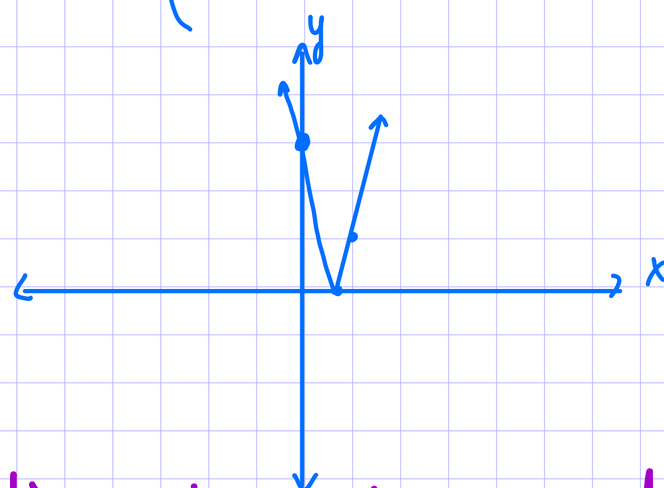
(From yesterday's Do Now)

9) Use the algebraic definition of absolute value to rewrite as a piecewise function and graph.

$$f(x) = |4x-3| = \begin{cases} 4x-3 & \text{if } 4x-3 \geq 0,\ x \geq \frac{3}{4} \\ -4x+3 & \text{if } x < \frac{3}{4} \end{cases}$$

$x = 0,\ f(0) = 3$
$x = 1,\ f(1) = 1$



10) Perform the indicated operation and state restrictions:

$$\frac{(2p-q)}{(2p-q)}\cdot\frac{p}{2p^2+3pq-2q^2} + \frac{-3q}{4p^2-4pq+q^2}\cdot\frac{(p+2q)}{(p+2q)}$$

$$\frac{p}{(2p-q)(p+2q)} + \frac{-3q}{(2p-q)^2}$$

$$\frac{2p^2-pq-3pq-6q^2}{(2p-q)^2(p+2q)}$$

this is factorable

$q \neq 2p$
$p \neq -2q$

$$\frac{2p^2-4pq-6q^2}{(2p-q)^2(p+2q)} = \frac{2(p^2-2pq-3q^2)}{(2p-q)^2(p+2q)}$$

Name: ____ Date: ______________

PCH: Geometric Approach to Absolute Value Ms. Loughran

Do Now:

1. A closed tin can with height h and radius r has volume 5 cubic centimeters. Express the surface area of the tin can as a function of r.

Geometric Definition of Absolute Value:

$|x|$ means x's distance from 0 on a number line

$|x-0|$

$|x-a|$ means x's distance from a on a number line

$|x+a|$

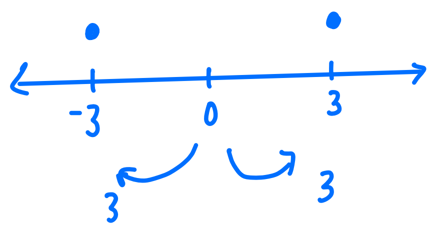
$|x-(-a)|$ means x's distance from $-a$ on a number line

Examples:

Solve each of the following using the geometric definition of absolute value.

