① Given $f(x) = \frac{2x-1}{x+5}$, find $f^{-1}(x)$

$$y = \frac{2x-1}{x+5}$$

$$x = \frac{2y-1}{y+5}$$

$$xy + 5x = 2y - 1$$

$$xy - 2y = -5x - 1$$

$$y(x-2) = -5x - 1$$

$$f^{-1}(x) = y = \frac{-5x-1}{x-2}$$

② If $f(x) = \frac{x}{2x-3}$, find $f^{-1}(2) = 2$

$$\frac{x}{2x-3} = 2$$

$$x = 4x - 6$$

$$-3x = -6$$

$$x = 2$$

③ Use the geometric definition of absolute value to solve and express the solution set in interval notation.

$$\left|4 - \frac{3}{5}x\right| \geq 12$$

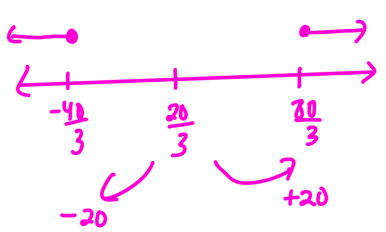
$-4 \cdot \frac{5}{3}$

$$\left|\frac{3}{5}x - 4\right| \geq 12$$

$$\frac{5}{3} \cdot \frac{3}{5}\left|x - \frac{20}{3}\right| \geq \cancel{12}^{\,4} \cdot \frac{5}{\cancel{3}}$$

$$\left|x - \frac{20}{3}\right| \geq 20$$

x's distance from $\frac{20}{3} \geq 20$



$$\left(-\infty, -\frac{40}{3}\right] \cup \left[\frac{80}{3}, \infty\right)$$

④ Use compositions to determine if f and g are inverse functions

$$f(x) = \frac{(x-6)^3}{2} + 4 \quad \text{and} \quad g(x) = 2\sqrt[3]{x-4} + 6$$

$$f(g(x)) = g(f(x)) = x$$

$$f\left(2\sqrt[3]{x-4} + 6\right) = \frac{\left(2\sqrt[3]{x-4} \cancel{+6} \cancel{-6}\right)^3}{2} + 4$$

$$\frac{\left(2\sqrt[3]{x-4}\right)^3}{2} + 4$$

$$\frac{\overset{4}{\cancel{8}}(x-4)}{\cancel{2}} + 4 \qquad \text{not inverses}$$

$$4(x-4) + 4$$

$$4x - 16 + 4 \neq x$$

Name ______________________

Review of Functions

Classwork/Homework 11-03

1.) Write the equation of the line (in slope-intercept form) that passes through the points (4, -5) and (2, 3).

$m = \frac{3-(-5)}{2-4} = \frac{8}{-2} = -4$

$y-3 = -4(x-2)$

$y-3 = -4x+8$

$y = -4x+11$

2.) Write the equation of the line (in slope-intercept form) that is perpendicular to the line in problem 1 and passes through the point $(-3,-1)$.

$m_\perp = +\frac{1}{4}$

$y+1 = \frac{1}{4}(x+3)$

$y+1 = \frac{1}{4}x + \frac{3}{4}$

$y = \frac{1}{4}x - \frac{1}{4}$

3.) Rewrite the equation $y = -\frac{2}{5}x + \frac{1}{3}$ in standard (general) form.

$15\left(y = -\frac{2}{5}x + \frac{1}{3}\right)$

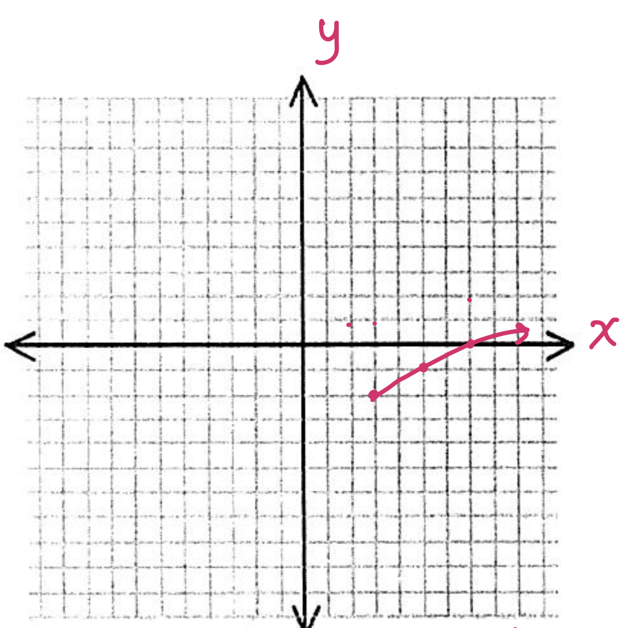
$15y = -6x + 5$

$6x + 15y = 5$

Sketch the graph of each function and write the domain and range in interval notation.

