

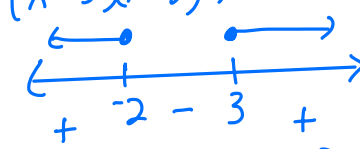
Do Now:

Find the domain of each of the following.

1. $y = 5 - x$
 $\{x | x \in \mathbb{R}\}$

2. $y = x^2 - x - 6$
 $\{x | x \in \mathbb{R}\}$

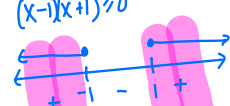

3. $y = \frac{1}{x^2 - x - 6}$
 $(x-3)(x+2)$
 $\{x | x \in \mathbb{R} / -2, 3\}$
 $\{x | x \in \mathbb{R}, x \neq -2, 3\}$
 $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

4. $y = \sqrt{x^2 - x - 6}$
 $x^2 - x - 6 \geq 0$
 $(x-3)(x+2) \geq 0$

 $\{x | x \leq -2 \vee x \geq 3\}$

5. $y = \frac{1}{\sqrt{x^2 - x - 6}}$
 $x^2 - x - 6 > 0$ see work #4

6. $y = \sqrt{12-x} - \frac{2x+1}{x-8}$
 $12-x \geq 0 \rightarrow x \leq 12$
 $-x \geq -12$
 $x \leq 12$
 $x \neq 8$
 $\{x | x \leq 12, x \neq 8\}$

$\{x | x < -2 \vee x > 3\}$

7. $y = \sqrt{x^2 - 1} + \sqrt{9 - x^2}$
 $x^2 - 1 \geq 0$
 $(x-1)(x+1) \geq 0$

 $9 - x^2 \geq 0$
 $(3-x)(3+x) \geq 0$

 $D: [-3, -1] \cup [1, 3]$
 $\{x | -3 \leq x \leq -1 \vee 1 \leq x \leq 3\}$

Domain of a composition of 2 functions:

$(f \circ g)(x)$ is defined whenever both $g(x)$ and $(f \circ g)(x)$ are defined.

$(g \circ f)(x)$ " " " " $f(x)$ and $(g \circ f)(x)$ " "

Examples:

$D_f \mathbb{R}$ $D_g \mathbb{R}$

1. Let $f(x) = x^2$ and $g(x) = x + 5$

(a) Find the function $f \circ g$ and state its domain.

$(f \circ g)(x)$
 $f(x+5) = (x+5)^2 \quad \mathbb{R}$

$D_{f \circ g} \{x | x \in \mathbb{R}\}$

(b) Find the function $g \circ f$ and state its domain.

$(g \circ f)(x)$
 $g(x^2) = x^2 + 5 \quad \mathbb{R}$

$D_{g \circ f} \{x | x \in \mathbb{R}\}$

2. Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4-x}$

$4-x \geq 0$
 $4 \geq x$
 $x \leq 4$

(a) Find the function $f \circ g$ and state its domain.

$f(g(x))$
 $f(\sqrt{4-x}) = \sqrt{\sqrt{4-x}} = ((4-x)^{\frac{1}{2}})^{\frac{1}{2}} = (4-x)^{\frac{1}{4}} = \sqrt[4]{4-x}$

$D_{f \circ g} \{x | x \leq 4\}$

(b) Find the function $g \circ f$ and state its domain. $x \geq 0$

$$g(f(x)) = \sqrt{4 - \sqrt{x}}$$

$$\begin{aligned} 4 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -4 \\ \sqrt{x} &\leq 4 \\ x &\leq 16 \end{aligned}$$

(c) Find the function $f \circ f$ and state its domain. $x \geq 0$

$$f(f(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x} \quad x \geq 0$$

$$D_{f \circ f} = \{x \mid x \geq 0\}$$

(d) Find the function $g \circ g$ and state its domain. $x \leq 4$

$$g(g(x)) = \sqrt{4 - \sqrt{4-x}}$$

$$\begin{aligned} 4 - \sqrt{4-x} &\geq 0 \\ -\sqrt{4-x} &\geq -4 \\ \sqrt{4-x} &\leq 4 \\ 4-x &\leq 16 \\ -x &\leq 12 \end{aligned}$$

$$D_{g \circ g} = \{x \mid -12 \leq x \leq 4\}$$

$$x \geq -12$$

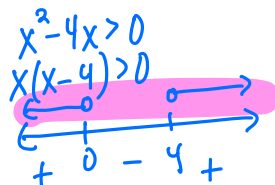
For questions 3 -5, find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

3. $f(x) = 6x - 5$, $g(x) = \frac{x}{2}$

4. $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

$D_f: x > 0$ $D_g: \mathbb{R}$
 5. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2 - 4x$

$(f \circ g)(x)$
 $f(x^2 - 4x) = \frac{1}{\sqrt{x^2 - 4x}}$



$D_{f \circ g} = (-\infty, 0) \cup (4, \infty)$

$(g \circ f)(x)$
 $g\left(\frac{1}{\sqrt{x}}\right) = \left(\frac{1}{\sqrt{x}}\right)^2 - \frac{4}{\sqrt{x}} = \frac{1}{x} - \frac{4}{\sqrt{x}}$ $x > 0$