1. $|x| = 3$

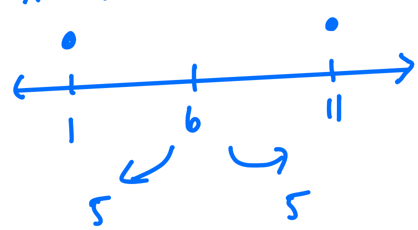
x's distance from 0 is 3



$\{\pm 3\}$

2. $|x-6| = 5$

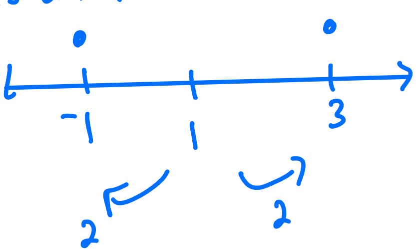
x's distance from 6 is 5



$\{1, 11\}$

3. $|x-1| = 2$

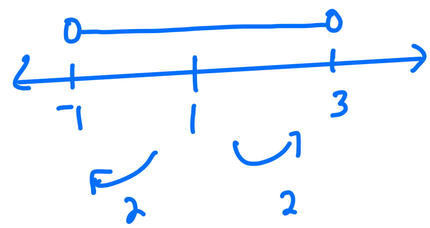
x's distance from 1 is 2



$\{-1, 3\}$

4. $|x-1| < 2$

x's distance from 1 < 2



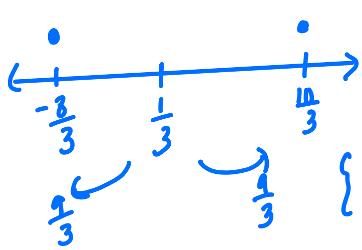
$(-1, 3)$

5. $|3x-1| = 9$

$3\left|x-\frac{1}{3}\right| = 9$

$\left|x-\frac{1}{3}\right| = \frac{9}{3}$

x's distance from $\frac{1}{3}$ is $\frac{9}{3}$



$\left\{-\frac{8}{3}, \frac{10}{3}\right\}$

6. $|6x+4| = -8$

$\emptyset$

or

$\{\ \}$

or

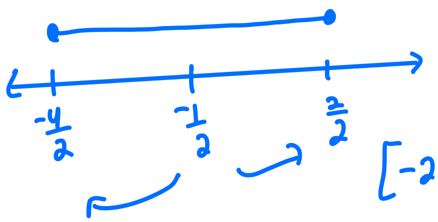
no solution

7. $|2x+1| \le 3$

$2\left|x+\frac{1}{2}\right| \le 3$

$\left|x+\frac{1}{2}\right| \le \frac{3}{2}$

x's distance from $-\frac{1}{2} \le \frac{3}{2}$



$[-2, 1]$

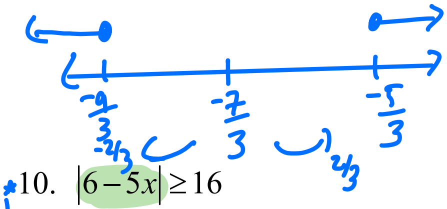
8. $|7+3x| \ge 2$

$|3x+7| \ge 2$

$3\left|x+\frac{7}{3}\right| \ge 2$

$\left|x+\frac{7}{3}\right| \ge \frac{2}{3}$

x's distance from $-\frac{7}{3} \ge \frac{2}{3}$



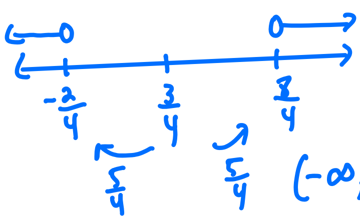
$(-\infty, -3] \cup [-\frac{5}{3}, \infty)$

9. $|4x-3| > 5$

$4\left|x-\frac{3}{4}\right| > 5$

$\left|x-\frac{3}{4}\right| > \frac{5}{4}$

x's distance from $\frac{3}{4} > \frac{5}{4}$



$(-\infty, -\frac{1}{2}) \cup (2, \infty)$

* $|x-y| = |y-x|$

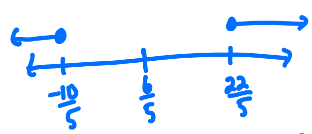
*10. $|6-5x| \ge 16$

$|5x-6| \ge 16$

$5\left|x-\frac{6}{5}\right| \ge 16$

$\left|x-\frac{6}{5}\right| \ge \frac{16}{5}$

x's distance from $\frac{6}{5} \ge \frac{16}{5}$



$(-\infty, -2] \cup [\frac{22}{5}, \infty)$

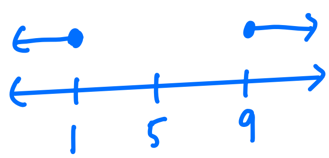
11. $\left|\frac{5-x}{4}\right| \ge 1$

$\left|\frac{x-5}{4}\right| \ge 1$

$4 \cdot \frac{1}{4}|x-5| \ge 1 \cdot 4$

$|x-5| \ge 4$

x's distance from $5 \ge 4$

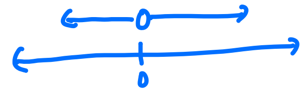


$(-\infty, 1] \cup [9, \infty)$

12. $|x+4| < -1$

$\emptyset$

absolute value can not be $\ominus$



13. $|x+4| > -1$

$(-\infty, \infty)$

14. $|x| > 0$

$\{x \mid x \in \mathbb{R}/0\}$

15. $\left|\frac{x+1}{2}\right| \le 4$

$\frac{1}{2}|x+1| \le 4$

$|x+1| \le 8$

x's distance from $-1 \le 8$

$[-9, 7]$

16. $3-|2x+4| \le 1$

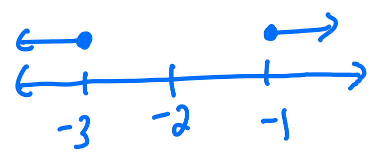
$-|2x+4| \le -2$

$|2x+4| \ge 2$

$2|x+2| \ge 2$

$|x+2| \ge 1$

x's distance from $-2 \ge 1$



$(-\infty, -3] \cup$

$[-1, \infty)$

Practice

1. $|x| \le 7$

2. $|t| \ge 5$

3. $|y-5| = 3$

Homework 10-31

Practice

Use compositions to prove if the given functions are inverses.