4.) $f(x) = -2 + \sqrt{x-3}$

$\sqrt{}$ right 3 ↓ 2



D: $[3, \infty)$

R: $[-2, \infty)$

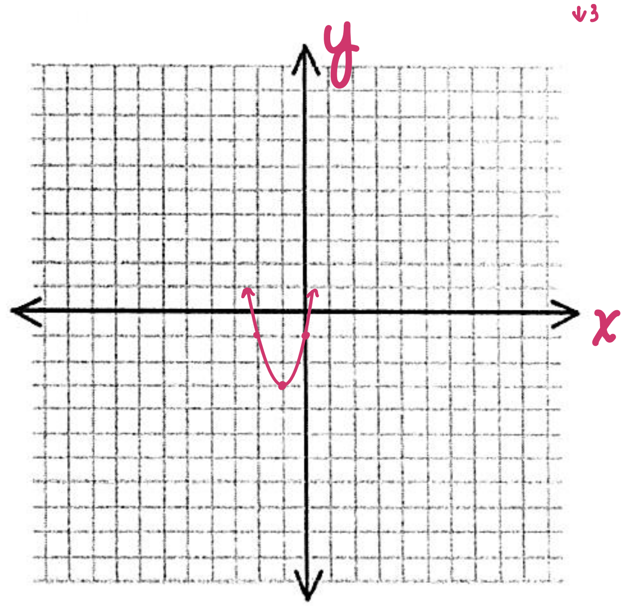
5.) $f(x) = 2x^2 + 4x - 1$

$f(x) = 2(x^2 + 2x + 1 - 1) - 1$

$f(x) = 2(x+1)^2 - 2 - 1$

$f(x) = 2(x+1)^2 - 3$

left one
vertical stretch
↓3



D: $(-\infty, \infty)$

R: $[-3, \infty)$

Find the inverse of each function.

6.) $f(x)=1+\sqrt[3]{2x+3}$

$x = 1+\sqrt[3]{2y+3}$

$(x-1)^3 = \left(\sqrt[3]{2y+3}\right)^3$

$(x-1)^3 = 2y+3$

$\dfrac{(x-1)^3-3}{2} = y = f^{-1}(x)$

7.) $f(x)=\dfrac{x+3}{x-2}$

$x = \dfrac{y+3}{y-2}$

$xy-2x = y+3$

$xy-y = 2x+3$

$y(x-1) = 2x+3$

$y = \dfrac{2x+3}{x-1} = f^{-1}(x)$

8.) $f(x)=\dfrac{2x-1}{x+2}$

$x = \dfrac{2y-1}{y+2}$

$xy+2x = 2y-1$

$xy-2y = -2x-1$

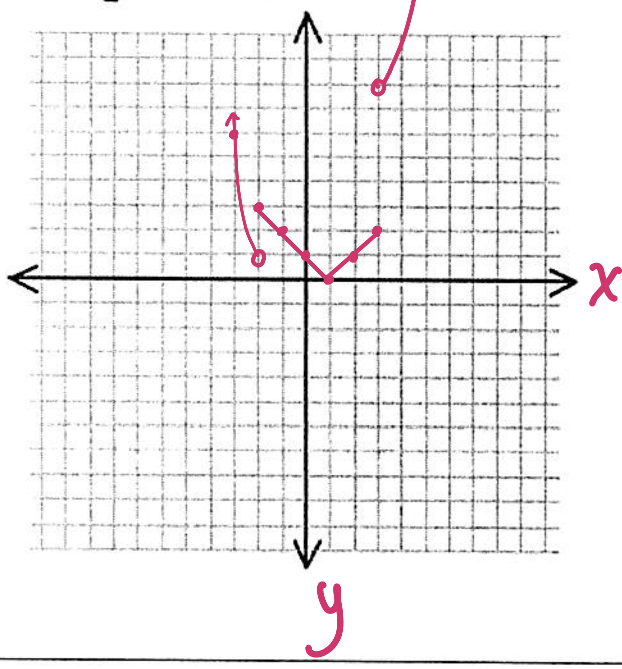
$y(x-2) = -2x-1$

$y = \dfrac{-2x-1}{x-2} = f^{-1}(x)$

9.) Graph the piecewise function:

$$f(x) = \begin{cases} x^2-3 & if\ x<-2 \\ |x-1| & if\ -2 \le x \le 3 \\ (x-1)^3 & if\ x>3 \end{cases}$$

(4,27)



10.) Write a piecewise function for the graph below.

