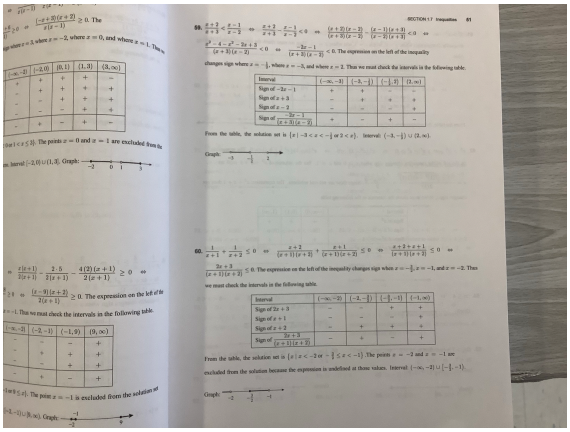
Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign of $x+3$	$-$	$+$	$+$
Sign of $x-1$	$-$	$-$	$+$
Sign of $(x+3)(x-1)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid x < -3 \text{ or } 1 < x\}$. Interval: $(-\infty, -3) \cup (1, \infty)$.

Graph: −3, 1

41. $x^2 < 4 \Leftrightarrow x^2 - 4 < 0 \Leftrightarrow (x+2)(x-2) < 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Sign of $x+2$	$-$	$+$	$+$
Sign of $x-2$	$-$	$-$	$+$
Sign of $(x+2)(x-2)$	$+$	$-$	$+$

From the table, the solution set is $\{x \mid -2 < x < 2\}$. Interval: $(-2, 2)$.

Graph: −2, 2