$f(g(n)) = f\left(\frac{-12-2n}{3}\right) = \frac{-5+\cancel{6}^{2}\left(\frac{-12-2n}{\cancel{3}}\right)}{5}$

$= \frac{-5-24-4n}{5} = \frac{-5-28n}{5} \neq n$

2) $g(n) = \frac{-12-2n}{3}$

$f(n) = \frac{-5+6n}{5}$

No

$f(g(x)) = -\frac{4}{7}\left(\frac{3}{2}x - \frac{3}{2}\right) - \frac{16}{7}$

$\frac{-12}{14}x + \frac{12}{14} - \frac{16}{7}$

$\frac{-6}{7}x + \frac{6}{7} - \frac{8}{7} \neq x$

4) $f(x) = -\frac{4}{7}x - \frac{16}{7}$

$g(x) = \frac{3}{2}x - \frac{3}{2}$

No

6) $f(n) = 2(n-2)^3$

$g(n) = \frac{4+\sqrt[3]{4n}}{2}$

Yes

8) $g(x) = -\frac{2}{x} - 1$

$f(x) = -\frac{2}{x+1}$

Yes

Find the inverse of each function.

10) $g(x) = \frac{1}{x} - 2$

$x = \frac{1}{y} - 2$

$x + 2 = \frac{1}{y}$

$\frac{1}{x+2} = y$

$g^{-1}(x) = \frac{1}{x+2}$

12) $g(x) = -4x + 1$

$x = -4y + 1$

$\frac{x-1}{-4} = y$

$g^{-1}(x) = \frac{x-1}{-4}$

14) $f(x) = x + 3$

$f^{-1}(x) = x - 3$

16) $f(x) = 4x$

$f^{-1}(x) = \frac{x}{4}$

18) $f(x) = -\frac{3}{-x-3} - 2 = \frac{3}{x+3} - 2$

$x = \frac{3}{y+3} - 2$

$x + 2 = \frac{3}{y+3}$

$(y+3)(x+2) = 3$

$y + 3 = \frac{3}{x+2}$

$y = \frac{3}{x+2} - 3$

$f^{-1}(x) = \frac{3}{x+2} - 3$

21) If $g(x) = 1 + \sqrt[3]{2x+1}$, find $g^{-1}(4) = 13$

$1 + \sqrt[3]{2x+1} = 4$

$\sqrt[3]{2x+1} = 3$

$2x + 1 = 27$

$2x = 26$

$x = 13$

Review of Functions

Name ____________________

Classwork/Homework 11-05

1.) Write the equation of the line (in slope-intercept form) that passes through the points (4, -5) and (2, 3).

$m = \frac{3-(-5)}{2-4} = \frac{8}{-2} = -4$

$y-3 = -4(x-2)$

$y-3 = -4x+8$

$y = -4x+11$

2.) Write the equation of the line (in slope-intercept form) that is perpendicular to the line in problem 1 and passes through the point $(-3,-1)$.

$m_\perp = +\frac{1}{4}$

$y+1 = \frac{1}{4}(x+3)$

$y+1 = \frac{1}{4}x + \frac{3}{4}$

$y = \frac{1}{4}x - \frac{1}{4}$

3.) Rewrite the equation $y = -\frac{2}{5}x + \frac{1}{3}$ in standard (general) form.

$15\left(y = -\frac{2}{5}x + \frac{1}{3}\right)$

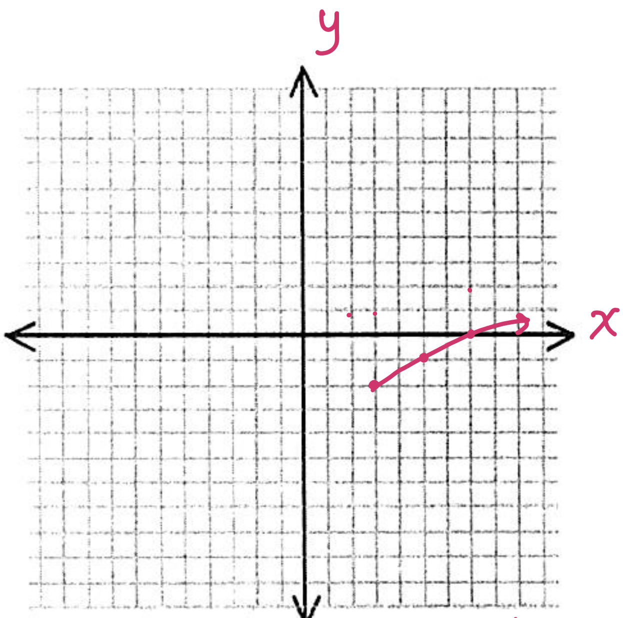
$15y = -6x + 5$

$6x + 15y = 5$

Sketch the graph of each function and write the domain and range in interval notation.

