Do Now: #5 from the Modeling with Functions Practice packet 3

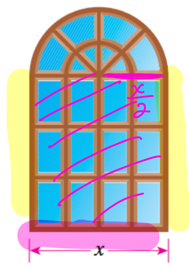
5. A Norman window has the shape of a rectangle surmounted by a semicircle, as shown in the figure. A Norman window with perimeter 30 ft is to be constructed. Find a function that models the area of the window as a function of x.

$A =$ semicircle + rectangle

$A = \frac{\pi r^2}{2} + lw$ need w in terms of x

$A(x) = \frac{\pi}{2}\left(\frac{x}{2}\right)^2 + x\left(15 - \frac{1}{4}\pi x - \frac{1}{2}x\right)$

$A(x) = \frac{\pi x^2}{8} + 15x - \frac{1}{4}\pi x^2 - \frac{1}{2}x^2$, $\left(0, \frac{60}{\pi+2}\right)$



$P = 30$

$30 = \frac{1}{2}\pi x + 2w + x$

$30 - \frac{1}{2}\pi x - x = 2w$

$15 - \frac{1}{4}\pi x - \frac{1}{2}x = w$

Restrictions

$x > 0$

$4\left(15 - \frac{1}{4}\pi x - \frac{1}{2}x > 0\right)$

$60 - \pi x - 2x > 0$

$60 > \pi x + 2x$

$60 > x(\pi + 2)$

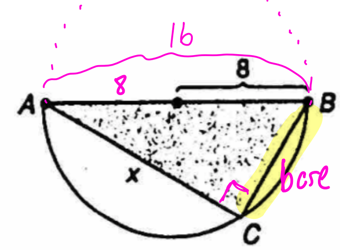
$\frac{60}{\pi + 2} > x$

From Practice Packet 4 …

2. Triangle ABC is inscribed in a semicircle of radius 8 so that one of its sides coincides with a diameter. Express the area of the triangle as a function of $x = AC$.

$A = \frac{1}{2}bh$ need base in terms of x

$A = \frac{1}{2}bx$



$\text{base}^2 + x^2 = 16^2$

$\text{base}^2 = 256 - x^2$

$\text{base} = \cancel{\pm}\sqrt{256 - x^2}$

$A(x) = \frac{1}{2}\sqrt{256 - x^2} \cdot x$, $(0, 16)$

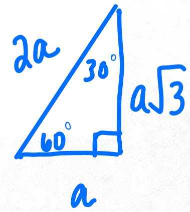
Restrictions

$x > 0$

$256 - x^2 > 0$ $(-16, 16)$

$-$ -16 $+$ 16 $-$

Homework 11-13

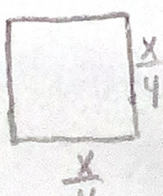


Name: Key

PCH: Modeling with Functions Practice Packet 3 Ms. Loughran

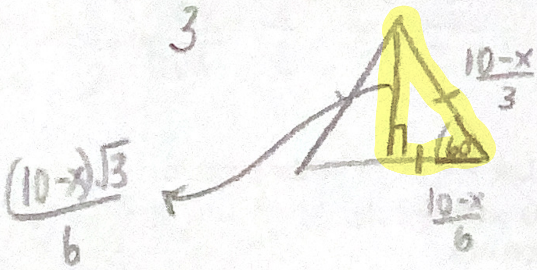
1. A piece of wire 10m long is cut into two pieces. One piece, of length x, is bent into the shape of a square. The other piece is bent into the shape of an equilateral triangle. Express the total area enclosed as a function of x.