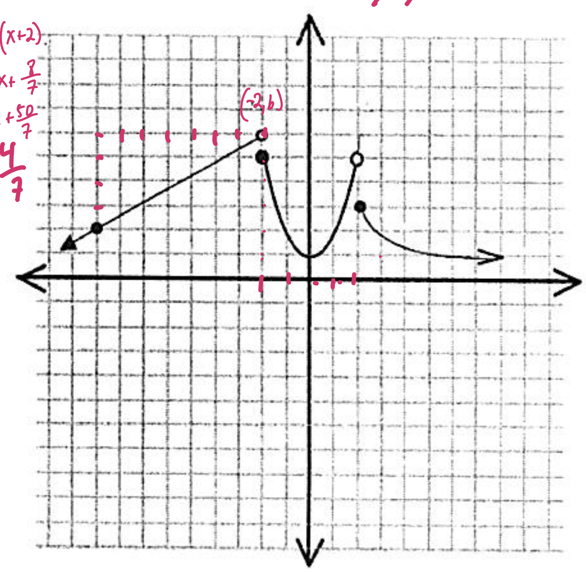
$$f(x) = \begin{cases} \frac{4}{7}x + \frac{50}{7} & x<-2 \\ x^2+1 & -2\le x<2 \\ -\sqrt{x-2}+3 & x\geq 2 \end{cases}$$

$y-6 = \frac{4}{7}(x+2)$

$y-6 = \frac{4}{7}x+\frac{8}{7}$

$y = \frac{4}{7}x+\frac{50}{7}$

$m = \frac{4}{7}$



Determine whether the functions are inverses of each other using composition. You must show your work.

11.) $f(x)=x^2-3$ $\quad$ $g(x)=\sqrt{x+3}$

$f(g(x))$

$f(\sqrt{x+3})$

$(\sqrt{x+3})^2-3$

$x+3-3$

x

$g(f(x))$

$g(x^2-3)$

$\sqrt{x^2-3+3}$

$\sqrt{x^2}$

x

yes

12.) $f(x) = \dfrac{\frac{4}{x}+3}{2}$ $g(x) = \dfrac{4}{2x-3}$

$f(g(x))$

$f\left(\frac{4}{2x-3}\right)$

$\dfrac{\frac{4}{\frac{4}{2x-3}}+3}{2}$

$\dfrac{\frac{4(2x-3)}{4}+3}{2}$

$\dfrac{2x\cancel{-3}\cancel{+3}}{2}$ $\frac{2x}{2}=x$

$g(f(x))$

$g\left(\dfrac{\frac{4}{x}+3}{2}\right)$

$\dfrac{4}{\cancel{2}\left(\frac{\frac{4}{x}+3}{\cancel{2}}\right)-3}$

$\dfrac{4}{\frac{4}{x}\cancel{+3}\cancel{-3}}$

$x \cdot \dfrac{4}{\frac{4}{\cancel{x}}}$ $\cancel{x}$

$\frac{4x}{4}=x$

Yes

Find the inverse of each function.

13.) $y = 3x^3 - 2$

$x = 3y^3 - 2$

$x+2 = 3y^3$

$\frac{x+2}{3} = y^3$

$\sqrt[3]{\frac{x+2}{3}} = y$

14.) $y = \dfrac{5}{3x+2}$

$x = \frac{5}{3y+2}$

$x(3y+2) = 5$

$3y+2 = \frac{5}{x}$

$3y = \frac{5}{x}-2$

$y = \dfrac{\frac{5}{x}-2}{3}$

$y = \dfrac{5-2x}{3x}$

Decompose each function h into the functions f and g so that $h(x) = f(g(x))$

15.) $h(x) = \sqrt[3]{x-4}$

$f(x) = \sqrt[3]{x}$

$g(x) = x-4$

16.) $h(x) = \dfrac{4}{3x+2}$

$f(x) = \frac{4}{x}$

$g(x) = 3x+2$

Answers may vary

17) If $f(x) = \dfrac{x}{2x-3}$, find $f^{-1}(-2)$.