4.) $f(x) = -2 + \sqrt{x-3}$

$\sqrt{}$ right 3 ↓ 2



D: $[3, \infty)$

R: $[-2, \infty)$

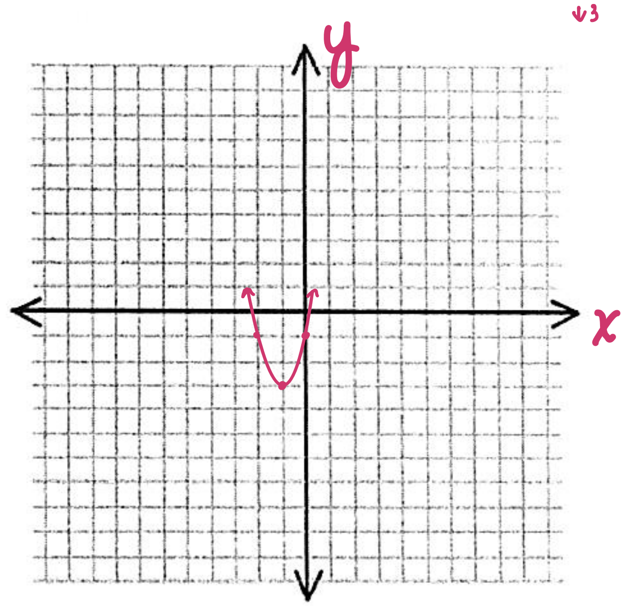
5.) $f(x) = 2x^2 + 4x - 1$

$f(x) = 2(x^2+2x+1-1)-1$

$f(x) = 2(x+1)^2-2-1$

$f(x) = 2(x+1)^2-3$

left one
vertical stretch
↓3



D: $(-\infty, \infty)$

R: $[-3, \infty)$

Find the inverse of each function.

6.) $f(x)=1+\sqrt[3]{2x+3}$

$x=1+\sqrt[3]{2y+3}$

$(x-1)^3=(\sqrt[3]{2y+3})^3$

$(x-1)^3=2y+3$

$\frac{(x-1)^3-3}{2}=y=f^{-1}(x)$

7.) $f(x)=\frac{x+3}{x-2}$

$x=\frac{y+3}{y-2}$

$xy-2x=y+3$

$xy-y=2x+3$

$y(x-1)=2x+3$

$y=\frac{2x+3}{x-1}=f^{-1}(x)$

8.) $f(x)=\frac{2x-1}{x+2}$

$x=\frac{2y-1}{y+2}$

$xy+2x=2y-1$

$xy-2y=-2x-1$

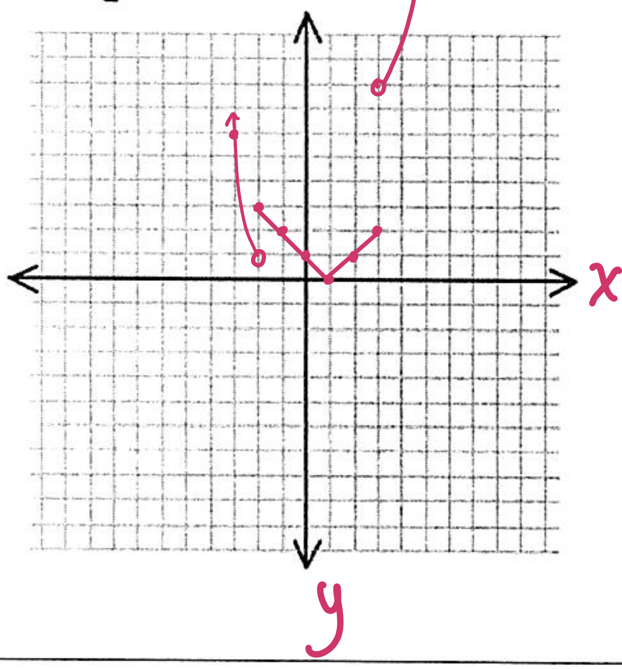
$y(x-2)=-2x-1$

$y=\frac{-2x-1}{x-2}=f^{-1}(x)$

9.) Graph the piecewise function:

$$f(x)=\begin{cases} x^2-3 & if\ \ x<-2 \\ |x-1| & if\ \ -2\le x\le 3 \\ (x-1)^3 & if\ \ x>3 \end{cases}$$

(4,27)



10.) Write a piecewise function for the graph below.