$x > 0$



$$A_{EA}(x) = \left(\frac{x}{4}\right)^2 + \frac{1}{2}\left(\frac{10-x}{3}\right)\left(\frac{(10-x)\sqrt{3}}{6}\right)$$

$$A_{EA}(x) = \frac{x^2}{16} + \frac{(10-x)^2\sqrt{3}}{36}, \quad 0 < x < 10$$



$(10-x)\sqrt{3} > 0$

$x < 10$

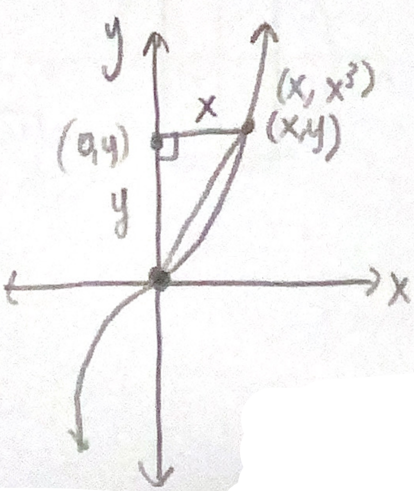
$10 - x > 0$

$-x > -10$

$x < 10$

2. A right triangle has one vertex on the graph of $y = x^3, x > 0$ at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$. Express the area of the triangle as a function of x.

$y = x^3$



$A = \frac{1}{2}bh$

$A(x) = \frac{1}{2}x(x^3)$ $\quad x > 0$

$A(x) = \frac{1}{2}x^4$

3. Express the volume V of a sphere as a function of its surface area S. If the surface area doubles, how does the volume change? multiplied by $2\sqrt{2}$

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

$r^2 = \frac{S}{4\pi}$

$V = \frac{1}{3} \cdot 4\pi r^2 \cdot r$

$r = \pm\sqrt{\frac{S}{4\pi}}$

$V = \frac{1}{3} S \cdot r$ ← need r in terms of S

$V = S \cdot \frac{r}{3}$

$S > 0$

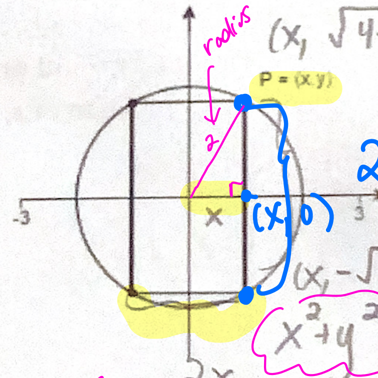
If $S=1$, $V = \frac{1}{6\sqrt{\pi}}$

If $S=2$, $V = \frac{2\sqrt{2}}{6\sqrt{\pi}}$

mult by $2\sqrt{2}$

$V(S) = S \cdot \frac{\sqrt{\frac{S}{4\pi}}}{3} = \frac{S}{3}\sqrt{\frac{S}{4\pi}} = \frac{S}{3} \cdot \frac{\sqrt{S}}{2\sqrt{\pi}} = \frac{S\sqrt{S}}{6\sqrt{\pi}}$

4. A rectangle is inscribed in a circle of radius 2. Let $P = (x, y)$ be the point in Quadrant I that is a vertex of the rectangle and is on the circle.
(a) Express the area of the rectangle as a function of x.
(b) Express the perimeter of the rectangle as a function of x.



radius 2

$(x, \sqrt{4-x^2})$

$2\sqrt{4-x^2}$

$(x, 0)$

$(x, -\sqrt{4-x^2})$

$x^2+y^2=4$ ← eq. of circle center at origin

$2x$

Using Pythagorean Thm

$x^2+y^2=2^2$

$x^2+y^2=4$

$y^2=4-x^2$

$y=\pm\sqrt{4-x^2}$

$x>0$

$4-x^2>0$

$-2 \quad + \quad 2$

$-2<x<2$

a) $A = lw$

$A(x) = 2x(2\sqrt{4-x^2}), \; 0<x<2$

b) $P = 2l + 2w$

$P(x) = 2(2x) + 2(2\sqrt{4-x^2})$

$P(x) = 4x + 4\sqrt{4-x^2}$ $\quad 0<x<2$

$x^2+y^2=4$

$y^2 = 4-x^2$

$y = \pm\sqrt{4-x^2}$