$\frac{x}{2x-3} = -2$

$-4x+6 = x$

$6 = 5x$

$\frac{6}{5} = x$

Exam 3B

⑨ $h(x) = -4$ ← constant

$h(2x-5) = -4$

$h(g(x))$ $\quad d_g \{x \mid x \leq 6\}$

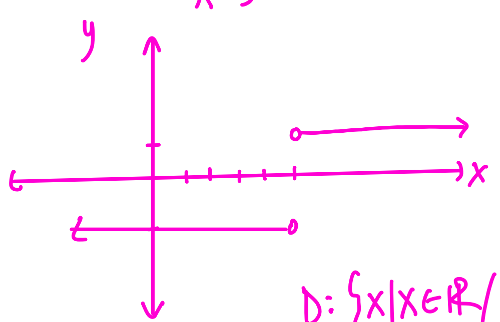
⑤ b) $h(\sqrt{6-x}) = \dfrac{1}{\sqrt{6-x}}$

$6-x>0$
$6>x$
$x<6$

$d_{h \circ g} \{x \mid x<6\}$

⑩ $y = \dfrac{|5-x|}{x-5} = \begin{cases} \dfrac{5-x}{x-5} = -1 & x<5 \\ \dfrac{-(5-x)}{x-5} = 1 & x>5 \end{cases}$

$5-x>0$
$5>x$
$x<5$



D: $\{x \mid x \in \mathbb{R}/5\}$
R: $\{\pm 1\}$

Exam 2A

② $f(x) = 3^x$

$f(x-1) - f(x) =$

$f(x-1) = 3^{x-1}$
$f(x) = 3^x$

$3^{x-1} - 3^x$

$3^x \cdot 3^{-1} - 3^x$

$3^x \cdot \frac{1}{3} - 3^x = 3^x\left(\frac{1}{3} - 1\right)$

$3^x(-2/3)$

9

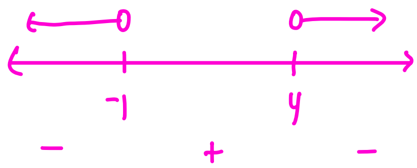
$$\frac{x+6}{x+1} < 2$$

IN

$$\frac{x+6}{x+1} - 2 < 0$$

$$\frac{x+6-\overset{-2x-2}{2(x+1)}}{x+1} < 0$$

$$\frac{-x+4}{x+1} < 0$$



$(-\infty,-1) \cup (4,\infty)$

Exam 3A

4

$h(x) = g(f(x))$ $\mathbb{R}$

$f(x) = 6 - x$

$g(x) = \frac{1}{\sqrt{x}}$

$$g(6-x) = \frac{1}{\sqrt{6-x}}$$

$6 - x > 0$

$6 > x$

$x < 6$

$d_h: \{x \mid x < 6\}$

Exam 1 B

$$\frac{(4x-5)^3 \cdot 2(2x+3) \cdot 2 - (2x+3)^2 \cdot 3(4x-5)^2 \cdot 4}{(4x-5)^6}$$

$$\frac{4(4x-5)^3(2x+3) - 12(2x+3)^2(4x-5)^2}{(4x-5)^6}$$

$$\frac{4(4x-5)^2(2x+3)\left(4x-5 \overset{-6x-9}{-3(2x+3)}\right)}{(4x-5)^6}$$

$-2(x+7)$

$$\frac{4\cancel{(4x-5)^2}(2x+3)(-2x-14)}{(4x-5)^{\cancel{6}\,4}} = \frac{-8(2x+3)(x+7)}{(4x-5)^4}$$

Exam 3B

(1) $\dfrac{x^3-8}{\sqrt{2}-\sqrt{x}} = \dfrac{(x-2)(x^2+2x+4)}{\sqrt{2}-\sqrt{x}} = \dfrac{\overset{-1}{\cancel{(\sqrt{x}-\sqrt{2})}}(\sqrt{x}+\sqrt{2})(x^2+2x+4)}{\cancel{\sqrt{2}-\sqrt{x}}}$