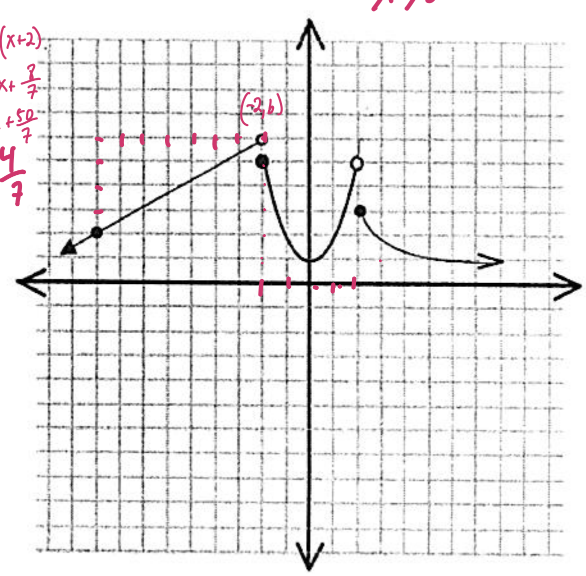
$$f(x)=\begin{cases} \frac{4}{7}x+\frac{50}{7} & x<-2 \\ x^2+1 & -2\le x<2 \\ -\sqrt{x-2}+3 & x\ge 2 \end{cases}$$

$y-6=\frac{4}{7}(x+2)$

$y-6=\frac{4}{7}x+\frac{8}{7}$

$y=\frac{4}{7}x+\frac{50}{7}$

$m=\frac{4}{7}$



Determine whether the functions are inverses of each other using composition. You must show your work.

11.) $f(x)=x^2-3$ $\qquad g(x)=\sqrt{x+3}$

$f(g(x))$

$f(\sqrt{x+3})$

$(\sqrt{x+3})^2-3$

$x+3-3$

x

$g(f(x))$

$g(x^2-3)$

$\sqrt{x^2-3+3}$

$\sqrt{x^2}$

x

yes

12.) $f(x) = \frac{\frac{4}{x} + 3}{2}$ $g(x) = \frac{4}{2x-3}$

$f(g(x))$

$f\left(\frac{4}{2x-3}\right)$

$\frac{\frac{4}{\frac{4}{2x-3}} + 3}{2}$

$\frac{\frac{4(2x-3)}{4} + 3}{2}$

$\frac{2x\cancel{-3}\cancel{+3}}{2}$ $\frac{2x}{2} = x$

$g(f(x))$

$g\left(\frac{\frac{4}{x}+3}{2}\right)$

$\frac{4}{\cancel{2}\left(\frac{\frac{4}{x}+3}{\cancel{2}}\right)-3}$

$\frac{4}{\frac{4}{x}\cancel{+3}\cancel{-3}}$

$x \cdot \frac{4}{\frac{4}{x}}$

$\cancel{x} \cdot \frac{4}{\frac{4}{\cancel{x}}}$

$\frac{4x}{4} = x$

Yes

Find the inverse of each function.

13.) $y = 3x^3 - 2$

$x = 3y^3 - 2$

$x+2 = 3y^3$

$\frac{x+2}{3} = y^3$

$\sqrt[3]{\frac{x+2}{3}} = y$

14.) $y = \frac{5}{3x+2}$

$x = \frac{5}{3y+2}$

$x(3y+2) = 5$

$3y+2 = \frac{5}{x}$

$3y = \frac{5}{x} - 2$

$y = \frac{\frac{5}{x} - 2}{3}$

$y = \frac{5-2x}{3x}$

Decompose each function h into the functions f and g so that $h(x) = f(g(x))$

15.) $h(x) = \sqrt[3]{x-4}$

$f(x) = \sqrt[3]{x}$

$g(x) = x-4$

16.) $h(x) = \frac{4}{3x+2}$

$f(x) = \frac{4}{x}$

$g(x) = 3x+2$

Answers may vary.

17) If $f(x) = \frac{x}{2x-3}$, find $f^{-1}(-2)$.

$\frac{x}{2x-3} = -2$

$-4x+6 = x$

$6 = 5x$

$\frac{6}{5} = x$