$D_{g \circ f} : \{x \mid x > 0\}$

$$(f \circ f)(x)$$

$$f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = \frac{1}{\sqrt[3]{x}} = \sqrt[3]{x} \quad x > 0$$

$$D_{f \circ f} = \{x \mid x > 0\}$$

$$(g \circ g)(x)$$

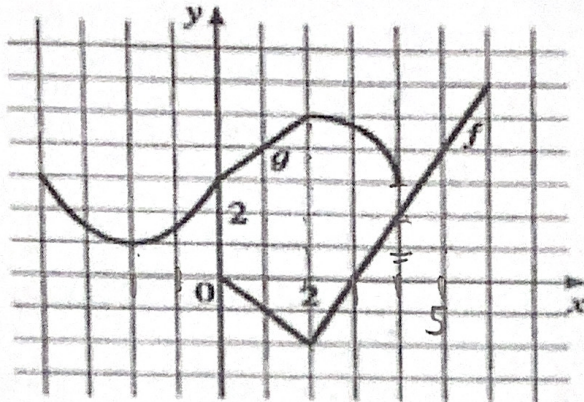
$$g(x^2 - 4x) = (x^2 - 4x)^2 - 4(x^2 - 4x) \quad \mathbb{R}$$

$$D_{g \circ g} = \{x \mid x \in \mathbb{R}\}$$

Homework 10-20

Practice:

For 1-6, use the given graphs of f and g to evaluate the expression.



1. $f(g(2))$
 $f(5)$
 4

2. $g(f(0))$
 $f(0) = 0$
 $g(0) = 3$

3. $(g \circ f)(4)$
 $f(4) = 2$
 $g(2) = 5$

4. $(f \circ g)(4)$
 $f(3)$
 0

5. $(g \circ g)(-2)$
 $g(1)$
 4

6. $(f \circ f)(4)$
 $f(2)$
 -2

7. For each of the following, find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

(a) $f(x) = 2x + 3$, $g(x) = 4x - 1$

(b) $f(x) = 6x - 5$, $g(x) = \frac{x}{2}$

$(f \circ g)(x)$
 $f(4x - 1)$

$(g \circ f)(x)$
 $g(2x + 3)$
 $4(2x + 3) - 1$
 $8x + 12 - 1$
 $8x + 11$

$(f \circ g)(x)$
 $f(\frac{x}{2})$
 $6(\frac{x}{2}) - 5$
 $3x - 5$

$(g \circ f)(x)$
 $g(6x - 5)$
 $\frac{6x - 5}{2}$ or $3x - \frac{5}{2}$

$2(4x - 1) + 3$
 $8x - 2 + 3$
 $8x + 1$

(c) $f(x) = x^3 + 2$, $g(x) = \sqrt[3]{x}$

(d) $f(x) = x^2$, $g(x) = \sqrt{x - 3}$

$(f \circ g)(x)$
 $f(\sqrt[3]{x})$
 $(\sqrt[3]{x})^3 + 2$
 $x + 2$

$(g \circ f)(x)$
 $g(x^3 + 2)$
 $\sqrt[3]{x^3 + 2}$

$(f \circ g)(x)$
 $f(\sqrt{x - 3})$
 $(\sqrt{x - 3})^2$
 $x - 3$

$(g \circ f)(x)$
 $g(x^2)$
 $\sqrt{x^2 - 3}$

(e) $f(x) = x^2$, $g(x) = x - 1$

$$\begin{aligned} (f \circ g)(x) \\ f(x-1) \\ (x-1)^2 \\ x^2 - 2x + 1 \end{aligned}$$

$$\begin{aligned} (g \circ f)(x) \\ g(x^2) \\ x^2 - 1 \end{aligned}$$

8. Find $f(g(h(x)))$

(a) $f(x) = x - 1$, $g(x) = \sqrt{x}$, $h(x) = x + 1$

$$\begin{aligned} f(g(h(x))) \\ f(\sqrt{x+1}) \\ \sqrt{x+1} - 1 \end{aligned}$$

(b) $f(x) = \frac{1}{x}$, $g(x) = x^3$, $h(x) = x^2 + 2$

$$\begin{aligned} f(g(h(x))) \\ f(x^2+2)^3 \\ \frac{1}{(x^2+2)^3} \end{aligned}$$

(c) $f(x) = x^4 + 1$, $g(x) = x - 5$, $h(x) = \sqrt{x}$

$$\begin{aligned} f(g(h(x))) \\ f(\sqrt{x} - 5) \\ (\sqrt{x} - 5)^4 + 1 \end{aligned}$$

$$(d) f(x) = \sqrt{x}, \quad g(x) = \frac{x}{x-1}, \quad h(x) = \sqrt[3]{x}$$

$$f(g(\sqrt[3]{x}))$$

$$f\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$$

$$\sqrt{\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$$