Do Now: #4 from yesterday's packet

For questions 3 -5, find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$d_f\ x \geq 0 \quad d_g\ x \leq 2$

4. $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

$(f \circ g)(x)$

$f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}} = \sqrt[4]{2-x} \qquad x \leq 2$

$d_{f \circ g}\ \{x \mid x \leq 2\}$

$(g \circ f)(x)$

$g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

$2-\sqrt{x} \geq 0$

$-\sqrt{x} \geq -2$

$\sqrt{x} \leq 2$

$x \leq 4$

$d_{g \circ f}\ \{x \mid 0 \leq x \leq 4\}$

$(f \circ f)(x)$

$f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x} \qquad x \geq 0$

$d_{f \circ f} : \{x \mid x \geq 0\}$

$(g \circ g)(x)$

$$g\left(\sqrt{2-x}\right) = \sqrt{2-\sqrt{2-x}}$$

$d_{g\circ g}\ \{x \mid -2 \le x \le 2\}$

$$2-\sqrt{2-x} \ge 0$$
$$-\sqrt{2-x} \ge -2$$
$$\sqrt{2-x} \le 2$$
$$2-x \le 4$$
$$-x \le 2$$
$$x \ge -2$$

Name: ___________________________________ Date: __________

PCH Decomposition of Functions Ms. Loughran

Do Now:

Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$, find $x \neq 0$

(a) $f(g(4))$ $= f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{4}} = \frac{1}{2}$

(b) $f(g(x))$ $= f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$ $\leftarrow x > 0$

(c) find $d_{f(g(x))}$ $\{x \mid x > 0\}$

A composite function is a function that brings together two or more functions. For instance, let h be given by

$$h(x) = \sqrt{x^2 + 2x + 2}$$

If we let $f(x) =$ $x^2 + 2x + 2$ and $g(x) =$ $\sqrt{x}$, then $(g \circ f)(x) =$ $\sqrt{x^2+2x+2} = h(x)$

Thus the given function h has been ***decomposed*** into the composition of the two functions f and g. Such decompositions are not unique. More than one decomposition is possible.

We could have decomposed h into $f(x) =$ $\sqrt{x+2}$ and $g(x) =$ $x^2 + 2x$

$f(g(x)) =$ $f(x^2 + 2x) = \sqrt{x^2 + 2x + 2}$

We are going to avoid using the identity function ($f(x) = x$) in our decompositions.

TRY:

Given: $h(x) = \dfrac{2}{\sqrt{x+1}-3}$

(a) Create functions f and g so that $h(x) = (f \circ g)(x)$.

$g(x) = \sqrt{x+1}$
$f(x) = \dfrac{2}{x-3}$

or

$g(x) = \sqrt{x+1} - 3$
$f(x) = \dfrac{2}{x}$

or

$g(x) = x+1$
$f(x) = \dfrac{2}{\sqrt{x}-3}$

(b) Create functions f, g, and k so that $h(x) = (f \circ g \circ k)(x)$.

$k(x) = x+1$
$g(x) = \sqrt{x} - 3$
$f(x) = \dfrac{2}{x}$

$k(x) = \sqrt{x+1}$
$g(x) = x-3$
$f(x) = \dfrac{2}{x}$

More Practice

Express the function in the form $f \circ g$

1. $F(x) = (x-9)^5$

$f(x) = x^5$
$g(x) = x-9$

or

$f(x) = (x-5)^5$
$g(x) = x-4$

2. $F(x) = \sqrt{x} + 1$

$f(x) = x+1$
$g(x) = \sqrt{x}$

or

$f(x) = x+2$
$g(x) = \sqrt{x} - 1$

3. $F(x) = \dfrac{x^2}{x^2+4}$

$f(x) = \dfrac{x}{x+4}$
$g(x) = x^2$

or

$f(x) = \dfrac{x-4}{x}$
$g(x) = x^2+4$

or

$f(x) = \dfrac{x^4}{x^4+4}$
$g(x) = \sqrt{x}$

4. $F(x) = \dfrac{1}{x+3}$

$f(x) = \dfrac{1}{x}$
$g(x) = x+3$

$f(x) = \dfrac{1}{x+1}$
$g(x) = x+2$

5. $F(x) = |1-x^3|$

$f(x) = |1-x|$
$g(x) = x^3$

$f(x) = |1+x|$
$g(x) = -x^3$

$f(x) = |x|$
$g(x) = 1-x^3$

6. $F(x) = \sqrt{1+\sqrt{x}}$

$f(x) = \sqrt{1+x}$
$g(x) = \sqrt{x}$

$f(x) = \sqrt{x}$
$g(x) = 1+\sqrt{x}$

Express the function in the form $f \circ g \circ h$

7. $F(x) = \dfrac{1}{x^2+1}$

$f(x) = \dfrac{1}{x}$
$g(x) = x+1$
$h(x) = x^2$

$f(x) = \dfrac{1}{x+2}$
$g(x) = x-1$
$h(x) = x^2$

8. Find *f* and *g* such that $h = f \circ g$, where $h(x) = \left(\dfrac{1}{3x-1}\right)^5$ and the inner function is fractional rational.