$-1(\sqrt{x}+\sqrt{2})(x^2+2x+4)$

Exam 2A

(12) $f(x)=\dfrac{4x}{x+1}$ find $\dfrac{f(x+h)-f(x)}{h}$

$h \neq 0$

$x \neq -1, -h-1$

$$\frac{\dfrac{4x+4h}{\cancel{x+h+1}}(x+1)\cancel{(x+h+1)} - \dfrac{4x}{\cancel{x+1}}\cancel{(x+1)}(x+h+1)}{h(x+1)(x+h+1)}$$

$$\frac{\cancel{4x^2}+\cancel{4xh}+\cancel{4x}+4h-\cancel{4x^2}-\cancel{4xh}-\cancel{4x}}{h(x+1)(x+h+1)}$$

$$\frac{4\cancel{h}}{\cancel{h}(x+1)(x+h+1)} = \frac{4}{(x+1)(x+h+1)}$$

Period 9

Exam 3A

& constant

(9) $h(x) = -3$

$h(3x-4) = -3$

(14) $f(x) = -9x - 9$ $\qquad (f \circ g)(x) = -9\sqrt{x} + 72$

$f(g(x))$

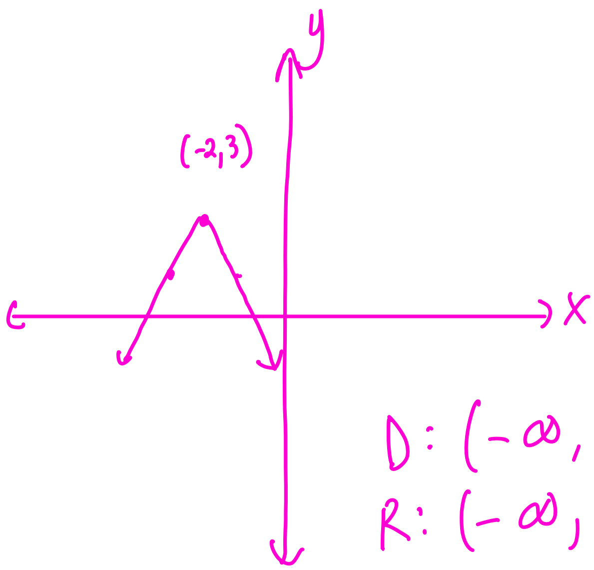
$-9g(x) - 9 = -9\sqrt{x} + 72$

$$\frac{-9g(x)}{-9} = \frac{-9\sqrt{x}}{-9} + \frac{81}{-9}$$

$g(x) = \sqrt{x} - 9$

(12) A

$$f(x) = -2|x+2| + 3 = \begin{cases} -2(x+2)+3 & x+2 \geq 0 \\ -2x-4+3 & x \geq -2 \\ -2x-1 & \\ -2(-x-2)+3 & x < -2 \\ 2x+4+3 & \\ 2x+7 & \end{cases}$$



D: $(-\infty, \infty)$

R: $(-\infty, 3]$

Exam 2A

(2) $f(x) = 3^x$

$f(x-1) - f(x) =$

$3^{x-1} - 3^x$

$3^x \cdot 3^{-1} - 3^x$

$\frac{1}{3} \cdot 3^x - 3^x$

$3^x\left(\frac{1}{3} - 1\right) = 3^x\left(-\frac{2}{3}\right)$

Exam 2B

(3) $f(x) = 3x^2 + 1$

$f(x+y) = 3x^2 + 24x + 49$

$f(x+y) = 3(x+y)^2 + 1 = 3x^2 + 6xy + 3y^2 + 1$

$\cancel{3x^2} + 6xy + 3y^2 + 1 = \cancel{3x^2} + 24x + 49$

$6xy = 24x$

$y = 4$

Exam 2A

(1) $2x-3y=5$

$(-8,4)$

$2x-5=3y$

$\frac{2}{3}x-\frac{5}{3}=y$

$y-4=-\frac{3}{2}(x+8)$

$m=\frac{2}{3}$

$m_{\perp}=-3/2$

Exam 1A

(9) $$\frac{(2x+3)^2\cdot 3(4x-5)^2\cdot 4-(4x-5)^3\cdot 2\cdot(2x+3)\cdot 2}{(2x+3)^4}$$

$$\frac{12(2x+3)^2(4x-5)^2-4(4x-5)^3(2x+3)}{(2x+3)^4}$$

$6x+9-4x+5$

$$\frac{4(2x+3)(4x-5)^2\left(3(2x+3)-(4x-5)\right)}{(2x+3)^4}$$

$2(x+7)$

$$\frac{4\cancel{(2x+3)}(4x-5)^2(2x+14)}{(2x+3)^{\cancel{4}\,3}}=\frac{8(4x-5)^2(x+7)}{(2x+3)^3}$$