$g(x) = \dfrac{1}{3x-1}$
$f(x) = x^5$

Homework 10-23

19 a) $g(-2) = 2-(-2)^2 = -2$

$f(-2) = 3(-2)-5 = -11$

b) $f(-2) = 3(-2)-5 = -11$

$g(-11) = 2-(-11)^2 = 2-121 = -119$

24 $g(f(0))$

$g(0) = 3$

27 $(g \circ g)(-2)$

$g(1) = 4$

29 $f(x) = 2x+3$ $\quad d_f : \mathbb{R}$

$g(x) = 4x-1$ $\quad d_g : \mathbb{R}$

$(f \circ g)$ $\quad d_{f \circ g} : \mathbb{R}$

$f(4x-1) = 2(4x-1)+3 = 8x-2+3 = 8x+1$

$(g \circ f)$ $\quad d_{g \circ f} : \mathbb{R}$

$g(2x+3) = 4(2x+3)-1 = 8x+12-1 = 8x+11$

$(f \circ f)$

$f(2x+3) = 2(2x+3)+3 = 4x+6+3 = 4x+9$ $\quad d_{f \circ f}: \mathbb{R}$

$(g \circ g)$

$g(4x-1) = 4(4x-1)-1 = 16x-4-1 = 16x-5$ $\quad d_{g \circ g}: \mathbb{R}$

(31) $f(x) = x^2$ $\quad d_f: \mathbb{R}$ $\qquad g(x) = x+1$ $\quad d_g: \mathbb{R}$

$(f \circ g)$

$f(x+1) = (x+1)^2$ $\quad d_{f \circ g}: \mathbb{R}$

$(f \circ f)$

$f(x^2) = (x^2)^2 = x^4$ $\quad d_{f \circ f}: \mathbb{R}$

$(g \circ f)$

$g(x^2) = x^2+1$ $\quad d_{g \circ f}: \mathbb{R}$

$(g \circ g)$

$g(x+1) = x+1+1 = x+2$ $\quad d_{g \circ g}: \mathbb{R}$

(33) $f(x) = \frac{1}{x}$ $\quad d_f: x \neq 0$ $\qquad g(x) = 2x+4$ $\quad d_g: \mathbb{R}$

$(f \circ g)$

$f(2x+4) = \frac{1}{2x+4}$ $\quad x \neq -2$ $\qquad d_{f \circ g}: x \neq -2$

$(g \circ f)$

$g\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right)+4 = \frac{2}{x}+4$ $\quad d_{g \circ f}: x \neq 0$

$(f \circ f)$

$f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ $\quad d_{f \circ f}: x \neq 0$

$(g \circ g)$

$g(2x+4) = 2(2x+4)+4 = 4x+8+12 = 4x+12$ $\quad d_{g \circ g}: \mathbb{R}$

(34) $f(x)=x^2$ $d_f: \mathbb{R}$ $g(x)=\sqrt{x-3}$ $d_g: x \geq 3$

$(f \circ g)$

$f(\sqrt{x-3}) = (\sqrt{x-3})^2 = x-3$ $d_{f \circ g}: x \geq 3$

$(g \circ f)$

$g(x^2) = \sqrt{x^2-3}$ $x^2-3 \geq 0$ $-\sqrt{3}$ $\sqrt{3}$ $d_{g \circ f}: x \leq -\sqrt{3} \vee x \geq \sqrt{3}$

$(f \circ f)$

$f(x^2) = (x^2)^2 = x^4$ $d_{f \circ f}: \mathbb{R}$

$(g \circ g)$

$g(\sqrt{x-3}) = \sqrt{\sqrt{x-3}-3}$

$\sqrt{x-3} - 3 \geq 0$

$\sqrt{x-3} \geq 3$

$x-3 \geq 9$

$x \geq 12$

$d_{g \circ g}$ $x \geq 12$

(37) $f(x)=\frac{x}{x+1}$ $d_f: x \neq -1$ $g(x)=2x-1$ $d_g: \mathbb{R}$

$(f \circ g)$

$f(2x-1) = \frac{2x-1}{2x-1+1} = \frac{2x-1}{2x}$ $x \neq 0$ $d_{f \circ g}$ $x \neq 0$

$(g \circ f)$

$g\left(\frac{x}{x+1}\right) = 2\left(\frac{x}{x+1}\right)-1 = \frac{2x}{x+1} - 1$ $d_{g \circ f}: x \neq -1$

$(f \circ f)$

$f\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \frac{x}{x+x+1} = \frac{x}{2x+1}$ $d_{f \circ f}: x \neq -1, -\frac{1}{2}$

$(g \circ g)$

$g(2x-1) = 2(2x-1)-1 = 4x-2-1 = 4x-3$ $d_{g \circ g}: \mathbb{R}$